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A note on economic order quantity using fuzzy optimization technique

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ABSTRACT

In this article, the authors attempt to find the economic order quantity under fuzzy environment. The ordering quantity, optimal order quantity, and the total profit are finding by use of Matlab. In this paper, we derive the conclusion by comparison of Crisp and Fuzzy Model.

Introduction

Fuzzy set concept has been widely used to treat the classical inventory model In the real world, imperfect products are inevitable during most production processes. It would be interesting to discuss an inventory model with imperfect products.

This Paper deals with inventory situation where all the items received are not of perfect quality. They are picked up during the screening process and are shipped in one lot due to economies of sales. Shortages are backordered. The defective rate, demand, holding cost, ordering cost and shortage cost are taken as triangular fuzzy numbers. Graded mean integration method is used for defuzzification of the total profit.

Preliminaries

In this section, we just recall the definitions and notations of fuzzy set theory initialed by Bellman, Zadeh and. Zimmerman [15].

Definition 2. 1

If X is a collection of objects denoted by X, then a fuzzy set $\tilde{A}$ in X is defined as a set of ordered pairs

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) / x \in X\}$$

where, $\mu_{\tilde{A}}(x)$ is called the membership function for the fuzzy set $\tilde{A}$. The membership function maps each element of X to a membership grade (or membership value) between 0 and 1.

Definition 2.2

A support of a Fuzzy set $\tilde{A}$ is the set of all points x in X such that $\mu_A(x) > 0$. (i.e.,)

Support $(\tilde{A}) = \{ x / \mu_{\tilde{A}}(x) > 0\}$.

Definition 2.3

The α-cut (or) α -level set of fuzzy set $\tilde{A}$ is a set consisting of those elements of the universe X whose membership values exceed the threshold level α .

(i.e.,)

$$\tilde{A}_\alpha = \{ x / \mu_{\tilde{A}}(x) \geq \alpha \}.$$

Definition 2.4

A Fuzzy set $\tilde{A}$ is convex if,

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda) x_2) \geq \min (\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)), \quad x_1, x_2 \in X \text{ and } \lambda \in [0,1].$$

Alternatively, a fuzzy set is convex, if all α -level set are convex.

Definition 2.5

A Fuzzy set $\tilde{A}$, is called triangular fuzzy number with Peak(or core) a_2 , left width $a_2 \geq 0$ and right width $a_3 \geq 0$ if its membership function has the following form

$$\mu_{\tilde{A}}(x) = \begin{cases} 1- (a_2 -x) / a_1 , & \text{if } a_2 - a_1 \leq x \leq a_2 \\ 1- (x -a_2) / a_3 , & \text{if } a_2 \leq x \leq a_2 + a_3 \\ 0 , & \text{if otherwise.} \end{cases}$$

And the set of all triangular fuzzy numbers is denoted by F(R), where in parametric form is $\tilde{A} =/ a_1 -(r -1) + a_2 , a_3(1 - r)$

Definition 2.6

A fuzzy number $\tilde{A}$ is said to be a L-R type fuzzy number if and only if ,

$$\mu_{\tilde{A}}(x) = \begin{cases} L((a_2 -x) / a_1 , & \text{if } a_2 - a_1 \leq x \leq a_2 \\ R((x -a_2) / a_3, & \text{if } a_2 \leq x \leq a_2 + a_3 \\ 0 , & \text{if otherwise.} \end{cases}$$

L is for left and R for right reference and a_2 is the core of $\tilde{A}$.

Symbolically, we write

$$\tilde{A} = (a_1, a_2 ,a_3)$$

If L(x) and R(x) be linear functions then the corresponding L-R number is said to be a triangular fuzzy number.

Definition 2.7

Let $\tilde{A}$ and $\tilde{B}$ are two triangular fuzzy number of L - R type $\tilde{A} =(a_1, a_2 ,a_3)$, $\tilde{B} =(b_1, b_2 ,b_3)$, then

- $\tilde{A} + \tilde{B} = (a_1, a_2 , a_3) + (b_1, b_2 , b_3) = (a_1+ b_1, a_2 + b_2, a_3 + b_3)$.
- $-\tilde{B} = - (b_1, b_2, b_3) = (b_3, -b_2 ,b_1)$.
- $\tilde{A} - \tilde{B} = (a_1, a_2 , a_3) - (b_1, b_2 , b_3) = (a_1+ b_3, a_2 - b_2, a_3 + b_1)$.

