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Generalized semi generalized closed sets in Bitopological spaces

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ABSTRACT
In this paper, we introduce generalized semi generalized closed sets (gsg-closed sets) in bitopological spaces and basic properties of these sets are analyzed. As an application this closed set we investigate the notion of (i,j)-T_{gsg} space. Further we define and study (i,j)-gsg continuous mappings in bitopological spaces and some of their properties have been investigated.

1. Introduction

A triple (X, τ_1, τ_2), where X is a non empty set and τ_1 and τ_2 are topologies on X is called a bitopological space and Kelly[4] initiated the systematic study of such spaces. Fukutake[2] introduced and investigated the notion of generalized closed sets in bitopological spaces and after that several authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces. Khedr et al[5] introduced semi-generalized closed sets and pairwise semi-generalized closed sets in bitopological spaces. Recently Lellis et al[6] introduced and studied the concepts of generalized semi generalized closed sets which lies between closed sets and g-closed sets in general topological spaces.

The purpose of this paper is to introduce a new class of closed sets called generalized semi generalized closed sets (gsg-closed sets), (i,j)-T_{gsg} space and (i,j)-gsg-continuous mappings in bitopological spaces and investigate some of their properties.

2. Preliminaries

If A is a subset of X with a topology τ, then the closure of A is denoted by τ-cl(A) or cl(A), the interior of A is denoted by τ-int(A) or int(A) and the complement of A in X is denoted by A^c.

Definition 2.1 A subset A of a topological space (X, τ) is called

i. a semiopen set[7] if $A \subseteq cl(int(A))$
ii. a preopen set[9] if $A \subseteq int(cl(A))$
iii. a semi preopen set[1] if $A \subseteq cl(int(cl(A))$

Definition 2.2 A subset A of a topological space (X, τ) is called a generalized semi generalized closed set (briefly gsg-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg-open in X.

The intersection of all semiclosed sets (resp. pre closed sets, semi pre closed sets) containing A is called the semi closure of A (resp. pre closure of A, smei pre closure of A) and is denoted by τ-scl(A) or scl(A) (resp. τ-pcl(A) or pcl(A), τ-spcl(A) or spcl(A)).

Throughout this paper X, Y and Z always represent nonempty bitopological spaces (X, τ_1, τ_2), (Y, σ_1, σ_2) and (Z, η_1, η_2) on which no separation axioms are assumed unless explicitly mentioned and the integers $i, j, k \in \{1,2\}$. For a subset A of X τ_i-cl(A) (res. τ_i-int(A), τ_i-scl(A), τ_i-pcl(A), τ_i-spcl(A)) denote the closure (resp. interior, semi closure, pre closure, semi pre closure) of A with respect to the topology τ_i. By (i,j) we mean the pair of topologies (τ_i, τ_j).

Definition 2.3 A subset A of a topological space (X, τ_1, τ_2) is called

i. (i,j)-g-closed[2] if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i open in X.
ii. (i,j)-sg-closed[5] if τ_j-scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i semi open in X.
iii. (i,j)-gs-closed[11] if τ_j-scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i open in X.
iv. (i,j)-gsp-closed if τ_j-spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i open in X.
v. (i,j)-gp-closed if τ_j-pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i open in X.
vi. (i,j)-ω-closed[3] if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i semi open in X.

The complements of the above sets are called their respective open sets.

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Definition 2.4 A topological space (X, τ_1, τ_2) is called

i. (i, j)-$T_{1/2}$ –space[2] if every (i, j)-g-closed set in it is τ_j-closed.

ii. (i, j)-T_ω -space[3] if every (i, j)- ω-closed set in it is τ_j-closed.

iii. (i, j)-T_b-space[11] if every (i, j)-gs-closed set in it is τ_j-closed.

