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Fuzzy possibilistic linear – programming approach for two echelon supply chain with multi-product and multi-time period

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ABSTRACT

In the changing market scenario, the study of the supply chain model in a fuzzy environment is gaining phenomenal importance around the globe. In such a scenario, it is the need of the hour that a real supply chain is operated in an uncertain environment and the omission of any effects of uncertainty leads to inferior supply chain designs. This work presents a fuzzy possibilistic linear programming approach to make strategic resource – planning decisions in multi-product and multi-time period two-echelon supply chains. In this model, demand rate, holding cost, transportation cost, packing charge and toll fees are considered as triangular fuzzy numbers. The objective of the proposed model is to provide an optimal inventory level for the warehouses and distribution centers and also, minimizing the total cost related to transportation, inventory carrying, packing and toll fees of the entire supply chain for a finite planning horizon. An industrial case demonstrates the feasibility of applying the proposed model to real world problem in a two-echelon supply chain under uncertain environment. In addition to that, if the efficiency of the transportation will be increased by using the light weight eco-friendly plastic pail rather than the traditional one, then the reduction of the total cost in the supply chain is also analyzed.

Introduction

Supply Chain Management (SCM) issues have long attracted interest from both practitioners and academics because of its ability to reap more benefits by efficiently managing it. (Anamaria & Rakesh, 1999 , Beamon, 1998 , Bilgen & Ozkarahan, 2004, Chen, Lin & Huang, 2006, Erengüc, Simpson & Vakharia, 1999 , Gen & Syanf, 2005, Petrovic, 2001, Petrovic, Roy & Petrovic, 1998). Supply Chain Management is a set of synchronized decisions and activities utilized to efficiently integrate suppliers, manufacturers, warehouses, transporters, retailers and customers so that the right product or service is distributed at the right quantities, to the right locations and at the right time, in order to minimize system-wide costs while satisfying customer service level requirements. The objective of SCM is to achieve sustainable competitive advantage. To stay competitive, organizations should improve customer service, reduction of costs across the supply chain and efficient use of resources available in the supply chain.

In this field, numerous researches are conducted. Williams, 1981, developed seven heuristic algorithms to minimize distribution and production costs in supply chain. Cohen and Lee, 1989, presented a deterministic, mixed integer, non-linear programming with economic order quantity technique to develop global supply chain plan. Özdamar and Yazgaç, 1997, developed a distribution/production system involving a manufacturer center and its warehouses. They try to minimize total costs such as inventory, transportation costs etc under production capacity and inventory equilibrium constraints. Yan et al., 2003, tried to contrive a network which involves suppliers, manufacturers, distribution centers and customers via a mixed integer programming under logic and material requirements constraints. Yilmaz, 2004, handled a strategic planning problem for three echelon supply chain involving suppliers, manufacturers and distribution centers to minimize transportation, distribution, production costs. Nagurney and Toyasaki, 2005, try to balance e-cycling in multi tiered supply chain process. Gen and Syarif, 2005, developed a hybrid genetic algorithm for a multi period multi product supply chain network design. Paksoy, 2005, developed a mixed integer linear programming to design a multi echelon supply chain network under material requirement constraints. Byrne and Hossain ,2005, presented an extended linear programming (LP) model to improve the hybrid approach proposed by Byrne and Bakir ,1999, and Lee and Kim,2002 based on the unit load concept of just in time. Park ,2005 acquired solutions for integrated production and distribution problems in a multi-plant, multi-retailer, multi-item, and multi-period logistic environment, and investigated the effectiveness of these solutions using a computational study for maximizing total profit. Lin et al., 2007, compared flexible supply chains and traditional supply chains with a hybrid genetic algorithm and mentioned advantages of flexible ones. Tuzkaya and Önüt, 2009, developed a model to minimize holding inventory and penalty cost for suppliers, warehouse and manufacturers based a holonic approach. Sourirajan et al., 2009, considered a two-stage supply chain with a production facility that replenishes a single product at retailers. The objective is to locate distribution centers in the network such that the sum of facility location, pipeline inventory, and safety stock costs is minimized. They used genetic algorithms to solve the model and compare their performance to that of a Lagrangian heuristic developed in earlier work. Ahumada and Villalobos, 2009, reviewed the main contributions in the field of production and distribution planning for agri-foods based on agricultural crops. Through their analysis of the current state of

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the research, they diagnosed some of the future requirements for modeling the supply chain of agri-foods. Gunasekaran and Ngai, 2009, have developed a unified framework for modeling and analyzing BTO-SCM and suggested some future research directions. Fahimnia, B., Lee Luong, Marian, R, 2009, developed a mixed integer formulation for a two-echelon supply network considering the real-world variables and constraints. A multi-objective genetic algorithm (MOGA) is then designed for the optimization of the developed mathematical model. P. Subramanian, N. Ramkumar, T.T. Narendran, 2010, formulated a generalised multi-echelon, single time-period, multi-product, closed loop supply chain as an integer linear programme (ILP). K. Balaji Reddy et al.,2011, developed single-echelon supply chain two stage distribution inventory optimization model. Related investigations for solving the deterministic manufacturing/distribution planning decision problems included those by, Beamon,1998, Bilgen and Ozkarahan ,2004, Chandra and Fisher 1994, Erengüc et al. ,1999, Jang et al. ,2002, Kim and Kim ,2001, Simpson and Vakharia ,1999, and Thomas and Griffin 1996, G.Barbarsoglu, D.Ozgur, 1999, C.J.Vidal, M.Goetschalckx, 1999. These conventional methods assume the goals and model inputs are deterministic.

In the crisp environment, all parameters in the total cost such as holding cost, setup cost, purchasing price, transportation cost, demand rate, production rate etc are known and have definite value without ambiguity. Some of the business situations fit such conditions but in most of the situations and in the day by day changing market scenario the parameters and variables are highly uncertain or imprecise, under such circumstances, uncertainties are treated as fuzzy parameters.

Chen and Lee, 2004, designed a supply chain scheduling model to solve multi-products, multi-stages and multi-periods mixed integer non-linear programming problems with uncertain demand. This helped to resolve conflicting objectives in compromised preference levels on product price from sellers and buyers viewpoints .Chen and Huang, 2006 , proposed a fuzzy model by combining fuzzy sets with Program Evaluation and Review Technique (PERT). They applied this model to calculate the total cycle time of an entire supply chain. Alieu et al, 2007, developed fuzzy genetic approach to solve aggregate production-distribution planning problems. Related investigations include Chen et al., 2006,H.Giannoccaro et al.,2003 ,M.Kumar et al., 2004 , T.F.Liang, 2007 , Y.Xie et al., 2006.

The possibility theory, developed by Zadeh in 1978, provided an effective methodology that considers parameter vagueness in various fields. Buckley, 1988 , designed a mathematical programming problem in which all parameters maybe fuzzy variables, specified based on their possibility distributions. He also elucidated this problem using the possibility theory. Furthermore Lai and Huang, 1992,developed an auxiliary multi-objective linear programming model for solving mathematical programming problems with imprecise goals and constraint coefficients. Hsu and Wang,2001, developed a mathematical programming model that integrated the possibility theory and Zimmerman’s fuzzy programming method for managing production planning decisions that involve ambiguous cost goals and uncertain demand in an assemble to order environment. Inuiguchi and Ramik, 2000, combined several existing techniques with new ideas in fuzzy mathematical programming techniques using concrete examples. Wang and Liang,2005, presented an interactive fuzzy programming technique for solving the aggregate production planning problems that arise with imprecise demand, cost coefficients and capacities. Wang and Shu, 2005, generated a fuzzy supply chain model by combining possibility theory and genetic algorithm to provide an alternative framework to handle supply chain uncertainties and determine inventory strategies Ozgen et al., 2008, recently developed an integration model of the analytical hierarchy process and multi-objective possibilistic linear programming technique to account for all tangible, intangible, quantitative and qualitative factors, which were then used to evaluate suppliers and determine the optimum order quantities assigned to each. Related works include Linuiguchi and Sakawa, 1995, 1996, Tanaka et al., 2000, and Tang et al., 2001.