Definition 2.8

Let $\tilde{A}$ and $\tilde{B}$ are two triangular fuzzy number of L-R type $\tilde{A} = (a_1, a_2 , a_3)$, $\tilde{B} = (b_1, b_2 ,b_3)$, then $\tilde{A}$ and $\tilde{B}$ are positive

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$\tilde{A} \bullet \tilde{B} = (a_1, a_2, a_3) \bullet (b_1, b_2 ,b_3) = (a_2 b_1+b_2 a_1 , a_2 b_2, a_2 b_3 +b_2 a_3)$.

If $\tilde{B}$ is positive and $\tilde{A}$ is negative

$\tilde{A} \bullet \tilde{B} = (a_3, - a_2 ,a_1) \bullet (b_1, b_2 ,b_3) = (b_2 a_3- a_2 b_1, - a_2 b_2, b_2 a_1-a_2 b_3)$.

If $\tilde{A}$ and $\tilde{B}$ are negative

$\tilde{A} \bullet \tilde{B} = (a_3, - a_2, a_1) \bullet (b_3,- b_2 , b_1) =(- a_2 b_3,- b_2 a_3, a_2 b_2, - a_2 b_1- b_2 a_1)$.

Definition 2.9

Let $\tilde{A} = (a_1, a_2 ,a_3)$ be a triangular fuzzy number of L-R type, then

$\tilde{A}^{-1} = 1/ \tilde{A} = (1/ a_1 ,1/ a_2 ,1/a_3)$.

Definition 2.10

Let $\tilde{A}$ and $\tilde{B}$ are two triangular fuzzy number of L-R type $\tilde{A} =(a_1, a_2 ,a_3)$,

$\tilde{B} = (b_1, b_2 , b_3)$, then

$\tilde{A}$ and $\tilde{B}$ are positive

$\tilde{A} / \tilde{B} = (a_1, a_2 ,a_3) / (b_1, b_2 ,b_3) =(a_2 / b_1+ a_1 /b_2 , a_2 / b_2, a_2 / b_3 + a_3 / b_2)$.

If $\tilde{B}$ is positive and $\tilde{A}$ is negative

$\tilde{A}/\tilde{B} = (a_3, - a_2 ,a_1)/(b_1, b_2 ,b_3)=(a_3 / b_2 - a_2/ b_1 , - a_2 / b_2, a_1/ b_2 - a_2 / b_3)$.

If $\tilde{A}$ and $\tilde{B}$ are negative

$\tilde{A}/\tilde{B} =(a_3, - a_2 ,a_1)/(b_3,- b_2 , b_1)=(- a_2 / b_3,- a_3 / b_2 , a_2 / b_2, - a_2 /b_1- a_1 / b_2)$.

Crisp Model

The behavior of inventory level is as shown in Fig.2.3.1 where T is the cycle length, py is the number of defectives withdrawn from inventory, t is the total screening time to avoid shortages within the screening time, p is restricted to $p \le 1 - \frac{D}{x}$

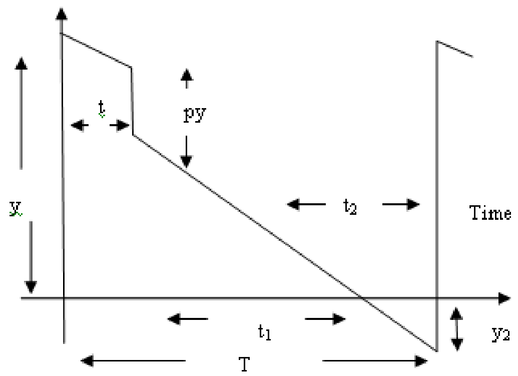

Fig. 2.1 The Behavior of Inventory Level

Total revenue is the sum of total sales of volume of good quality and imperfect quality items. Total revenue = (1- p) ys + pyv (2.1)