Definition 2.5 A map f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is called

i. τ_j-σ_k-continuous[8] if the inverse image of every σ_k-closed in (Y, σ_1, σ_2) is τ_j-closed in (X, τ_1, τ_2).

ii. (i, j)- g-σ_k-continuous[8] if the inverse image of every σ_k-closed in (Y, σ_1, σ_2) is (i, j)-g-closed in (X, τ_1, τ_2).

iii. (i, j)- sg-σ_k-continuous[5] if the inverse image of every σ_k-closed in (Y, σ_1, σ_2) is (i, j)-sg-closed in (X, τ_1, τ_2).

iv. (i, j)- gsp-σ_k-continuous if the inverse image of every σ_k-closed in (Y, σ_1, σ_2) is (i, j)-gsp-closed in (X, τ_1, τ_2).

v. (i, j)-gs-σ_k-continuous[11] if the inverse image of every σ_k-closed in (Y, σ_1, σ_2) is (i, j)-gs-closed in (X, τ_1, τ_2).

vi. (i, j)-ω-σ_k-continuous[3] if the inverse image of every σ_k-closed in (Y, σ_1, σ_2) is (i, j)- ω-closed in (X, τ_1, τ_2).

vii. (i, j)- gp-σ_k-continuous if the inverse image of every σ_k-closed in (Y, σ_1, σ_2) is (i, j)-gp-closed in (X, τ_1, τ_2).

3. Generalized sg-closed sets in bitopological space

In this section we introduce the concept of (i, j)-gsg-closed sets in bitopological spaces and discuss some of the related properties.

Definition 3.1 A subset A of a bitopological space (X, τ_1, τ_2) is said to be a (i, j)-generalized semi generalized closed set (briefly (i, j)-gsg-closed) if τ_j-cl(A) ⊆ U whenever A ⊆ U and U is τ_i-sg-open in X.

Proposition 3.1 Every τ_j-closed set is (i, j)-gsg-closed.

Proof **:** Let A be any τ_j-closed set and U be any τ_i-sg-open set containing A. Then τ_j-cl(A) = A ⊆ U. Hence A (i, j)-gsg-closed.

Proposition 3.2 Every (i, j)-gsg-closed set is (i, j)-g-closed.

Proof **:** Let A be any (i, j)-gsg-closed set and U be any τ_i-open set containing A. Since every τ_i-open set is τ_i-sg-open set and A is (i, j)-gsg-closed set, then τ_j-cl(A) ⊆ U. Hence A (i, j)-g-closed.

Proposition 3.3 Every (i, j)-gsg-closed set is (i, j)- ω-closed.

Proof **:** Let A be any (i, j)-gsg-closed set and U be any τ_i-semi open set containing A. Since every τ_i-semi open set is τ_i-sg-open set and A is (i, j)-gsg-closed set, then τ_j-cl(A) ⊆ U. Hence A (i, j)- ω-closed.

Proposition 3.4 Every (i, j)-gsg-closed set is (i, j)- sg-closed.

Proof : Let A be any (i, j)-gsg-closed set and U be any τ_i-semi open set containing A. Since every τ_i-semi open set is τ_i-sg-open set and A is (i, j)-gsg-closed set, thenτ_j-scl(A) ⊆ τ_j-cl(A) ⊆ U. Hence A is (i, j)- sg-closed.

Proposition 3.5 Every (i, j)-gsg-closed set is (i, j)- gs-closed, (i, j)- gsp-closed and (i, j)- gp-closed.

Proof **:** Let A be any (i, j)-gsg-closed set and U be any τ_i-open set containing A. Then τ_j-scl(A) ⊆ τ_j-cl(A) ⊆ U. Hence A is (i, j)- gs-closed.(resp. τ_j-spcl(A) ⊆ τ_j-cl(A) , τ_j-pcl(A) ⊆ τ_j-cl(A)).

The following examples show that the reverse implications of the above propositions are not true.

Example 3.1

Let $X = \{a, b, c\}$, $\tau_1 = \{\varphi, \{a\}, X\}$, $\tau_2 = \{\varphi, \{a, b\}, X\}$. Then $\{b, c\}$ is $(1,2)$-gsg-closed but not τ_2-closed.

Example 3.2

Let $X = \{a, b, c\}$, $\tau_1 = \{\varphi, \{a\}, X\}$, $\tau_2 = \{\varphi, \{a\}, \{b\}, \{a, b\}, X\}$. Then $\{a, b\}$ is $(1,2)$-g-closed but not $(1,2)$-gsg-closed.

Example 3.3

Let $X = \{a, b, c\}$, $\tau_1 = \{\varphi, \{a, b\}, X\}$, $\tau_2 = \{\varphi, \{a\}, \{c\}, \{a, c\}, X\}$. Then $\{a\}$ is $(1,2)$-ω-closed but not $(1,2)$-gsg-closed.