Supply Chain Management is a field that is usually been studied from a market and product perspective rather than from transport point of view. Transport processes are essential parts of the supply chain. They perform the flow of materials that connects an enterprise with its suppliers and with its customers. The integrated view of transport, production and inventory holding process is the characteristic of the modern supply chain management concept (B. Fleischmann, 2005). Only a good coordination between each component would bring the benefits to a maximum.

Transportation occupies one-third of the amount in the logistics and transportation systems influence the performance of logistics system hugely. Without well developed transportation systems, logistics could not bring its advantages into full play.

Some authors have recently worked in the development of the supply chain and transport relationship. (Stank and Goldsby, 2000, Potter and Lalwani, 2005, Disney and Jowill, 2003, Childhouse P., Towill D. R. 2003, Bask A.H., 2001). However, Supply Chain Management and transport are areas that should be discussed more in depth, in order to do so it is necessary to develop a framework that allow holistic analysis from a system perspective.

Packaging is a co-ordinated system of preparing goods for safe, efficient, cost effective transport, distribution, storage, retailing, consumption and recovery reuse or disposal combined with maximizing consumer value, sales and hence profit proper packaging is required by all freight carriers to ensure safety shipment. In supply chain packaging costs represents a significant part.

A toll is one of the fairest revenue sources. For the use of elevated road, user fee shall be collected from all vehicles. David Jackson of Business Day, in an article published on 19th March 2011, said that, the new tolling system pat strain on business. Logistics costs will increase by about 1% following the increased toll road tariffs, companies may have to review their entire distribution network strategy as they seek viable and cost efficient alternative to the toll road system.

Packing cost and toll fees are also uncertain in the day by day changing market scenario. According to our survey on the current literature, none of the previous models has considered the major cost elements packing and toll fees in two-echelon supply model with multi-product and multi-time period under uncertain environment. This characteristic makes the developed model adaptable to a wider manufacturing and distribution scenarios.

This paper presents a fuzzy possibilistic linear programming approach to solve multi-product and multi-time period two-echelon supply chain models in which demand rate, holding cost, transportation cost, packing charge and toll fees are considered as triangular fuzzy numbers. The objective of the proposed model is to provide an optimal inventory level for the warehouses and distribution centers and also minimizing the total cost of the entire supply chain for a finite planning horizon.

Also the study utilizes one of the leading paint company as a case study to demonstrate the practicality of the proposed approach. In addition to that, if the efficiency of the transportation is increased by using the light weight eco-friendly plastic pail rather than the traditional one, then the reduction of the total cost in the supply chain is also analyzed.

This paper is organized as follows : Section 2, describes the problem, details the assumptions and formulates the imprecise multi-product and multi-time period two-echelon supply chain model. Section 3 develops the fuzzy programming approach for solving the proposed model. Section 4 represents an industrial case for implementing the feasibility of applying the proposed approach to real situations. Finally, section 5 presents the conclusion.

Problem Formulation

Problem Description, Assumptions and Notation

The proposed model is a two-echelon supply chain model with N_w Warehouses, N_D Distribution centers and N_r retailers with limited capacities. In this model multi-product is being distributed from the warehouse to distribution centers in the system. The imprecise demand for the product will be forecasted before beginning of every period and will be used as the reference for warehouses to transfer stocks from them to distribution centers in a particular period h. Further the multi-product is being distributed from the distribution centers to retailers according to their imprecise forecasted demand for the product before beginning of every period and will be used as the reference for distribution centers to transfer stocks from them to retailers in a particular period h. The unit cost coefficients and related parameters are normally fuzzy because some of the information is incomplete and / or uncontrollable. This paper focuses on presenting a fuzzy possibilistic linear programming approach that optimizes a two-echelon supply chain models with multi-product and multi-time period in uncertain environments. The aim of this approach is to minimize the total cost associated with inventory holding cost, transportation cost, packing charge and toll fees.

The following list of assumptions is the basis for this study's fuzzy mathematical programming model.

1. The objective functions are fuzzy and have imprecise aspiration levels.
2. All the objective functions and constraints are linear equations.
3. Transportation costs on a given route are directly proportional to the units shipped.
4. Packing charges vary considerably depending on the item and how much material it takes to get it packed.
5. Toll fees considered per shipment.
6. Capacity of the warehouse and distribution centers in each period cannot exceed their maximum available levels.
7. The adoption of the pattern of triangular membership function represents the fuzzy numbers.

Assumption 1 relates to the fuzziness of the objective functions in practical decision problems in supply chains. It incorporates the variation in the Decision Maker Judgments regarding the solution for fuzzy optimization problems in a framework of imprecise aspiration levels. Assumptions 2 to 5 indicate that linearity, proportionality properties must be technically satisfied as a standard LP form. Assumption 6 represents the limit on the maximum available warehouse and distribution centers capacities in a normal business operation. Assumption 7 addresses the simplicity and flexibility of the fuzzy arithmetic operations. The use of triangular fuzzy numbers represents the imprecise data and therefore enhances computational efficiency and facilitates data acquisition.

This study uses the following notation.

Index Sets

i Warehouses $i = 1, 2, \ldots, N_w$.
j Distribution centers $j = 1, 2, \ldots, N_D$.
k Retailers $k = 1, 2, \ldots, N_r$.
l Product Type $l = 1, 2, \ldots, N_{pr}$.
h Time period $h = 1, 2, \ldots, H$.
S Stages $S = 1, 2$.

Objective Functions

$\tilde{Z}_{h1}$ Total cost in the first stage of period h

Z_{h1}^{m} modal value of $\tilde{Z}_h$ in stage 1

Z_{h1}^{0} lower bound value of $\tilde{Z}_h$ in stage 1

Z_{h1}^{p} upper bound value of $\tilde{Z}_h$ in stage 1

Z_{1h1} modal value of the objective function in stage 1

Z_{2h1} difference between the modal value of the objective function and its lower bound in stage 1

Z_{3h1} difference between the upperbound of the objective function and its modal value in stage 1

$\tilde{Z}_{h2}$ Total cost in the second stage of period h

Z_{h2}^{m} modal value of $\tilde{Z}_h$ in stage 2

Z_{h2}^{0} lower bound value of $\tilde{Z}_h$ in stage 2

Z_{h2}^{p} upper bound value of $\tilde{Z}_h$ in stage 2

Z_{1h2} modal value of the objective function in stage 2

Z_{2h2} difference between the modal value of the objective function and its lower bound in stage 2

Z_{3h2} difference between the upperbound of the objective function and its modal value in stage 2

$\tilde{Z}_h$ Total cost in period h, is equal to sum of $\tilde{Z}_{h1}$ and $\tilde{Z}_{h2}$

Decision Variables

IW_{hli} inventory level of product l by warehouse i in period h.