Total cost = ordering cost + purchasing cost + screening cost + holding cost + backordering cost

$$= K + cy + dy + h \; h\left(\frac{1}{2}\frac{(y-py-y_2)^2}{D}+\frac{py^2}{x}\right)+\frac{1}{2}\frac{by_2^2}{D} \quad (2.2)$$

Net profit per unit time is given by net profit = $\frac{\text{Total revenue - Total cost}}{T}$

Net profit per unit time is given by

$$TP(y) = D\left(s-v+\frac{hy}{x}\right)+D\left(\frac{v-c-d-\frac{hy}{x}-\frac{K}{y}}{1-p}\right)\frac{hy(1-p)}{2}+$$

$$hy^2-\frac{1}{2}\frac{hy_2^2}{(1-p)y}-\frac{1}{2}\frac{by_2^2}{(1-p)y} \quad (2.3)$$

The objective is to maximize the net profit. By taking the first derivative of TP with respect to y_2 and equating to zero we have

$$y_2=\frac{hy(1-p)}{h+b} \quad (2.4)$$

$\frac{\partial}{\partial y}TP(y)=0$ gives

$$\frac{1}{y^2}\left(\frac{kD}{1-p}+\frac{1}{2}\left(\frac{y_2^2(h+b)}{(1-p)}\right)\right)=\frac{h}{2}(1-p)+\frac{hDp}{x(1-p)} \quad (2.5)$$

Substituting the value of y_2 in (2.5) we have

$$\frac{kD}{y^2}=\frac{h}{2}(1-p)^2+\frac{hDp}{x}-\frac{h^2(1-p)^2}{2(h+b)}$$

$$\therefore y=\sqrt{\frac{kD}{\left(\frac{h}{2}(1-p)^2+\frac{hDp}{x}-\frac{h^2(1-p)^2}{2(h+b)}\right)}} \quad (2.6)$$

$\frac{\partial^2}{\partial y^2}(TP)$ is negative

$$\therefore y^0=\sqrt{\frac{kD}{\left(\frac{h}{2}(1-p)^2+\frac{hDp}{x}-\frac{h^2(1-p)^2}{2(h+b)}\right)}} \quad (2.7)$$

Validity of the Model

In case, all items are of perfect quality the result becomes $y^0=\sqrt{\frac{2kD(h+b)}{bh}}$ which is the traditional inventory model with shortages.

Notations and Assumptions

Assumptions

i. Production is instantaneous

ii. The demand is known

iii. Lead time is zero

iv. Shortages are allowed

v. Each lot received contains percentage defectives p. A 100% screening process of the lot is conducted at a rate x units per unit time with unit screening cost d, upon receiving the order

vi. Items of poor quality are kept in stock and sold prior to receiving the next shipment as a single batch at a discounted price v per unit, where as good quality item is sold at the regular prices per unit.

Notations

k - Ordering cost , C - Unit purchasing cost, h - inventory holding cost / unit / unit time, D - Demand (known and fixed), p - percentage of defective, x -screening rate (x > D), v - imperfect item selling rate/unit, s-price of perfect quality item s > v, y - Ordering quantity, y_1 - on hand inventory, y_2 -shortage quantity, T - ordering cycle, b-shortage cost / unit / unit time, d-screening cost / item, TP(y) - totalprofit, y^0- optimal order quantity, $\tilde{D}$-

fuzzy demand, $\tilde{p}$ - fuzzy defective percentage, $\tilde{h}$- fuzzy holding cost, $\tilde{K}$-fuzzy ordering cost, $\tilde{B}$-fuzzy total profit, $\tilde{T}P(y)$ - fuzzy total profit, $P\left(\tilde{T}P(y)\right)$-defuzzified value of $\tilde{T}P(y)$.

Methodology

In this Section, function principle and graded mean integration representation method are applied to calculate the optimal economic production quantity in the fuzzy inventory model, when the quantities are fuzzy numbers, we can use these model to obtain the solution by use of Matlab.

Fuzzy Number

Any fuzzy subset of the real line R, whose membership function μ_A satisfied the following conditions is a generalized fizzy number $\tilde{A}$.