Example 3.4

Let $X = \{a, b, c\}$, $\tau_1 = \{\varphi, \{a\}, X\}$, $\tau_2 = \{\varphi, \{a\}, \{b, c\}, X\}$. Then $\{a, b\}$ is $(1,2)$-gs-closed but not $(1,2)$-gsg-closed.

Example 3.5

Let $X = \{a, b, c, d\}$, $\tau_1 = \{\varphi, \{a, b, c\}, X\}$, $\tau_2 = \{\varphi, \{a, d\}, \{a, b, d\}, X\}$. Then $\{b\}$ is $(1,2)$-sg-closed but not $(1,2)$-gsg-closed.

Example 3.6

Let $X = \{a, b, c\}$, $\tau_1 = \{\varphi, \{a\}, \{a, b\}, X\}$, $\tau_2 = \{\varphi, \{a\}, \{b\}, \{a, b\}, X\}$. Then $\{a\}$ is $(1,2)$-gsp-closed but not $(1,2)$-gsg-closed.

Example 3.7

Let $X = \{a, b, c\}$, $\tau_1 = \{\varphi, \{c\}, X\}$, $\tau_2 = \{\varphi, \{a\}, \{a, b\}, X\}$. Then $\{a\}$ is $(1,2)$-gp-closed but not $(1,2)$-gsg-closed.

Definition 3.2 A subset A of a bitopological space (X, τ_1, τ_2) is said to be (i, j)-generalized semi generalized open set (briefly (i, j)-gsg-open) if A^c is (i, j)-gsg-closed in (X, τ_1, τ_2).

Theorem 3.1 In a bitopological space (X, τ_1, τ_2)

i. Every τ_j-open set is (i, j)-gsg-open.

ii. Every (i, j)-gsg-open set is (i, j)-g-open and (i, j)- ω-open.

iii. Every (i,j)-gsg-open set is (i,j)- sg-open and (i,j)- gs-open.
iv. Every (i,j)-gsg-open set is (i,j)- gsp-open and (i,j)- gp-open.

Theorem 3.2 If A and B are (i,j)-gsg-closed sets in X then A∪B (i,j)-gsg-closed.

Proof: Let U be any τ_i-sg-open set containing A and B. Then A∪B ⊆ U. Then A ⊆ U and B ⊆ U. Since A and B are (i,j)-gsg-closed, τ_j-cl(A) ⊆ U and τ_j-cl(B) ⊆ U. Now, τ_j-cl(A∪B) =τ_j-cl(A)∪τ_j-cl(B) ⊆ U and so τ_j-cl(A∪B) ⊆ U. Hence A∪B (i,j)-gsg-closed.

Theorem 3.3 If a set A is (i,j)-gsg-closed then τ_j-cl(A)− A contains no non empty τ_i-closed set.

Proof: Let A be any (i,j)-gsg-closed and F be a τ_i closed set such that F⊆ τ_j-cl(A) − A. Since A is (i,j)-gsg-closed, we have τ_j-cl(A) ⊆ F^c. Then F ⊆ τ_j-cl(A)∩(τ_j-cl(A))^c = ϕ. Hence F is empty.

The converse of the above theorem is not true as seen from the following example.

Example 3.8

Let $X = \{a,b,c\}$, $\tau_1 = \{\varphi, \{c\}, X\}$, $\tau_2 = \{\varphi, \{a\}, \{a,b\}, X\}$. If $A = \{a\}$ then τ_2-cl(A)) − A $=\{b,c\}$ does not contain non empty τ_i-closed set. But $\{a\}$ is not $(1,2)$-gsg-closed.

Theorem 3.4 A set A is (i,j)-gsg-closed if and only if τ_j-cl(A)− A contains no non empty (i,j)-sg-closed set.

Proof: Let A be any (i,j)-gsg-closed and D be a (i,j)-sg-closed set, such that D⊆ τ_j-cl(A)− A. Since A is (i,j)-gsg-closed, we have τ_j-cl(A) ⊆ D^c. Then D ⊆ τ_j-cl(A)∩(τ_j-cl(A))^c = ϕ. Thus D is empty.

