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Effect of Radiative heat transfer on shock waves in two –dimensional flow of a non-ideal gas

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ABSTRACT

In present paper an attempt is made to study the effect of radiative heat transfer term on the propagation of a weak shock in a two dimensional steady supersonic flow of non ideal gas along a curved wall. The flow phenomenon in characteristic plane is investigated and its behavior at the first frozen Mach wave is discussed .The position and condition of breakdown of the solution and condition necessary for no shock formation on the first frozen Mach line is also obtained and discussed. Using perturbation technique governing equation up to second order of approximation are obtained and from there it is concluded that for a set of boundary conditions at the shock waves homogeneous parts are same for first and second order approximation.

Introduction

The assumption that the medium is an ideal gas is no more valid when the flow takes place in the extreme condition. Anisinov and Spiner[1]have studied a problem of point explosion in low density non ideal gas by taking the equation of state in a simplified from which describes the behavior of medium satisfactorily .Robert and Wu[5] have studied the gas that obeys a simplified Vander Wall's equation of state. Vishwakarma et-al[8] have investigated the one dimensional unsteady self similar flow behind a strong shock ,driven out by a cylindrical or spherical piston in a medium which is assumed to be non ideal and which obey the simplified Vander Wall's equation of state as consider by Robert and Wu[6] . Pandey K. and Pathak P.P.[4] have considered the growth and decay behavior of sonic waves in non ideal gases.

Considerable attention has been given to the development of real gas flow theories which have applications in the fields of entry physics, combustion and radiation .As a result of increasing flight speeds in space exploration and the very high temperature attained by gases in motion the study of thermal radiation effects in uses becomes increasingly more important in aerodynamics problems. Since the formulation of a problems in radiative gas dynamics involves greater complexities a number of investigators have used a number of simplification according to their problems .The problem in radiation gas dynamics become manageable only with help of linearization and use of the differential approximation. Pai and Hsieh[2] using optically thin gas approximation have studied the supersonic flows over weak compression and expansion corner and over biconvex airfoil by means of a coordinate stretching in the characteristics variable. Takigami and Hasimoto[7] using a perturbation technique have study the shock-wave in steady two-dimensional flow of a non-equilibrium gas along a curved wall. In present case we have assumed that in the flow field the thermal radiation is everywhere in thermodynamic equilibrium and is optically thin. Using method suggested by Takigami and Hasimoto[7] we see that although the equations describing the relaxation phenomenon of[] do not have one to one correspondence with the equations describing the non-ideal radiative equilibrium flow considered in the present case , but qualitative results are strikingly similar .

Governing Equations And Boundary Conditions

Equations governing the non ideal two dimensional motion of non ideal gas when radiative heat transfer term is taken into account are given by Robert and Wu.[5] and Pai [3],

$$u\rho_{,x} + v\rho_{,y} + \rho\left(u_{,x} + v_{,y}\right) = 0, \quad (2.1)$$

$$\rho u u_{,x} + \rho v u_{,y} + p_{,x} = 0, \quad (2.2)$$

$$\rho u v_{,x} + \rho v v_{,y} + p_{,y} = 0, \quad (2.3)$$

$$u p_{,x} + v p_{,y} - a_f^{\,2}\rho(u_{,x} + v_{,y}) + \frac{Q(\gamma-1)}{(1-b\rho)} = 0, \quad (2.4)$$

where x and y are cartesian coordinates parallel and perpendicular to the oncoming uniform flow,with origin at the corner or the beginning of the bend, u and v are the velocity components along x and y axis p, ρ are pressure, density ,b is the internal volume of the gas molecules and Q denotes radiative heat transfer term given by $Q = 4k\sigma T^4$ where k is the Plank mean absorption coefficient depending on the density ρ , σ is the Stefan-Boltzmann constant ,and T is the uniform temperature. Comma followed by an index specifies partial differentiation with respect to that index. Effective speed of sound a^2 is defined by

$$a^2 = \frac{\gamma p}{\rho(1-b\rho)}$$

When the Mach number $M = \left[(u^2+v^2)/a^2\right]^{1/2}$ is greater than unity, the system of equation (2.1) to equation (2.4) possesses three families of real characteristics ,the outgoing and incoming Mach waves and streamlines. In terms of u, v and a the streamlines and outgoing mach waves have slopes in the x, y plane determined by

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$$\frac{dy}{dx}=\frac{v}{u}, \tag{2.5}$$

and ,
$$\frac{dy}{dx}=\frac{\left[uv+a(u^2+v^2-a^2)^{1/2}\right]}{(u^2-a^2)}=\lambda \tag{2.6}$$

respectively.