- μ_A is a continuous mapping from R to the closed interval [0, 1]
- $\mu_A = 0, -\infty < x \le a_1$,
- $\mu_A = L(x)$ is strictly increasing on $[a_1, a_2]$
- $\mu_A = w_A,\ a_2 \le x \le a_3$
- $\mu_A = R(x)$ is strictly decreasing on $[a_3, a_4]$
- $\mu_A = 0,\ a_4 \le x < \infty$

Where $0 < w_A \le 1$ and a_1, a_2, a_3 and a_4 are real numbers. Also this type of generalized fuzzy number be denoted $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)_{LR}$; when $w_A = 1$, it can be simplified as $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$.

Triangular Fuzzy Number

The fuzzy set $\tilde{A} = (a_1, a_2, a_3)$ where $a_1 < a_2 < a_3$ and defined on R, is called the triangular fuzzy number, if the membership function of $\tilde{A}$ is given by

$$\mu_A = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & a_2 \le x \le a_3 \\ 0, & \text{otherwise} \end{cases}$$

The Function Principle

The function principle was introduced by Chen[2] to treat fuzzy arithmetical operations. This principle is used for the operation of addition, subtraction, multiplication and division of fizzy numbers.

Suppose $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers.

Then

i. Addition of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} + \tilde{B} = (a_1+b_1,\ a_2+b_2,\ a_3+b_3)$ where a_1, a_2, a_3, b_1, b_2, b_3 are any real numbers

ii. Multiplication of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \times \tilde{B} = (c_1, c_2, c_3)$ where $T = \{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$ $c_1 = \min T$, $c_2 = a_2b_2$, $c_3 = \max T$ If a_1, a_2, a_3, b_1, b_2, b_3 are all non zero positive real numbers, then $\tilde{A} \times \tilde{B} = (a_1b_1, a_2b_2, a_3b_3)$

iii. $-\tilde{B} = (-b_3, -b_2, -b_1)$ then the subtraction of $\tilde{A}$ and $\tilde{B}$ is

iv. $\tilde{A} - \tilde{B} = (a_1-b_3,\ a_1-b_2,\ a_3-b_1)$ where a_1, a_2, a_3, b_1, b_2, b_3 are any real numbers

v. $\dfrac{1}{\tilde{B}} = \tilde{B}^{-1} = (1/b_3, 1/b_2, 1/b_1)$ where b_1, b_2, b_3 are all non zero positive real number, then the division of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}/\tilde{B} = (a_1/b_3, a_2/b_2, a_3/b_1)$

vi. For any real number K, $K\tilde{A} = (Ka_1, Ka_2, Ka_3)$ if $K > 0$ $K\tilde{A} = (Ka_3, Ka_2, Ka_1)$ if $K < 0$

Graded Mean Integration Representation Method

Chen et al.. introduced graded mean integration representation method based on the integral value of graded mean h-level of a generalized fuzzy numbers for defuzzifing generalized fuzzy number. Now, we describe a generalized fuzzy number as following.

Suppose A is a generalized fuzzy number. It is described as any fuzzy subset of the real line R, whose membership function μ_A satisfies the following conditions:

- μ_A is a continuous mapping from R to the closed interval [0, 1],
- $\mu_A = 0,\ -\infty < x \le a_1$,
- $\mu_A = L(x)$ is strictly increasing on $[a_1, a_2]$,
- $\mu_A = \omega_A,\ a_2 \le x \le a_3$,
- $\mu_A = R(x)$ is strictly decreasing on $[a_3, a_4]$,
- $\mu_A = 0,\ a_4 \le x < \infty$,

where $0 < \omega_A \le 1$, and a_1, a_2, a_3, and a_4 are real numbers.

Also this type of generalized fuzzy numbers be denoted as $\tilde{A} = (a_1, a_2, a_3, a_4; \omega_A)_{LR}$, when $\omega_A = 1$, it can be simplified as $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$.

In graded mean integration representation method, L^{-1} and R^{-1} are the inverse functions of L and R respectively, and the graded mean h-level value of generalized fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4; \omega_A)_{LR}$ is $h(L^{-1}(h) + R^{-1}(h))/2$. Then the graded mean integration representation of A is $P(\tilde{A})$ with grade ω_A where

$$p(\tilde{A}) = \frac{\int_0^{\omega_A} h\left(\frac{L^{-1}(h) + R^{-1}(h)}{2}\right) dh}{\int_0^{\omega_A} h\, dh} \qquad (2.4.4.1)$$

With $0 < h \le \omega_A$ and $0 < \omega_A \le 1$.