TW_{hlij} units distributed of product l from warehouse i to distribution center j in period h.

ID_{hlj} Inventory level of product l by distribution center j in period h.

TD_{hljk} units distributed of product l from distribution center j to retailer k in period h.

Parameters

$\widetilde{ICW}_{hli}$ unit inventory carrying cost of product l by warehouse i in period h.

$\tilde{TCW}_{hlij}$ unit transportation cost of product h from warehouse i to distributor j in period h.

$\tilde{PC}_{hli}$ packing charge of product l by warehouse i in period h.

$\tilde{TFW}_{hij}$ Toll fees from warehouse i to distribution center j in period h.

NSW_{hij} Number of shipments form warehouse i to distribution center j in period h is equal to $\frac{1}{10} X \sum_{l=1}^{Npr} TW_{hlij}$ for traditional pail, $\frac{1}{12} X \sum_{l=1}^{Npr} TW_{hlij}$ for light weight pail

$\tilde{ICD}_{hlj}$ unit inventory carrying cost of product l by distribution center j in period h.

$\tilde{TCD}_{hljk}$ unit transportation cost of product l from distribution center j to retailer k in period h.

$\tilde{TFD}_{hjk}$ Toll fees from distribution center j to retailer k in period h.

NSD_{hjk} Number of shipments form distribution center j to retailer k in period h is equal to $\frac{1}{10} X \sum_{l=1}^{Npr} TD_{hljk}$ for traditional pail, $\frac{1}{12} X \sum_{l=1}^{Npr} TD_{hljk}$ for light weight pail

$\tilde{FD}_{lhj}$ Forecasted demand of product l of distribution center j in period h.

$\tilde{AD}_{lhj}$ Actual demand of product l of distribution center j in period h.

$\tilde{D}_{lhk}$ Demand of product l of retailer k in period of h.

C_{hi} Capacity of ith warehouse in period h.

C_{hj} Capacity of kth retailer in period h.

 Cut level of fuzzy set.

w_t Corresponding weight of prominent point t of triangular fuzzy number.

$f_i(Z_{ihs})$ Corresponding linear membership function of the objective function Z_i in period h, in stage S.

Original multi-product and multi-time period two-echelon supply chain model

The proposed model deals with optimizing the inventory levels of the two-echelon multi-product and multi-time period supply chain models. The diagrammatic representation of the proposed model for the period h is shown in Fig.1.

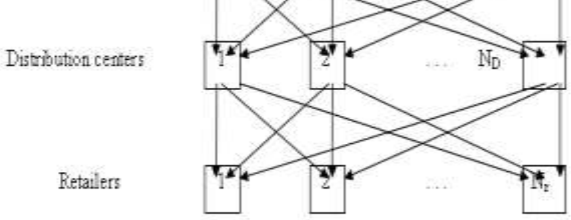


Figure 1 : Two-echelon Supply Chain Model

First Stage Problem

The imprecise objectives function for the first stage is given below :

$$\tilde{Z}_{h1} = \sum_{i=1}^{N_w}\sum_{l=1}^{N_{pr}} IW_{hli} \text{ x } \tilde{ICW}_{hli} + \sum_{i=1}^{N_w}\sum_{j=1}^{N_D}\sum_{l=1}^{N_{pr}} TW_{hlji} \text{ x } \tilde{TCW}_{hlji} + \sum_{i=1}^{N_w}\sum_{j=1}^{N_D}\sum_{l=1}^{N_{pr}} TW_{hlji} \text{ x } \tilde{PC}_{hli} + \sum_{i=1}^{N_w}\sum_{j=1}^{N_D} NSW_{hij} \text{ x } \tilde{TFW}_{hij} \quad \ldots\ldots\ldots (2.2.1)$$

where $\tilde{ICW}_{hli}$, $\tilde{TCW}_{hlji}$, $\tilde{PC}_{hli}$, $\tilde{TFW}_{hij}$ denote the fuzzy cost coefficients.

In the first stage we have the following constraints

1)The sum of the units of product l transferred from a warehouse to all distribution centers should be less than or equal to the warehouse inventory for a particular period h.

$$\sum_{j=1}^{N_D} TW_{hlji} \le IW_{hli},\ i = 1, 2, \ldots N_w,\ l = 1, 2, \ldots, N_{pr} \quad \ldots (2.2.2)$$

2. $\sum_{l=1}^{N_{pr}} IW_{hli} \le C_{hi}$, ie. Inventory at the warehouse should be less than or equal to the warehouse capacity in period h. . . . (2.2.3)

3. The sum of the units of product l transferred from all warehouses to a particular distribution center should be greater than or equal to the imprecise forecasted demand of that particular distributor in period h.

$$\sum_{l=1}^{N_w} TW_{hlji} \ge \tilde{FD}_{lhj},\ l = 1, 2, \ldots N_{pr},\ j = 1, 2, \ldots, N_D \quad \ldots (2.2.4)$$

4. The total distribution centers imprecise forecasted demand for a period h should be less than or equal to all warehouse inventory of product l, in that particular period

$$\sum_{l=1}^{N_w} IW_{hli} \ge \sum_{j=1}^{N_D} \tilde{FD}_{lhj},\ l = 1, 2, \ldots N_{pr} \quad \ldots (2.2.5)$$

5. The total number of units transferred from all warehouse to a particular distribution center should be less than or equal to that distribution center capacity

$$\sum_{i=1}^{N_w}\sum_{l=1}^{N_{pr}} TW_{hlji} \le C_{hj},\ j = 1, 2, \ldots N_D \quad \ldots (2.2.6)$$

Second Stage Problem

The imprecise objective function for the second stage is given below :

$$\tilde{Z}_{h2} = \sum_{j=1}^{N_D}\sum_{l=1}^{N_{pr}} ID_{hlj} \text{ x } \tilde{ICD}_{hlj} + \sum_{j=1}^{N_D}\sum_{k=1}^{N_r}\sum_{l=1}^{N_{pr}} TD_{hljk} \text{ x } \tilde{TCD}_{hljk} + \sum_{j=1}^{N_D}\sum_{k=1}^{Nr} NSD_{hjk} \text{ x } \tilde{TFD}_{hjk} \quad \ldots (2.2.7)$$

where $\tilde{ICD}_{hlj}$, $\tilde{TCD}_{hljk}$, $\tilde{TFD}_{hjk}$ denote the fuzzy cost coefficients.

In the second stage we have the following constraints

1) The inventory of product l at a particular distribution center at the end of stage one is equal to the total number units received from all warehouse by that distribution center minus the actual demand of that particular distribution center in period h.

$$ID_{hlj} = \sum_{i=1}^{N_w} TW_{hlji} - \tilde{AD}_{lhj},\ l = 1, 2, \ldots N_{pr} \quad \ldots (2.2.8)$$

(If this value is negative, $ID_{hlj} = 0$)

2) The sum of the units of product l transferred from all distribution centers to a particular retailer should be greater than or equal to the demand of that particular retailer in period h.

$$\sum_{j=1}^{N_D} TD_{hljk} \geq \tilde{D}_{hlk},\ l = 1, 2, \ldots N_{pr},\ K = 1, 2, \ldots N_r \quad \ldots (2.2.9)$$

3. The sum of the units of product l transferred from a particular distribution center to all retailer should be less than or equal to the actual demand of that particular distribution center in period h.

$$\sum_{K=1}^{N_r} TD_{hljk} \leq \tilde{AD}_{lhj},\ j = 1, 2, \ldots N_D,\ l = 1, 2, \ldots, N_{pr} \quad \ldots (2.2.10)$$

4. The total number of units transferred from all distribution centers to a particular retailer should be less than or equal to that retailer capacity

$$\sum_{j=1}^{N_D}\sum_{l=1}^{N_{pr}} TD_{hljk} \leq C_{hk},\ K = 1, 2, \ldots N_r \quad \ldots (2.2.11)$$

Arithmetic operations on triangular fuzzy numbers

The arithmetic operations on triangular fuzzy numbers are reviewed from D.Dubois,H.Prade,1978.