Introducing the characteristic parameters α and β, which are functions of x and y, we have,

$$\frac{dy}{dx}=\frac{y_{,\beta}}{x_{,\beta}}=\lambda \text{ ,when } \alpha(x,y)= \text{ const.} \tag{2.7}$$

and
$$\frac{dy}{dx}=\frac{y_{,\alpha}}{x_{,\alpha}}=\frac{v}{u}, \quad \text{when } \beta(x,y)=\text{const.} \tag{2.8}$$

Using transformation relations,

$$\alpha,_x=\frac{y,_\beta}{J},\alpha,_y=\frac{x,_\beta}{J},\beta,_x=-\frac{y,_\alpha}{J},\beta,_y=\frac{x,_\alpha}{J} \tag{2.9}$$

The basic equations (2.1) to equation (2.4) can be expressed in terms of characteristic variable α and β in following form

$$u\rho_{,\alpha}(\lambda-v/u)x_{,\beta}+\rho[u_{,\alpha}\lambda x_{,\beta}-(v/u)x_{,\alpha}x_{,\beta}+x_{,\alpha}v_{,\beta}-x_{,\beta}v_{,\alpha}]=0 \tag{2.10}$$

$$\rho\left(\lambda-v/u\right)ux,_\beta u,_\alpha+\lambda x,_\beta p,_\alpha-\left(v/u\right)x,_\alpha p,_\beta=0 \tag{2.11}$$

$$\rho\left(\lambda-v/u\right)ux,_\beta v,_\alpha-x,_\beta p,_\alpha+x,_\alpha p,_\beta=0 \tag{2.12}$$

$$\left(\lambda-v/u\right)ux,_\beta p,_\alpha+\rho a^2\left(\lambda x,_\beta u,_\alpha-\frac{v}{u}x,_\alpha u,_\beta-x,_\beta v,_\alpha+x,_\alpha v,_\beta\right)$$
$$+\frac{Q(\gamma-1)}{(1-b\rho)}\left(\lambda-\frac{v}{u}\right)x,_\alpha x,_\beta=0 \tag{2.13}$$

where $J=x_{,\alpha}y_{,\beta}-x_{,\beta}y_{,\alpha}$ is the Jacobean of the transformation ,which does not vanish or does not become infinity any where and using equation (2.7) & equation (2.8) we find that

$$J=\left(\lambda-\frac{v}{u}\right)x,_\alpha x,_\beta \; . \tag{2.14}$$

So that a breakdown of the solution in terms of the characteristic parameter will arise if either $x_{,\alpha}$ or $x_{,\beta}$ is zero.

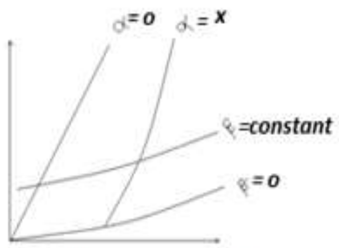

Fig.1 Schematic diagram of the flow field.

If the shape of the wall is described by equation $y=F(x)$, an equation (2.7) and equation (2.8) show that α is constant along an outgoing Mach wave and β is constant along a stream line. We assume for convenience that the first Mach line and wall are respectively $\alpha=0$ and $\beta=0$,The requirement that the flow should be tangential to the surface of the wall implies that,

$$x=\alpha,\; y=F(\alpha),\; \frac{v}{u}=F^{'}(\alpha) \text{ at } \beta=0. \tag{2.15}$$

Thus flow region is divided into two parts, one is of the uniform flow and the other is of the flow disturbed by the presence of the curved wall. The boundary of these two regions is the first Mach line or shockwave. As the flow variables are continuous across the first Mach line, the boundary conditions are given by,

$$p=p_0,\rho=\rho_0,u=u_0,v=0,y=\beta,x,_\beta=x,_{\beta_0} \text{ , at } \alpha=0, \tag{2.16}$$

where subscripts 0 denotes the free stream condition which are assumed to be in thermodynamic equilibrium. Rankine-Hugoniot relations in this case are given by,

$$\rho(u-v\cot\delta)=\rho_0u_0,$$
$$p-p_0=\rho_0u_0(u_0-u),v=(u_0-u)\cot\delta, \tag{2.17}$$

where δ is the shock angle.