If $\tilde{A} = (a_1, a_2, a_3)$ is a triangular number then the grated mean integration representation of $\tilde{A}$ by above formula is

$$p(A)=1/2\;\frac{\int_0^1 h[a_1+h(a_2-a_1)+a_3-h(a_3-a_2)]dh}{\int_0^1 h\,dh}=\frac{a_1+4a_2+a_3}{6}$$

Fuzzy Model

Let

$\tilde{D}=(D_1,D_2,D_3)$, $\tilde{p}=(p_1,p_2,p_3)$, $\tilde{h}=(h_1,h_2,h_3)$, $\tilde{k}=(k_1,k_2,k_3)$ $\tilde{b}=(b_1,b_2,b_3)$ be triangular fuzzy numbers.

Net profit per unit time in fuzzy sense is given by

$$\tilde{TP}(y)=\tilde{D}\left(s-v+\frac{\tilde{h}y}{x}\right)+\tilde{D}\left(\frac{v-c-d-\frac{\tilde{h}y}{x}-\frac{\tilde{K}}{y}}{1-\tilde{p}}\right)\frac{\tilde{h}y(1-\tilde{p})}{2}+$$

$$\tilde{h}y_2-\frac{1}{2}\frac{\tilde{h}y_2^2}{(1-\tilde{p})y}-\frac{1}{2}\frac{\tilde{b}y_2^2}{(1-\tilde{p})y} \qquad (2.5.1)$$

$$=\left[\left\{D_1\left(s-v+\frac{h_1y}{x}\right)+D_1\left(v-c-d-\frac{h_3y}{x}-\frac{K_3}{y}\right)\left(\frac{1}{(1-p_1}\right)\right.\right.$$

$$\left.-\frac{h_3y(1-p_3)}{2}+h_1y_2-\frac{1}{2}\frac{h_3y_2^2}{(1-p_1)y}-\frac{1}{2}\frac{b_3y_2^2}{(1-p_1)y}\right\},$$

$$\left\{D_2\left(s-v+\frac{h_2y}{x}\right)+D_2\left(v-c-d-\frac{h_2y}{x}-\frac{K_2}{y}\right)\left(\frac{1}{(1-p_2)}\right)\right.$$

$$\left.-\frac{h_2(1-p_2)}{2}+h_2y_2-\frac{1}{2}\frac{h_2y_2^2}{(1-p_2)y}-\frac{1}{2}\frac{b_2y_2^2}{(1-p_2)y}\right\},$$

$$\left\{D_3\left(s-v+\frac{h_3y}{x}\right)+D_3\left(v-c-d-\frac{h_1y}{x}-\frac{K_1}{y}\right)\left(\frac{1}{(1-p_3)}\right)\right.$$

$$\left.\left.-\frac{h_1y(1-p_1)}{2}+h_3y_2-\frac{1}{2}\frac{h_1y_2^2}{(1-p_3)y}-\frac{1}{2}\frac{b_1y_2^2}{(1-p_3)y}\right\}\right] \qquad (2.5.2)$$

$$P\left(\tilde{TP}(y)\right)=\frac{1}{6}\left[\left\{D_1\left(s-v+\frac{h_1y}{x}\right)+D_1\left(v-c-d-\frac{h_3y}{x}-\frac{K_3}{y}\right)\left(\frac{1}{1-p}\right)\right.\right.$$

$$\left.\frac{-h_3y(1-p_3)}{2}+h_1y_2-\frac{1}{2}\frac{h_3y_2^2}{(1-p_1)y}-\frac{1}{2}\frac{b_3y_2^2}{(1-p_1)y}\right\}$$

$$+4\left\{D_2\left(s-v+\frac{h_2y}{x}\right)+D_2\left(v-c-d-\frac{h_2y}{x}-\frac{K_2}{y}\right)\left(\frac{1}{1-p_2}\right)\right.$$

$$\left.-\frac{h_2y(1-p_2)}{2}+h_2y_2-\frac{1}{2}\frac{h_2y_2^2}{(1-p_2)y}-\frac{1}{2}\frac{b_2y_2^2}{(1-p_2)y}\right\},$$

$$+\left\{D_3\left(s-v+\frac{h_3y}{x}\right)+D_3\left(v-c-d-\frac{h_1y}{x}-\frac{K_1}{y}\right)\left(\frac{1}{1-p_3}\right)\right.$$

$$\left.\left.-\frac{h_1y(1-p_1)}{2}+h_3y_2-\frac{1}{2}\frac{h_1y_2^2}{(1-p_3)y}-\frac{1}{2}\frac{b_1y_2^2}{(1-p_3)y}\right\}\right] \qquad (2.5.3)$$