1. The addition of two fuzzy numbers $T_1=(a,b,c)$, $T_2=(d,e,f)$ is defined to be

$T_1 \oplus T_2 = (a+d,b+e,c+f)$

2. The sclar multiplication of T_1 by real number $\lambda > 0$ is defined by $\lambda.T = (\lambda a, \lambda b, \lambda c)$

Solution Methodology

Model of the Imprecise Data

This paper assumes that a DM has already adopted the triangular fuzzy number to represent all the imprecise data in the original multi-product, multi-time period two-echelon supply chain model formulated above. In real world situations, a DM is familiar with estimating the values of the upperbound (optimistic), lower bound (pessimistic), and modal value (most likely) parameters. The pattern of triangular fuzzy number is commonly adopted due to its ease in defining the maximum and minimum limits of deviation of an imprecise number from its central value. The primary advantages of the triangular fuzzy number are the simplicity and flexibility of the fuzzy arithmetic operations. Practically, a DM can construct a triangular fuzzy number based on the following three prominent data :

- The upper bound value (the most optimistic value) has a very low likelihood of belonging to the set of available values (membership degree – 0 if normalized).
- The modal value (the most possible value) definitely belongs to the set of available values (membership degree = 1 if normalized).
- The lower bound value (the most pessimistic value) has a very low likelihood of belonging to the set of available values (membership degree = 0 if normalized)

Treatment of the Imprecise Constraints

Recalling Constraints (2.2.4), (2.2.5), (2.2.8), (2.2.9),(2.2.10) in the original multi-product and multi-time period two-echelon supply chain model, consider the situations in which the forecast demand $\tilde{FD}_{lhj}$, the actual demand $\tilde{AD}_{lhj}$ for each distribution center and demand $\tilde{D}_{hlk}$ for each retailer have a distribution of triangular fuzzy number. The main treatment is to obtain a crisp representative number for an imprecise demand. The study applies the weighted averaging method to convert $\tilde{FD}_{lhj}$, $\tilde{AD}_{lhj}$, $\tilde{D}_{hlk}$ into a crisp number in the deffuzzification process (Y.J.Lai, C.L.Hwang,1992,J.Ramik, J.Rimanek, 2005, H.Tanaka, H.Ichihasni, K.Asai, 1985). If the α-cut level is given, the auxiliary expression of constraints (2.2.4), (2.2.5), (2.2.8), (2.2.9) and (2.2.10) can be represented as follows :

$$\sum_{i=1}^{N_w} TW_{hlji} \geq W_1 FD^0_{lhj,\alpha} + W_2 FD^m_{lhj,\alpha} + W_3 FD^p_{lhj,\alpha}$$

for $l = 1, 2, \ldots, N_{pr}$, $j = 1, 2, \ldots, N_D$, $h = 1, 2, \ldots, H$. ... (3.2.1)

$$\sum_{i=1}^{N_w} IW_{hli} \geq W_1 FD^0_{lhj,\alpha} + W_2 FD^m_{lhj,\alpha} + W_3 FD^p_{lhj,\alpha}$$

for $l = 1, 2, \ldots, N_{pr}$, $h = 1, 2, \ldots, H$, $j = 1, 2, \ldots, N_D$. ... (3.2.2)

$$ID_{hlj} = \sum_{i=1}^{N_w} TW_{hlji} - \left(W_1 AD^0_{lhj,\alpha} + W_2 AD^m_{lhj,\alpha} + W_3 AD^p_{lhj,\alpha}\right)$$

for $l = 1, 2, \ldots, N_{pr}$, $j = 1, 2, \ldots, N_D$, $h = 1, 2, \ldots, H$ ... (3.2.3)

$$\sum_{j=1}^{N_D} TD_{hljk} \geq W_1 D^0_{hlk,\alpha} + W_2 D^m_{hlk,\alpha} + W_3 D^p_{hlk,\alpha}$$

for $l = 1, 2, \ldots, N_{pr}$, $k = 1, 2, \ldots, N_r$, $h = 1, 2, \ldots, H$. ... (3.2.4)

$$\sum_{K=1}^{N_r} TD_{hljk} \leq \left(W_1 AD^0_{lhj,\alpha} + W_2 AD^m_{lhj,\alpha} + W_3 AD^p_{lhj,\alpha}\right)$$

for $j = 1, 2, \ldots N_D$, $l = 1, 2, \ldots, N_{pr}$, $h = 1,2, \ldots\ldots H$ ....(3.2.5)

where $0 \leq w_t \leq 1$ and $\Sigma w_t = 1$ ($t = 1, 2, 3$) and w_1, w_2 and w_3 represent the corresponding weights of the lower bound value, the modal value, and the upperbound value of the imprecise demand respectively. This work specifies $w_2 = \frac{4}{6}$, $w_1 = w_3 = \frac{1}{6}$ and $\alpha = 0.5$ for constraints (3.2.1), (3.2.2), (3.2.3) (3.2.4)and (3.2.5). The model value is generally the most important one, and thus should be assigned greater weight. However, the upper and lowerbound values, which provide the boundary solutions of the imprecise demand, should be assigned smaller weights (Y.J.Lai, C.L.Hwang, 1992 , T.F.Liang, 2006 , R.C. Wang, T.F. Liang, 2005).

Developing an auxiliary multi-objective linear programming model (MOLPM)

The objective function formulated in the original multi-product and multi-time period two-echelon supply chain model has the distribution of a triangular fuzzy number. Geometrically, this fuzzy objective function is fully defined by three prominent points $\left(Z^0_{h1},\ 0\right)\left(Z^m_{h1},\ 1\right)$ and $\left(Z^p_{h1},\ 0\right)$ and can be minimized by pushing the three prominent points leftwards. Since the vertical co-ordinates of the prominent points are fixed at either 1 or 0, it is only necessary to consider the three horizontal co-ordinates. The solving concept developed in the study involves simultaneous minimize of modal value of the objective function Z_{hs}^{m}, maximizing the difference between the modal value of the objective function and its lower bound, $\left(Z_{hs}^{m} - Z_{hs}^{0}\right)$, and minimizing the difference between the upper bound of the objective function and its modal value, $\left(Z_{hs}^{p} - Z_{hs}^{m}\right)$ in period h and stage s. (H.M.Hsu, 2001, Y.J.Lai, C.L.Hwang, 1992 ; D. Ozgen, S.Onut, B.Gulsun, U.R.Tuzkaya, G.Tuzkaya, 2008 ; R.C.Wang, T.F.Liang, 2004). The last two objectives are actually relative measures based on the modal value of the objective function. The resulting three new objective functions still guarantee the declaration of moving the triangular fuzzy number toward the left. Fig.2. presents the concept used to solve the imprecise objective function.