Behaviour at The First Mach Wave

Assuming that the wall is a continuous curved bend and we must presume that all of the quantities as well as u and v are continuous in the neighborhood of the first frozen Mach line $\alpha=0$, by hypothesis,

$$p_{,\beta}=\rho_{,\beta}=u_{,\beta}=v_{,\beta}=0 \text{ at } \alpha=0\;. \tag{3.1}$$

Thus the equation (2.11) & equation (2.12) give

$$p_{,\alpha}=-\rho_0u_0u_{,\alpha}=\rho_0u_0\lambda_0v_{,\alpha}, \text{ at } \alpha=0 \tag{3.2}$$

Differentiating equation (2.13) with respect to α and using the foregoing result, we find that at the first Mach wave α =0

$$p_{,\alpha\beta}+\rho_0\lambda_0u_0v_{,\alpha\beta}+v_{,\beta}x_{,\beta}\left[\frac{Q(\gamma-1)(1+\lambda^2{}_0)}{(1-b\rho)_0\lambda_0u^2{}_0}\left\{\frac{\rho(1+\lambda^2{}_0)\lambda_0b}{(1-b\rho)_0}-1\right\}\right]=0 \tag{3.3}$$

Differentiating equation (3.2) with respect to β and from there when we substitute $p_{,\alpha\beta}$ in equation equation (3.3) after certain manipulation we have

$$v_{,\alpha\beta}+v_{,\alpha}x_{,\beta}\Lambda=0 \;, \tag{3.4}$$

where

$$\Lambda=\frac{Q(\gamma-1)(1+\lambda^2{}_0)}{2\rho_0u_0\lambda^2{}_0u^2{}_0(1-b\rho)_0}\left\{\frac{\rho_0(1+\lambda^2{}_0)\lambda_0b}{(1-b\rho)_0}-1\right\}$$

Equation (3.4) can be integrated with respect to β on the line of constant $\alpha(=0)$ which gives

$$v_{,\alpha}=v_{,\alpha0}\exp\{-\Lambda(x-x_0)\}\;, \tag{3.5}$$

Which gives the dependence of $v_{,\alpha}$ at the first Mach line on x, and $v_{,\alpha0}$ is the value of $v_{,\alpha}$ at α =0 which arises when x is equal to x_0.

To compute $\partial v/\partial x$ at the first Mach line , we invoke equation (2.9) and equation (2.14) which give

$$\partial v/\partial x=v_{,\alpha}/x_{,\alpha} \;. \tag{3.6}$$

It follows from equation (2.7) and equation (2.8) and the boundary conditions equation (3.1) at α =0, that

$$\lambda_{,\alpha}x_{,\beta0}+\lambda_0x_{,\alpha\beta}=0 \tag{3.7}$$

Since a depends on p, we have

$$a_{,\alpha} = [\partial a/\partial p]p_{,\alpha} = \lambda_0(u_0/2a_0)(\Gamma_{f0}-1)v_{,\alpha} \quad (3.8)$$

where $\Gamma = 1+\rho(\partial a^2/\partial p)$.Using equation (3.7) and equation (3.8), in equation (2.6) we find that

$$\lambda_{,\alpha} = \frac{(1+\lambda^2{}_0)^2}{u_0\lambda_0}\left(\frac{\Gamma+1}{2}\right)v_{,\alpha} \quad (3.9)$$

Substituting $\lambda_{,\alpha}$ from equation 3.9 into 3.7 and making use of equation 3.5 we have

$$x_{,\alpha\beta} = -\frac{(1+\lambda_0^2)^2}{u_0\lambda_0}\frac{(\Gamma+1)}{2}x_{,\beta 0}v_{,\alpha 0}\exp\left\{-\Lambda\left(x-x_0\right)\right\} \quad (3.10)$$

Integrating equation (3.10) with respect to β on the line of constant $\alpha(=0)$, we obtain that

$$x_{,\alpha} = x_{,\alpha 0}\left\langle 1-(\xi_0/u_0)(\partial v/\partial x)_0\left[1-\exp\left\{-\Lambda(x-x_0)\right\}\right]\right\rangle \quad (3.11)$$

where $\xi_0 = \frac{(1+\lambda^2{}_0)^2}{\lambda_0}\frac{(\Gamma+1)}{2\Lambda}$.