Differentiating (2.5.3) partially with respect to y_2 and equating to zero for maximum profit we have,

$$\frac{\partial P}{\partial y_2}\left(\tilde{TP}(y)\right)=\frac{1}{6}\left[\left\{h_1-\frac{h_3y_2}{(1-p_1)y}-\frac{b_3y_2}{(1-p_1)y}\right\}+4\left\{h_2-\frac{h_2y_2}{(1-p_2)y}-\frac{b_2y_2}{(1-p_2)y}\right\}\right.$$

$$\left.+\left\{h_3-\frac{h_1y_2}{(1-p_3)y}-\frac{b_1y_2}{(1-p_3)y}\right\}\right]=0$$

$$\text{(i.e.,)}\,(h_1+4h_2+h_3)-\frac{y_2}{y}\left(\frac{(h_3+b_3)}{1-p_1}+\frac{4(h_2+b_2)}{1-p_2}+\frac{(h_1+b_1)}{1-p_3}\right)=0$$

$$y_2=\frac{y(h_1+4h_2+h_3)}{\left(\frac{h_3+b_3}{1-p_1}+\frac{4(h_2+b_2)}{1-p_2}+\frac{(h_1+b_1)}{1-p_3}\right)} \qquad (2.5.4)$$

Now differentiating (2.5.4) partially with respect to y and equating to zero we have

$$\frac{D_1h_1+4D_2h_2+D_3h_3}{x}-\frac{1}{x}\left(\frac{D_1h_3}{1-p_1}+\frac{4D_2h_2}{1-p_2}+\frac{D_3h_1}{1-p_3}\right)$$

$$+\frac{1}{y^2}\left(\frac{D_1K_3}{1-p_1}+\frac{4D_2K_2}{1-p_2}+\frac{D_3K_1}{1-p_3}\right)-\frac{1}{2}(h_3(1-p_3)+4h_2(1-p_2)+h_1(1-p_1))$$

$$+\frac{(h_1+4h_2+h_3)^2}{2\left(\frac{h_3+b_3}{1-p_1}+\frac{4(h_2+h_2)}{1-p_2}+\frac{h_1+b_1}{1-p_3}\right)}=0 \qquad (2.5.5)$$

$$y=\sqrt{\frac{\frac{D_1K_3}{1-p_1}+\frac{4D_2K_2}{1-p_2}+\frac{D_3K_1}{1-p_3}}{\left[\begin{array}{l}\frac{1}{x}\left(\frac{D_1h_3}{1-p_1}+\frac{4D_2h_2}{1-p_2}+\frac{D_3h_1}{1-p_3}\right)+\frac{1}{2}(h_3(1-p_3)+4h_2(1-p_2)+h_1(1-p_1))\\ -\left(\frac{D_1h_1+4D_2h_2+D_3h_3}{x}\right)-\frac{(h_1+4h_2+h_3)^2}{2\left(\frac{h_3+b_3}{1-p_1}+\frac{4(h_2+h_2)}{1-p_2}+\frac{h_1+b_1}{1-p_3}\right)}\end{array}\right]}} \qquad (2.5.6)$$

$$\frac{\partial P}{\partial y_2}\left(\tilde{TP}(y)\right)=-\frac{2}{y^3}\left(\frac{D_1K_3}{1-p_1}+\frac{4D_2K_2}{1-p_2}+\frac{D_3K_1}{1-p_3}\right)\text{ which is negative.}$$