Conversely, Suppose that τ_j-cl(A)− A contains no non empty (i,j)-sg-closed set. Let A ⊆ G and G is (i,j)-sg-open. If τ_j-cl(A) ⊈ G then τ_j-cl(A) ∩ G^c is non empty. Since τ_j-cl(A) is closed set and G^c is (i,j)-sg-closed, we have τ_j-cl(A) ∩ G^c is non empty (i,j)-sg-closed set of τ_j-cl(A)− A which is a contradiction. Therefore τ_j-cl(A) ⊆ G. Hence A is (i,j)-gsg-closed.

Theorem 3.5 If a set A is (i,j)-gsg-closed then τ_i-cl($\{x\}$)∩ A ≠ ϕ holds for each $x \in \tau_j$-cl(A).

Proof: If τ_i-cl($\{x\}$) ∩ A = ϕ for some $x \in \tau_j$-cl(A), then A ⊆ (τ_i-cl($\{x\}$))^c. Since A is (i,j)-gsg-closed, we have τ_j-cl(A) ⊆ (τ_i-cl($\{x\}$))^c. This shows that $x \notin \tau_j$-cl(A).This contradicts the assumption.

The converse of the above theorem is not true as seen from the following example.

Example 3.9

Let $X = \{a,b,c\}$, $\tau_1 = \{\varphi, \{a\}, X\}$, $\tau_2 = \{\varphi, \{a\}, \{b,c\}, X\}$. For a subset $A = \{a,b\}$ is not a $(1,2)$-gsg-closed set, but τ_1-cl($\{x\}$)∩ A ≠ ϕ , for each $x \in \tau_2$-cl(A).

Theorem 3.6 If A is a (i,j)-gsg-closed set of (X, τ_1, τ_2) such that A ⊆ B ⊆ τ_j-cl(A), then B is also an (i,j)-gsg-closed of (X, τ_1, τ_2).

Proof: Let U be τ_i-sg-open set such that B ⊆ U. As A is (i,j)-gsg-closed and A ⊆ U, we have τ_j-cl(A) ⊆ U. Now B ⊆ τ_j-cl(A) which gives, τ_j-cl(B) ⊆ τ_j-cl{τ_j-cl(A)}= τ_j-cl(A) ⊆ U. Thus τ_j-cl(B) ⊆ U.

Theorem 3.7 Let A ⊆ Y ⊆ X and suppose that A is (i,j)-gsg-closed in X. Then A is (i,j)-gsg-closed relative to Y.

Theorem 3.8 In a bitopological space (X, τ_1, τ_2), SGO(X, τ_i) ⊆ {F ⊆ X: F^c ∈τ_j} if and only if every subset of X is an (i,j)-gsg-closed set.

Proof: Suppose that SGO(X, τ_i) ⊆ {F ⊆ X: F^c ∈τ_j}.Let A be a subset of X and U be τ_i-sg-open set such that A ⊆ U. Then τ_j-cl(A) ⊆ τ_j-cl(U) = U. Hence A is a (i,j)-gsg-closed set.

Conversely, suppose that every subset of X is a (i,j)-gsg-closed set. Let U ∈ SGO(X, τ_i). Since U is a (i,j)-gsg-closed set, τ_j-cl(U) ⊆ U. Therefore U ∈ {F ⊆ X: F^c ∈τ_j} and hence SGO(X, τ_i) ⊆ {F ⊆ X: F^c ∈τ_j}.

Theorem 3.9 If A is τ_i-sg-open and (i,j)-gsg-closed in X then A is τ_j-closed in X.

Proof: Since A is τ_i-sg-open and (i,j)-gsg-closed in X, then τ_j-cl(A) ⊆ A and hence A is τ_j-closed in X.

Theorem 3.10 For each point x of (X, τ_1, τ_2), either a singleton set $\{x\}$ is τ_i-sg-closed or $\{x\}^c$ is (i,j)-gsg-closed in X.

Proof: If set $\{x\}$ is not τ_i-sg-closed in X then $\{x\}^c$ is not τ_i-sg-open in X and the only τ_i-sg-open set containing $\{x\}^c$ is the space X itself. Then τ_j-cl($\{x\}^c$) ⊆ X and so $\{x\}^c$ is (i,j)-gsg-closed in X.

Theorem 3.11 If a subset A of (X, τ_1, τ_2) is (i,j)-gsg-closed in X, then τ_j-cl(A)−A is (i,j)-gsg-open.