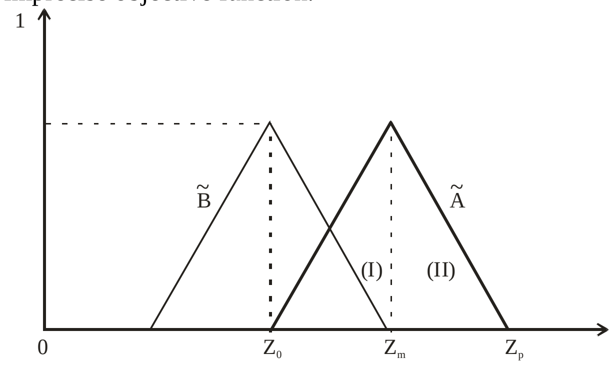


Figure 2. The concept to solve $\tilde{Z}^{o}$: $\tilde{B}$ is preferred to $\tilde{A}$

The resulting three new objective functions for stage 1 are as follows :

$$Z_{1h1} = Z_{h1}^{m} = \sum_{i=1}^{N_w}\sum_{l=1}^{N_{pr}} IW_{hli} \text{ x } ICW_{hli}^{m} + \sum_{i=1}^{N_w}\sum_{j=1}^{N_D}\sum_{l=1}^{N_{pr}} TW_{hlji} \text{ x } TCW_{hlji}^{m} + \sum_{i=1}^{N_w}\sum_{j=1}^{N_D}\sum_{l=1}^{N_{pr}} TW_{hlji} \text{ x } PC_{hli}^{m} + \sum_{i=1}^{N_w}\sum_{j=1}^{N_D} NSW_{hij} \text{ x } TFW_{hij}^{m} \quad ...(3.3.1)$$

$$Z_{2h1} = \left(Z_{h1}^{m} - Z_{h1}^{0}\right) = \sum_{i=1}^{N_w}\sum_{l=1}^{N_{pr}} IW_{hli} \text{x}\left(ICW_{hli}^{m} \text{-} ICW_{hli}^{0}\right) + \sum_{i=1}^{N_w}\sum_{j=1}^{N_D}\sum_{l=1}^{N_{pr}} TW_{hlji} \text{ x } \left(TCW_{hli}^{m} \text{-} TCW_{hli}^{0}\right) + \sum_{i=1}^{N_w}\sum_{j=1}^{N_D}\sum_{l=1}^{N_{pr}} TW_{hlji} \text{x}\left(PC_{hli}^{m} \text{-} PC_{hli}^{0}\right) + \sum_{i=1}^{N_w}\sum_{j=1}^{N_D} NSW_{hij} \text{x}\left(TFW_{hij}^{m} \text{-} TFW_{hij}^{0}\right) \quad ...(3.3.2)$$

$$Z_{3h1} = \left(Z_{h1}^{p} - Z_{h1}^{m}\right) = \sum_{i=1}^{N_w}\sum_{l=1}^{N_{pr}} IW_{hli} \text{x}\left(ICW_{hli}^{p} \text{-} ICW_{hli}^{m}\right) + \sum_{i=1}^{N_w}\sum_{j=1}^{N_D}\sum_{l=1}^{N_{pr}} TW_{hlji} \left(TCW_{hlji}^{p} \text{-} TCW_{hlji}^{m}\right) + \sum_{i=1}^{N_w}\sum_{j=1}^{N_D}\sum_{l=1}^{N_{pr}} TW_{hlji} \text{x}\left(PC_{hli}^{p} \text{-} PC_{hli}^{m}\right) + \sum_{i=1}^{N_w}\sum_{j=1}^{N_D} NSW_{hij} \text{x}\left(TFW_{hij}^{p} \text{-} TFW_{hij}^{m}\right) \quad ...(3.3.3)$$

The three new objective functions for stage 2 are as follows

$$Z_{1h2} = Z_{h2}^{m} = \sum_{j=1}^{N_D}\sum_{l=1}^{N_{pr}} ID_{hlj} \text{ x } ICD_{hlj}^{m} + \sum_{j=1}^{N_D}\sum_{K=1}^{N_r}\sum_{l=1}^{N_{pr}} TD_{hljk} \text{ x } TCD_{hljk}^{m} + \sum_{j=1}^{N_D}\sum_{k=1}^{Nr} NSD_{hjk} \text{ x } TFD_{hjk}^{m} \quad ...(3.3.4)$$

$$Z_{2h2} = \left(Z_{h2}^{m} - Z_{h2}^{0}\right) = \sum_{j=1}^{N_D}\sum_{l=1}^{N_{pr}} ID_{hlj} \text{ x } \left(ICD_{hlj}^{m} \text{-} ICD_{hlj}^{0}\right) + \sum_{j=1}^{N_D}\sum_{K=1}^{N_r}\sum_{l=1}^{N_{pr}} TD_{hljk} \text{ x } \left(TCD_{hljk}^{m} \text{-} TCD_{hljk}^{0}\right) + \sum_{j=1}^{N_D}\sum_{k=1}^{Nr} NSD_{hjk} \text{x } \left(TFD_{hjk}^{m} \text{-} TFD_{hjk}^{0}\right) \quad ..(3.3.5)$$

$$Z_{3h2} = \left(Z_{h2}^{p} - Z_{h2}^{m}\right) = \sum_{j=1}^{N_D}\sum_{l=1}^{N_{pr}} ID_{hlj} \text{ x } \left(ICD_{hlj}^{p} \text{-} ICD_{hlj}^{m}\right) + \sum_{j=1}^{N_D}\sum_{K=1}^{N_r}\sum_{l=1}^{N_{pr}} TD_{hljk} \text{ x}\left(TCD_{hljk}^{p} \text{-} TCD_{hljk}^{m}\right) + \sum_{j=1}^{N_D}\sum_{k=1}^{Nr} NSD_{hjk} \text{x } \left(TFD_{hjk}^{p} \text{-} TFD_{hjk}^{m}\right) \quad .(3.3.6)$$

Solving the Auxiliary Multi-Objective Linear Programming (MOLP) Problem

When using the fuzzy decision-making concept of Bellman and Zadeh, 1970 ; with the linear membership function, the above auxiliary MOLP problem can be converted into an equivalent ordinary single-goal LP form that represents the fuzzy goals of a DM. First specify the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) for each of the three objective functions of Stage 1 as follows :

$$Z_{1h1}^{PIS} = \text{Min } Z_{h1}^{m} \qquad Z_{1h1}^{NIS} = \text{Max } Z_{h1}^{m}$$

$$Z_{2h1}^{PIS} = \text{Max } \left(Z_{h1}^{m} - Z_{h1}^{0}\right) \qquad Z_{2h1}^{NIS} = \text{Min } \left(Z_{h1}^{m} - Z_{h1}^{0}\right)$$

$$Z_{3h1}^{PIS} = \text{Min } \left(Z_{h1}^{p} - Z_{h1}^{m}\right) \qquad Z_{3h1}^{NIS} = \text{Max } \left(Z_{h1}^{p} - Z_{h1}^{m}\right)$$