Consequently if

$$(\partial v/\partial x)_0 > u_0/\xi_0,$$

The characteristics will pile up at the first Mach line to from a shock wave at $x = x_c$ is given by

$$x_c = x_0 + \Lambda^{-1}\log[1-(u_0/\xi_0)(\partial v/\partial x)^{-1}{}_0]^{-1} \quad (3.12)$$

where $x_{,\alpha}$ and the Jacobean J vanish and we can write

$$x_c = x_0 + (u_0/\xi_0\Lambda)(\partial v/\partial x)^{-1}{}_0 \quad . \quad (3.13)$$

From equation (3.5), equation (3.6) and equation (3.11) the value of $(\partial v/\partial x)$ at the first Mach line may be given by

$$\partial v/\partial x = \frac{(\partial v/\partial x)_0\exp\left\{-\Lambda\left(x-x_0\right)\right\}}{1-(\xi_0/u_0)(\partial v/\partial x)_0\left[1-\exp\left\{-\Lambda(x-x_0)\right\}\right]} \quad (3.14)$$

Consequently, if $(\partial v/\partial x)_0$ is negative , $(\partial v/\partial x)_0$ at the first Mach line increases monotonically to zero as x increase. If $(\partial v/\partial x)_0$ is positive and has a magnitude greater than $u_0/\xi_0, \partial v/\partial x$ at first Mach wave will steepen up into a shock wave at $x = x_c$.

However ,if $(\partial v/\partial x)_0 = u_0/\xi_0$ then $(\partial v/\partial x)_{\alpha=0}$ remains constant . Finally if $(\partial v/\partial x)_0$ is positive and has a magnitude less than $u_0/\xi_0, (\partial v/\partial x)_{\alpha=0}$ decreases monotonically to zero.

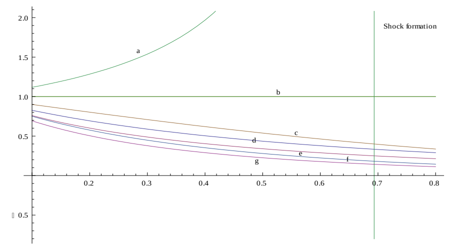

Figure -2 The value of $\partial v/\partial x$ at the first Mach line

To relate $(\partial v/\partial x)_0$ with the coefficient of second order derivative $F''(x) = 0$ of the shape of the wall y=F(x) , we take $x_0 = 0$,thus equation (3.14) can be rewritten in the following form,

$$\frac{\partial v}{\partial x} = \frac{u_0 F''(0)\exp(-\Lambda x)}{1-\xi_0 F''(0)\left\{1-\exp(-\Lambda x)\right\}} \quad . \quad (3.15)$$

In particular if $F''(0) \to -\infty$,we have simply

$$\frac{\partial v}{\partial x} = \frac{-u_0/\xi_0\exp(-\Lambda x)}{1-\exp(-\Lambda x)} \quad , \quad (3.16)$$

which gives the partial derivative of the y component of velocity with respect to x at the first Mach line of a Prandtl - Meyer flow.

The condition necessary for no shock to form on the first Mach line is given by

$$F''(0) < 1/\xi_0 \quad . \quad (3.17)$$

From equation (3.16) we see that even if the wall shape is the compressible corner at x=0 (i.e. $F''(0) > 0$),the shock wave is not always formed on the first Mach line.

Perturbation Equations and Boundary Conditions

Let us assume that the wall be a concave or convex with a small deflection angle and be described by the equation $y = \varepsilon R(x)$, where ($\varepsilon = 1$) may be taken as the slope of the wall at $x = 0$ and $R(x) : o(1)$, so that the boundary conditions (2.23) becomes,

$$x = \alpha \ , \ y = \varepsilon R(\alpha) \ , \ \frac{v}{u} = \varepsilon R'(\alpha) \ , \quad \text{at } \beta = 0 \quad (4.1)$$

As ε is a small parameter, we consider a solution of the form,

$$\psi(\alpha,\beta) = \psi_0(\alpha,\beta) + \varepsilon\psi_1(\alpha,\beta) + \varepsilon^2\psi_2(\alpha,\beta) + ... \quad (4.2)$$

where $\psi(\alpha,\beta)$ denotes any one of the dependent variables concerned. Substituting equation (4.2) into equation (2.12), (2.13), (2.14),and equation (3.1) to equation (3.5) and collecting the terms of like order in ε, we obtain a set of equations governing the variable of order ε as given below.