4. Application of (i,j)-gsg-closed sets

In this section as an application of (i,j)-gsg-closed sets, we introduce (i,j)-T_{gsg} -space in bitopological spaces and investigate some of its properties

Definition 4.1 A topological space (X,τ_1, τ_2) is called a (i,j)-T_{gsg}-space if every (i,j)-gsg-closed set in it is τ_j-closed.

Proposition 4.1 Every (i,j)-$T_{1/2}$ -space is a (i,j)-T_{gsg} -space.

Proof: Let (X,τ_1,τ_2) be a (i,j)-$T_{1/2}$-space and let A be a (i,j)-gsg-closed set in X. By proposition 3.2, A is a (i,j)-g-closed in X. Since X is a (i,j)-$T_{1/2}$-space, A is τ_j-closed in X. Hence (X,τ_1,τ_2) is a (i,j)-T_{gsg} -space.

Proposition 4.2 Every (i,j)-T_ω -space is a (i,j)-T_{gsg}-space.

Proof: Let (X,τ_1,τ_2) be a (i,j)-T_ω-space and let A be a (i,j)-gsg-closed set in X. By proposition 3.3, A is a (i,j)-ω-closed in X. Since X is a (i,j)-T_ω-space, A is τ_j-closed in X. Hence (X,τ_1,τ_2) is a (i,j)-T_{gsg} –space.

Proposition 4.3 Every (i,j)-T_b -space is a (i,j)-T_{gsg} -space.

Proof: Let (X,τ_1,τ_2) be a (i,j)-T_b-space and let A be a (i,j)-gsg-closed set in X. By proposition 3.5, A is a (i,j)-gs-closed set in X. Since X is a (i,j)-T_b-space, A is τ_j-closed in X. Hence (X,τ_1,τ_2) is a (i,j)-T_{gsg} –space

Example 4.1 In Example 3.1, the space (X,τ_1,τ_2) is a $(1,2)$-T_{gsg}-space but not a $(1,2)$-T_b-space and $(1,2)$-$T_{1/2}$-space.

Theorem 4.1 A bitopological space (X,τ_1,τ_2) is a (i,j)-T_{gsg} -space if and only if every singleton set $\{x\}$ of X is either τ_i-sg-closed or τ_j-open.

Proof: Suppose that $\{x\}$ is not τ_i-sg-closed. Then $\{x\}^c$ is (i,j)-gsg-closed by theorem 3.9. Since X is a (i,j)-T_{gsg} –space, we have $\{x\}^c$ is τ_j-closed. Hence $\{x\}$ is τ_j-open in X.

Conversely, Let A be a (i,j)-gsg-closed set of X. Clearly A $\subseteq\tau_j$-cl(A). Let $x \in\tau_j$-cl(A).Then by hypothesis $\{x\}$ is either τ_i-sg-closed or τ_j-open.

Case 1: Suppose $\{x\}$ is τ_i-sg-closed. If $x\notin$A, then $\{x\}\subseteq\tau_j$-cl(A)−A, which is a contradiction to theorem 3.4. Therefore $x\in$A.

Case 2: Suppose $\{x\}$ is τ_j-open. Since $x \in\tau_j$-cl(A), $\{x\}\cap$A $\neq \phi$. Therefore $x\in$ A.Thus in both the cases, we have $x\in$ A. So τ_j-cl(A) $\subseteq$ A . Therefore A=τ_j-cl(A) or A is τ_j-closed. Hence (X,τ_1,τ_2) is a (i,j)-T_{gsg} -space.

5. (i,j)-gsg-continuous mapping

In this section we introduce the concept of (i,j)-gsg-continuous mapping bitopological spaces and discuss some of the related properties.

Definition 5.1 A mapping $f: (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is said to be (i,j)-gsg-σ_k-continuous if the inverse image of every σ_k-closed in Y is (i,j)-gsg-closed in X.

Theorem 5.1 If a mapping $f: (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is (i,j)-gsg-σ_k-continuous then f is (i,j)-sg-σ_k-continuous.

Proof: Let V be any σ_k-closed in Y. Since f is (i,j)-gsg-σ_k-continuous, $f^{-1}(V)$ is (i,j)-gsg-closed in X. Then by proposition 3.4, $f^{-1}(V)$ is (i,j)-sg-closed in X. Hence f is (i,j)-sg-σ_k-continuous.

Theorem 5.2 If a mapping $f: (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is (i,j)-gsg-σ_k-continuous then f is (i,j)-gsp-σ_k-continuous.