Therefore y^0 is as follows,

$$y^0=\sqrt{\frac{\frac{D_1K_3}{1-p_1}+\frac{4D_2K_2}{1-p_2}+\frac{D_3K_1}{1-p_3}}{\left[\begin{array}{l}\frac{1}{x}\left(\frac{D_1h_3}{1-p_1}+\frac{4D_2h_2}{1-p_2}+\frac{D_3h_1}{1-p_3}\right)+\frac{1}{2}(h_3(1-p_3)+4h_2(1-p_2)+h_1(1-p_1))\\ -\left(\frac{D_1h_1+4D_2h_2+D_3h_3}{x}\right)-\frac{(h_1+4h_2+h_3)^2}{2\left(\frac{h_3+b_3}{1-p_1}+\frac{4(h_2+h_2)}{1-p_2}+\frac{h_1+b_1}{1-p_3}\right)}\end{array}\right]}} \qquad (2.5.7)$$

Numerical Example

A Textile company has an annual demand of an item to be approximately 60,000 units, the ordering cost is around \$ 120 / cycle, cost of holding the item is nearly \$7/unit/year. The company likes to supply only perfect quality item, so it has an equipment to screen the items which screens at a rate of 1 unit / min. It sells the good quality items at a rate of \$75 / unit and defective item at a rate of \$27/unit. The defective rate is nearly 0.03. The company entertains shortages and cost of shortage is approximately \$ 12/unit/year. The company wants to determine the economic order quantity.

Crisp Model

Here, D = 60000 units / year, K = \$ 120 / cycle, H = \$7 / unit year,
x = 1 unit / min. (x = 175200 units / year), d = \$0.7 / unit, c = \$27 / unit
s = \$ 75 / unit, v = \$25 / unit, p = 0.03, b = \$12 / unit / year

Fuzzy Model

Let $\tilde{D} = (59000, 60000, 61000)$, $\tilde{K} = (115, 120, 125)$, $\tilde{h} = (6, 7, 8)$, $\tilde{p} = (.025, .03, .035)$, $\tilde{b} = (11, 12, 13)$

Conclusion

This paper we discussed the fuzzy model of Inventory problem using Matlab. Our model extend the approach of Nagoorgani,A., and Maheswari,S.,[6].

i. In crisp model from the table 2. 1 it is observed that the economic order quantity, back order quantity and total profit increase when demand increases if all the costs and defective rate remain the same.

ii. In crisp model from the table 2. 2 it is observed that the economic production quantity, shortage quantity and total profit increase when percentage of imperfect quality alone changes when all other items are kept same.

iii. In fuzzy model from the table 2..3 it is observed that the economic order quantity, back order quantity and total profit increase when demand increases if all the costs and defective rate remain the same.

iv. In fuzzy model from the table 2..4 it is observed that the economic production quantity, shortage quantity and total profit increase when percentage of imperfect quality alone changes when all other items are kept same.

From the above results, we conclude that the solution by the fuzzy model is very much closer to the crisp model.

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Table 2.1

D	y_2	y	TP
60000	653.6983	1829.20	2824900
61000	658.9516	1843.90	2872000
62000	664.1335	1858.40	2919200
63000	669.2796	1872.80	2966300
64000	674.3899	1887.10	3013500
65000	679.4646	1901.30	3060600

Table 2.2

p	y_2	y	TP
.031	653.3100	1830.0	2824700
.032	652.9211	1830.8	2824500
.033	652.5316	1831.6	2824300
.034	652.1415	1832.4	2824200
.035	651.7508	1833.2	2824000
.036	651.3596	1834.0	2823800

Table 2.3

D	y_2	y	TP
(59000,60000,61000)	654.5256	1831.2	2824900
(60000,61000,62000)	659.7799	1845.9	2871000
(61000,62000,63000)	664.9983	1860.5	2919200
(62000,63000,64000)	670.1453	1874.9	2966300
(63000,64000,65000)	675.2566	1889.2	3013500
(64000,65000,66000)	680.3321	1903.4	3060600

Table 2.4

P	y_2	y	TP
(.026,.031,.036)	654.1366	1832.0	2824700
(.027,.032,.037)	653.7470	1832.8	2824500
(.028,.033,.038)	653.3568	1833.6	2824300
(.029,.034,.039)	652.9661	1834.4	2824200
(.030,.035,.040)	652.5741	1835.2	2824000