Then the corresponding linear membership functions of three objective functions of Stage 1 are defined by

$$f_1(Z_{1h1}) = \begin{cases} 1 & \text{if } Z_{1h1} < Z_{1h1}^{PIS} \\ \dfrac{Z_{1h1}^{NIS} - Z_{1h1}}{Z_{1h1}^{NIS} - Z_{1h1}^{PIS}} & \text{if } Z_{1h1}^{PIS} \le Z_{1h1} \le Z_{1h1}^{NIS} \\ 0 & \text{if } Z_{1h1} > Z_{1h1}^{NIS} \end{cases} \quad ......(3.4.1)$$

$$f_2(Z_{2h1}) = \begin{cases} 1 & \text{if } Z_{2h1} > Z_{2h1}^{PIS} \\ \dfrac{Z_{2h1} - Z_{2h1}^{NIS}}{Z_{2h1}^{PIS} - Z_{2h1}^{NIS}} & \text{if } Z_{2h1}^{NIS} \le Z_{2h1} \le Z_{2h1}^{PIS} \\ 0 & \text{if } Z_{2h1} < Z_{2h1}^{NIS} \end{cases} \quad ....(3.4.2)$$

$$f_3(Z_{3h1}) = \begin{cases} 1 & \text{if } Z_{3h1} < Z_{3h1}^{PIS} \\ \dfrac{Z_{3h1}^{PIS} - Z_{3h1}}{Z_{3h1}^{NIS} - Z_{3h1}^{PIS}} & \text{if } Z_{3h1}^{PIS} \le Z_{3h1} \le Z_{3h1}^{NIS} \\ 0 & \text{if } Z_{3h1} > Z_{3h1}^{NIS} \end{cases} \quad \ldots\ldots\ldots\ldots(3.4.3)$$

Applying the minimum operator to aggregate fuzzy sets, the equivalent ordinary single-goal LP model for Stage 1 can be derived as follows :

Max L

Such that $L \le f_g(Z_{gh1})$; g = 1, 2, 3 ; h = 1, 2, . . . H

$$\sum_{i=1}^{N_w} TW_{hlji} \ \ge \ W_1FD^0_{lhj,\alpha} + W_2FD^m_{lhj,\alpha} + W_3FD^p_{lhj,\alpha}$$

$$\sum_{i=1}^{N_w} IW_{hli} \ \ge \ W_1FD^0_{lhj,\alpha} + W_2FD^m_{lhj,\alpha} + W_3FD^p_{lhj,\alpha}$$

and equations (2.2.2), (2.2.3), (2.2.6).

$0 \le L \le 1$ where the auxiliary variable L can be interpreted as representing the overall DM satisfaction with the determined goal values.

Similarly, specify the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) for each of the three objective functions of Stage 2 as follows:

$$Z_{1h2}^{PIS} = \text{Min } Z_{h2}^m \qquad Z_{1h2}^{NIS} = \text{Max } Z_{h2}^m$$

$$Z_{2h2}^{PIS} = \text{Max}\left(Z_{h2}^m - Z_{h2}^0\right) \qquad Z_{2h2}^{NIS} = \text{Min}\left(Z_{h2}^m - Z_{h2}^0\right)$$

$$Z_{3h2}^{PIS} = \text{Min}\left(Z_{h2}^p - Z_{h2}^m\right) \qquad Z_{3h2}^{NIS} = \text{Max}\left(Z_{h2}^p - Z_{h2}^m\right)$$

Then the corresponding linear membership functions of three objective functions of Stage 2 and defined by :

$$f_1(Z_{1h2}) = \begin{cases} 1 & \text{if } Z_{1h2} < Z_{1h2}^{PIS} \\ \dfrac{Z_{1h2}^{NIS} - Z_{1h2}}{Z_{1h2}^{NIS} - Z_{1h2}^{PIS}} & \text{if } Z_{1h2}^{PIS} \le Z_{1h2} \le Z_{1h2}^{NIS} \\ 0 & \text{if } Z_{1h2} > Z_{1h2}^{NIS} \end{cases} \quad \ldots\ldots..(3.4.4)$$

$$f_2(Z_{2h2}) = \begin{cases} 1 & \text{if } Z_{2h2} > Z_{2h2}^{PIS} \\ \dfrac{Z_{2h2} - Z_{2h2}^{NIS}}{Z_{2h2}^{PIS} - Z_{2h2}^{NIS}} & \text{if } Z_{2h2}^{NIS} \le Z_{2h2} \le Z_{2h2}^{PIS} \\ 0 & \text{if } Z_{2h2} < Z_{2h2}^{NIS} \end{cases} \quad \ldots\ldots\ldots(3.4.5)$$

$$f_3(Z_{3h2}) = \begin{cases} 1 & \text{if } Z_{3h2} < Z_{3h2}^{PIS} \\ \dfrac{Z_{3h2}^{NIS} - Z_{3h2}}{Z_{3h2}^{NIS} - Z_{3h2}^{PIS}} & \text{if } Z_{3h2}^{PIS} \le Z_{3h2} \le Z_{3h2}^{NIS} \\ 0 & \text{if } Z_{3h2} > Z_{3h2}^{NIS} \end{cases} \quad \ldots\ldots..(3.4.6)$$

Applying the minimum operator to aggregate fuzzy sets, the equivalent ordinary single goal LP model for Stage 2 can be derived as follows :

Max L

Show that $L \le f_g(Z_{gh2})$; g = 1, 2, 3 ; h = 1, 2, . . . H

$$ID_{hlj} = \sum_{i=1}^{N_w} TW_{hlji} - \left(W_1AD^0_{lhj,\alpha} + W_2AD^m_{lhj,\alpha} + W_3AD^p_{lhj,\alpha}\right)$$

$$\sum_{j=1}^{N_D} TD_{hljk} \ \ge \ W_1D^0_{hlk,\alpha} + W_2D^m_{hlk,\alpha} + W_3D^p_{hlk,\alpha}$$

$$\sum_{K=1}^{N_r} TD_{hljk} \le \left(W_1AD^0_{lhj,\alpha} + W_2AD^m_{lhj,\alpha} + W_3AD^p_{lhj,\alpha}\right)$$

And equation (2.2.11).

$0 \le L \le 1$ where the auxiliary variable can be interpreted as representing the overall DM satisfaction with the determined goal values.

Solution procedure

Step 1. Formulate the original multi-product and multi-time period two echelon supply chain model according to Eqs. (2.2.1) – (2.2.11)

Step 2. Model all the imprecise data using the triangular fuzzy numbers.

Step 3. Given the a-cut level, convert the imprecise constraint (2.2.4), (2.2.5), (2.2.8), (2.2.9) and (2.2.10) into a crisp one using the weighted averaging method,as Eq. (3.2.1), (3.2.2), (3.2.3) (3.2.4)and (3.2.5)show.

Step 4. Develop the three new objective functions for the imprecise objective function using Eqs. (3.3.1)- (3.3.3) and (3.3.4) - (3.3.6) for stages 1 & 2 respectively.

Step 5. Specify the corresponding linear membership functions for the three fuzzy objective functions using Eqs. (3.4.1)-(3.4.3) and (3.4.4) –(3.4.6) for stages 1 & 2 respectively. Then convert the MOLP problem into an equivalent ordinary single-goal LP model using the minimum operator to aggregate fuzzy sets.

Step 6. Interactively solve and modify the ordinary single-goal LP model.