Proof: Let V be any σ_k-closed in Y. Since f is (i,j)-gsg-σ_k-continuous, $f^{-1}(V)$ is (i,j)-gsg-closed in X. Then by proposition 3.5, $f^{-1}(V)$ is (i,j)-gsp-closed in X. Hence f is (i,j)-gsp-σ_k-continuous.

Theorem 5.3 If a mapping $f: (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is (i,j)-gsg-σ_k-continuous then f is (i,j)-gp-σ_k-continuous, (i,j)-gs-σ_k-continuous and (i,j)-ω-σ_k-continuous.

Theorem 5.4 If a mapping $f: (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is (i,j)-gsg-σ_k-continuous if and only if inverse image of each σ_k-open set of Y is (i,j)-gsg-open in X.

Proof: Let f be (i,j)-gsg-σ_k-continuous. If V is any σ_k-open set of Y then V^c is σ_k-closed in Y. Since f is (i,j)-gsg-σ_k-continuous, $f^{-1}(V^c) = (f^{-1}(V))^c$ is (i,j)-gsg-closed in X. Hence $f^{-1}(V)$ is (i,j)-gsg-open in X.

Conversely, Let V be any σ_k-closed in Y. By hypothesis $f^{-1}(V^c)$ is (i,j)-gsg-open in X. Then $f^{-1}(V)$ is (i,j)-gsg-closed in X. Hence f is (i,j)-gsg-σ_k-continuous.

Theorem 5.5 If $f: (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is (i,j)-gsg-σ_k-continuous and $g: (Y,\sigma_1,\sigma_2) \to (Z,\eta_1,\eta_2)$ is (i,j)-g-σ_k-continuous and (Y,σ_1,σ_2) is (i,j)-$T_{1/2}$-space. Then $g\circ f: (X,\tau_1,\tau_2) \to (Z,\eta_1,\eta_2)$ is (i,j)-gsg-σ_k-continuous.

Proof: Let V be any η_k-closed in Z. Since g is (i,j)-g-σ_k-continuous and Y is (i,j)-$T_{1/2}$-space, $g^{-1}(V)$ is σ_j-closed in Y. Since f is (i,j)-gsg-σ_k-continuous, $f^{-1}(g^{-1}(V))$ is (i,j)-gsg-closed in X. Hence $g\circ f$ is (i,j)-gsg-σ_k-continuous.

Theorem 5.6 Let $f: (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be a map:

i. If (X,τ_1,τ_2) is an (i,j)-$T_{1/2}$-space then f is (i,j)-g-σ_k-continuous if and only if it is (i,j)-gsg-σ_k-continuous.

ii. If (X,τ_1,τ_2) is an (i,j)-T_{gsg}-space then f is τ_j-σ_k-continuous if and only if it is (i,j)-gsg-σ_k-continuous.

Proof:

i. Let V be any σ_k-closed in Y. Since f is (i,j)-g-σ_k-continuous, $f^{-1}(V)$ is (i,j)-g-closed in X. But (X,τ_1,τ_2) is an (i,j)-$T_{1/2}$-space, which implies , $f^{-1}(V)$ is τ_j-closed. By proposition 3.1, $f^{-1}(V)$ is (i,j)-gsg-closed in X. Hence f is (i,j)-gsg-σ_k-continuous .

Conversely, suppose that f is (i,j)-gsg-σ_k-continuous. Let V be any σ_k-closed in Y. Then $f^{-1}(V)$ is (i,j)-gsg-closed in X.

By proposition 3.2, f^{-1}(V) is (i,j)-g-closed in X. Hence f is (i,j)-g-σ_k-continuous.

ii. Let V be any σ_k-closed in Y. Since f is τ_j-σ_k-continuous , f^{-1}(V) is τ_j-closed in X. By proposition 3.1, f^{-1}(V) is (i,j)-gsg-closed in X. Hence f is (i,j)-gsg-σ_k-continuous .

Conversely, suppose that f is (i,j)-gsg-σ_k-continuous. Let V be any σ_k-closed in Y. Then f^{-1}(V) is (i,j)-gsg-closed in X. But (X, τ_1,τ_2) is an (i,j)-Tgsg-space, which implies f^{-1}(V) is τ_j-closed in X. Hence f is τ_j-σ_k-continuous.

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