Implementation and computational analysis

Case Description

The company chosen for the application of the proposed methodology in this work is a leading paint company located in the Southern part of India. This study focuses only on the emulsion paints which are the fast moving paint products. Emulsion paints are coming in 3 different packs such as small, retailer, bulk. The forecasted demand of these three products are different in three different time periods. Moreover, holding cost,transportation cost and toll fees for warehouse and distribution centers, forecasted demand for each distribution center and retailer and packing charge for warehouse are an imprecise value. The company has planned to build a mathematical model to minimize the total cost of the supply chain. The total cost can be minimized by optimizing the inventory levels at warehouse and distribution centers. Also optimizing the transportation cost, packing charge and toll fees in the supply chain. This can be done by transporting the product with proper packaging through the optimum route and by maintaining an optimum inventory level at warehouse and distribution centers. This study assumes the holding cost, transportation cost, toll fees for warehouse and distribution centers and packing charge for warehouse are same for all the three products in three different time periods. Usually a truck can carry maximum load of 10 tons per shipment. But if 16% eco friendly light weight plastic pail is used rather than the traditional one, then a truck can carry maximum load of 12 tons per shipment. So the number of shipments can be reduced. In this study, the reduced cost in the supply chain due to this is also analysed. The following data is collected for validating the above proposed model.

Number of warehouse N_w	-	1
Number of distribution centers N_D	-	2

Number of Retailers N_r	-	4
Number of Products	-	3
Number of Timeperiods	-	3

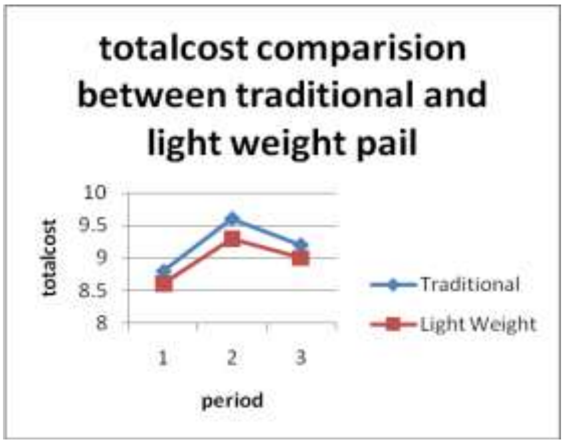
The input data for warehouse distribution centers and retailers are given in Tables 1 – 8.

Results

After solving the model for two stages, using Lindo Software, the optimal solutions for the case study are obtained. These results are tabulated in tables 9 – 15.

Computation Analysis

The Optimal Total Cost in Table 15 shows that there is 2.25% significant decrease in total cost of the two echelon, multi-product, multi-time period supply chain due to the usage of eco-friendly light weight plastic pail rather than the traditional one. The following graph proves that the total cost incurred while using light weight pail is comparatively less than the traditional one. So it is better to adopt eco- frielndly light weight pail in future.



Moreover the proposed approach is desirable because it yields an efficient and preferable solution. The proposed approach is based on Zimmermann's fuzzy programming method, which assumes that the minimum operator is the proper representation of a human DM who aggregates fuzzy sets using logical 'and' operations. Zimmermann's fuzzy programming method, which uses the linear membership function and the minimum operator, generates efficient solutions for fuzzy multi-objective programming problems The primary feature of the proposed approach is that a DM can adjust the search direction interactively during the solution procedure, until the yielded solution satisfies the DM's preferences and results in the preferred efficient solution. The following table summarizes the optimal results when applying ordinary single goal LP for minimizing the total cost .

Compare this result with table 15, the proposed approach is better than the crisp model. As a result, although increased cost, fuzzy model provides a better balanced distribution flows between facilities.

Conclusion

In real-world problems in supply chains, environmental coefficients, and related parameters are often imprecise due to incomplete and/or unavailable information over the intermediate planning horizon. This work presents a fuzzy possibilistic linear programming approach for solving multi-product, multi-time period two-echelon supply chain models. The proposed model provided an optimal inventory level for warehouses and distribution centers and minimizing the fuzzy total cost related to inventory, transportation, toll fees and packing charge of the entire supply chain. An industrial case study is utilized to demonstrate the feasibility of applying the proposed approach to practical problems in a supply chain. The proposed approach yields more efficient solution and several significant managerial implications for practical applications. The reduction of the total cost in the supply chain due to the usage of eco-friendly lightweight plastic pail, rather than the traditional plastic pail is also analyzed in the present work. In particular, the proposed computational methodology can be easily extended to any other situation and can handle real life decision making problems in uncertain environment.

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Table 1 : Input Data for Warehouse

Inventory carrying cost per unit in Rs.		(25,30,35)
Warehouse Capacity in Units		650 ton
Packing Charge/pack	Small Pack	(2,3,4)
	Retail Pack	(4,5,6)

Table 2 : Transportation Cost from Warehouse to Distribution Centers per unit in Rs.

	Distribution Centers	
	D_1	D_2
Transportation Cost in Rupees from Warehouse to	(800,1000,1100)	(1200,1400,1500)

Table 3 : Toll fees from Warehouse to Distribution Centers per shipment in Rs.

	Distribution Centers	
	D_1	D_2
Toll Fees from Warehouse to Distribution Centers	(1670,1770,1800)	(1990,2090,2150)

Table 4 : Distribution Centers Demand

Period	Product	Distribution Centers			
		D_1		D_2	
		Forecasted	Actual	Forecasted	Actual
h_1	S	(115,125,135)	(110,120,134)	(130,150,170)	(134,142,155)
	R	(85,100,110)	(80,90,103)	(112,120,130)	(97,108,122)
	B	(20,25,40)	(10,15,22)	(25,30,45)	(17,20,26)
h_2	S	(62,75,90)	(48,60,76)	(78,90,103)	(69,78,91)
	R	(165,180,200)	(159,172,190)	(180,200,225)	(175,189,203)
	B	(18,25,34)	(15,18,23)	(25,30,35)	(12,15,20)
h_3	S	(25,35,50)	(19,28,38)	(30,45,65)	(27,34,45)
	R	(125,140,160)	(122,130,142)	(150,165,180)	(150,158,167)
	B	(80,90,100)	(75,84,95)	(90,100,120)	(84,93,104)

Table 5 : Input Data for Distribution Centers

	Distribution Centers	
	D_1	D_2
Inventory carrying cost per unit in Rs.	(17,20,26)	(22,25,30)
Capacity in Units	300	400

Table 6 : Transportation cost from Distribution Centers to Retailers per unit in Rs.

Distribution Centers	Retailers			
	R_1	R_2	R_3	R_4
D_1	(138,150,162)	(225,234,250)	(450,468,480)	(535,540,550)
D_2	(430,444,460)	(530,537,550)	(110,120,135)	(180,189,197)

Table 7 : Toll fees from Distribution Centers to Retailers per shipment

Distribution Centers	Retailers			
	Nr_1	Nr_2	Nr_3	Nr_4
D_1	(160,170,180)	220,250,270)	(400,430,450)	(520,560,600)
D_2	(490,510,530)	(580,595,610)	(160,170,180)	(320,340,350)

Table 8 : Retailer Demand

Period	Product	Retailer			
		R_1	R_2	R_3	R_4
h_1	S	(63,68,75)	(47,52,59)	(75,78,85)	(59,64,70)
	R	(48,52,57)	(32,38,46)	(54,60,68)	(43,48,54)
	B	(7,10,15)	(3,5,7)	(10,12,16)	(7,8,10)
h_2	S	(24,30,38)	(24,30,38)	(42,47,55)	(27,31,36)
	R	(92,100,110)	(67,72,80)	(105,113,120)	(70,76,83)
	B	(8,10,13)	(7,8,10)	(8,10,13)	(4,5,7)
h_3	S	(7,11,15)	(12,17,23)	(19,23,28)	(8,11,17)
	R	(54,59,65)	(68,71,77)	(98,102,107)	(52,56,60)
	B	(34,39,45)	(41,45,50)	(55,60,66)	(29,33,38)

Table 8a : Input Data for Retailers

	Retailers			
	R_1	R_2	R_3	R_4
Retailers Capacity in Units	150	140	190	130

Table 9 : Optimal Warehouse Stock

Period	Product	Number of Units
h_1	S	$IW_{111} = 275$
	R	$IW_{121} = 219.667$
	B	$IW_{131} = 56.66$
h_2	S	$IW_{211} = 165.24$
	R	$IW_{221} = 380.839$
	B	$IW_{231} = 55.16$
h_3	S	$IW_{311} = 80.839$
	R	$IW_{321} = 305.42$
	B	$IW_{331} = 190.83$

Table 10 : Optimal number of units transferred from warehouse to distribution centers

Period	Product	Number of Units	
		D_1	D_2
h_1	S	$TW_{1111} = 125$	$TW_{1112} = 150$
	R	$TW_{1211} = 99.5$	$TW_{1212} = 120.17$
	B	$TW_{1311} = 25.83$	$TW_{1312} = 30.83$
h_2	S	$TW_{2111} = 75.16$	$TW_{2112} = 90.08$
	R	$TW_{2211} = 180.42$	$TW_{2212} = 200.42$
	B	$TW_{2311} = 25.17$	$TW_{2312} = 30$
h_3	S	$TW_{3111} = 35.41$	$TW_{3112} = 45.41$
	R	$TW_{3211} = 140.42$	$TW_{3212} = 165$
	B	$TW_{3311} = 90$	$TW_{3312} = 100.83$

Table 11 : Optimal inventory in units at distribution centers

Period	Product	Distribution Centers	
		D_1	D_2
h_1	S	$ID_{111} = 4.67$	$ID_{112} = 7.59$
	R	$ID_{121} = 9.25$	$ID_{122} = 11.92$
	B	$ID_{131} = 10.67$	$ID_{132} = 10.58$
h_2	S	$ID_{211} = 14.83$	$ID_{212} = 11.75$
	R	$ID_{221} = 8.01$	$ID_{222} = 11.42$
	B	$ID_{231} = 7.01$	$ID_{232} = 14.84$
h_3	S	$ID_{311} = 7.34$	$ID_{312} = 11.09$
	R	$ID_{321} = 10.08$	$ID_{322} = 6.92$
	B	$I_{331} = 5.84$	$I_{332} = 7.67$

Table 12 : Optimal number of units transferred from Distribution Centers to Retailers

Distribution Center	Period	Product	Retailer			
			R_1	R_2	R_3	R_4
D_1	h_1	S	$TD_{1111} = 68.16$	$TD_{1112} = 52.06$	$TD_{1113} = 0$	$TD_{1114} = 0$
		R	$TD_{1211} = 0$	$TD_{1212} = 0$	$TD_{1213} = 42.16$	$TD_{1214} = 48.08$
		B	$TD_{1311} = 10$	$TD_{1312} = 5$	$TD_{1313} = 10.16$	$TD_{1314} = 0$
	h_2	S	$TD_{2111} = 30.16$	$TD_{2112} = 30.16$	$TD_{2113} = 0$	$TD_{2114} = 0.01$
		R	$TD_{2211} = 0$	$TD_{2212} = 0$	$TD_{2213} = 96.33$	$TD_{2214} = 76.08$
		B	$TD_{2311} = 0$	$TD_{2312} = 8.08$	$TD_{2313} = 10.08$	$TD_{2314} = 0$
	h_3	S	$TD_{3111} = 11$	$TD_{3112} = 5.83$	$TD_{3113} = 0$	$TD_{3114} = 11.25$
		R	$TD_{3211} = 0$	$TD_{3212} = 0$	$TD_{3213} = 102.08$	$TD_{3214} = 28.25$
		B	$TD_{3311} = 0$	$TD_{3312} = 45.08$	$TD_{3313} = 39.08$	$TD_{3314} = 0$
D_2	h_1	S	$TD_{1121} = 0$	$TD_{1122} = 0$	$TD_{1123} = 78.33$	$TD_{1124} = 64.08$
		R	$TD_{1221} = 0$	$TD_{1222} = 38.167$	$TD_{1223} = 18$	$TD_{1224} = 0$
		B	$TD_{1321} = 10.16$	$TD_{1322} = 0$	$TD_{1323} = 2.01$	$TD_{1324} = 8.08$
	h_2	S	$TD_{2121} = 0$	$TD_{2122} = 0$	$TD_{2123} = 47.25$	$TD_{2124} = 31.07$
		R	$TD_{2221} = 100.16$	$TD_{2222} = 72.25$	$TD_{2223} = 16.59$	$T_{2224} = 0$
		B	$TD_{2321} = 10.08$	$TD_{2322} = 0$	$TD_{2323} = 0$	$T_{2324} = 5.08$
	h_3	S	$TD_{3121} = 0$	$TD_{3122} = 11.25$	$TD_{3123} = 23.08$	$TD_{3124} = 0$
		R	$TD_{3221} = 59.08$	$TD_{3222} = 71.25$	$TD_{3223} = 0$	$TD_{3224} = 27.75$
		B	$TD_{3321} = 39.08$	$TD_{3322} = 0$	$TD_{3323} = 21$	$TD_{3324} = 33.08$

Table 13 : Optimal number of shipments from warehouse to distribution centers

Type	Period	Distribution Centers	
		D_1	D_2
Traditional	h_1	25.03	30.10
	h_2	28.08	32.05
	h_3	26.58	31.12
Light Weight	h_1	20.77	24.98
	h_2	23.30	26.60
	h_3	22.06	25.83

Table 14 : Optimal number of Shipments from Distribution Centers to Retailers

Distribution Center	Type	Period	Retailer			
			R_1	R_2	R_3	R_4
D_1	Traditional	h_1	6.82	5.72	5.23	4.81
		h_2	3.02	3.82	10.64	7.61
		h_3	1.10	5.09	14.12	3.95
	Lightweight	h1	5.66	4.75	4.34	3.99
		h2	2.50	3.17	8.83	6.32
		h3	0.913	4.23	11.72	3.28
D_2	Traditional	h_1	6.22	3.82	9.83	7.22
		h_2	11.02	7.23	6.38	3.17
		h_3	9.82	8.25	4.41	6.08
	Lightweight	h_1	5.17	3.17	8.16	5.99
		h_2	9.15	6	5.3	3
		h_3	8.15	6.85	3.66	5.05

Table 15 : Optimal Total Cost in Rs.

Period	Type	
	Traditional	Light Weight
h_1	(743012.4,890762.4,901182.4)	(723957.4,870660.9,881291.9)
h_2	(817390,964690,975540)	(796200,943300,954120)
h_3	(782510,929910,942000)	(763610,908910,920980)

Period	Type	
	Traditional	Light Weight
h_1	8,88,737.40	8,68,649.85
h_2	9,61,343.40	9,39,595.74
h_3	9,29,746.20	9,08,765.19