Equations governing terms of zeroth order are

$$y_{0,\beta} = \lambda_0 x_{0,\beta}, y_{o,\alpha} = 0, \quad (4.3)$$

where $\lambda_0 = \frac{a_0}{(u^2{}_0 - a^2{}_0)^{1/2}} = \tan\mu_{f_0}$, μ_{f_0} being the Mach angle in the free stream.

Equations governing the terms of first order are

$$p_{1,\alpha} + \rho_0 u_0 u_{1,\alpha} = 0,$$

$$\rho_0 u_0 x_{0,\beta} v_{1,\alpha}\lambda_0 - x_{0,\beta} p_{1,\alpha} + x_{0,\alpha} p_{1,\beta} = 0,$$

$$\lambda_0 u_0 x_{0,\beta} p_{1,\alpha} + \rho_0 a_0^2(\lambda_0 x_{0,\beta} u_{1,\alpha} - x_{0,\beta} v_{1,\alpha} + x_{0,\alpha} v_{1,\beta}) + Q_1(1-b_0\rho_0)(\gamma-1)\lambda_0\rho_0 a_0^2 x_{0,\alpha} x_{0,\beta} = 0,$$

$$y_{1,\beta} = \lambda_0 x_{1,\beta} + \lambda_1 x_{0,\beta}, \quad y_{1,\alpha} = (v_1/u_0)x_{0,\alpha},$$

$$\lambda_1 = \left[\frac{u_0 v_1 a}{(u^2{}_0 - a_0{}^2)} - \frac{a_{f0}u_0u_1}{\left(u_0{}^2 - a_0{}^2\right)^{3/2}} + \frac{u^2{}_0 a_1}{\left(u^2{}_0 - a_0{}^2\right)^{3/2}}\right], \quad (4.4)$$

where,

$a_1 = (\partial a/\partial p)_0\, p_1$.

In similar manner, it follows from equation (2.16) and equation (4.1) that ,

$$x_0(\alpha,0)=\alpha\,,\; y_0(0,\beta)=\beta \qquad (4.5)$$

$$p_1=\rho_1=u_1=v_1=y_1=0 \text{ at } \alpha=0 \qquad (4.6)$$

$$x_1=0\,,\; y_1=R(\alpha)\,,\; v_1=u_0R'(\alpha) \text{ at } \beta=0 \qquad (4.7)$$

Substituting equation (4.2) into the Rankine-Hugoniot relation equation (2.17) we obtain a set of boundary conditions at the shock wave. The shock relations of order ε are given by,

$$\rho_0[u_1-v_1\cot\delta_0]+u_0\rho_{,p_0}p_1=0\,,$$

$$p_1+\rho_0u_0u_1=0\;,$$

$$v_1+u_1\cot\delta_0=0\;. \qquad (4.8)$$

Equation (4.8) are a homogeneneous system in p_1, u_1 and v_1 . If these are not simultaneously zero at the shock wave, the determinant of the coefficients of the system must vanish. It follows that $\cot\delta_0=\left(\frac{u^2{}_0}{a^2{}_0}-1\right)^{1/2}$, that is , the slope of the shock wave is equal to that of the Mach wave in free stream to zeroth –order in ε .Making use of the relation $\cot\delta_0=\left(\frac{u^2{}_0}{a^2{}_0}-1\right)^{1/2}$ from equation (4.8) we obtain the following relations

$$p_1=\frac{\rho_0u_0a_0}{(u^2{}_0-a^2{}_0)^{1/2}}\;. \qquad (4.9)$$

Likewise, collecting the term of ε^2 , we obtain

$$\rho_2u_0+\rho_1(u_1-v_1\cot\delta_0)+\rho_0(u_2-v_2\cot\delta_0)-\rho_0v_1\cot\delta_1=0\;, \qquad (4.10)$$

$$p_2+\rho_0u_0u_2=0\;, \qquad (4.11)$$

$$v_2+u_2\cot\delta_0+u_1\cot\delta_1=0\;. \qquad (4.12)$$

Results &Discussion

Behavior of the wave at the first Mach wave is discussed which gives the condition necessary for no shock formation and condition of breakdown of solution. Using perturbation technique governing equation up to second order of approximation are obtained and from there it is concluded that for a set of boundary conditions at the shock wave homogeneous parts are same for first and second order approximation. For expansion wave $(\partial v/\partial x)$ decreases and damped out ultimately. For compression wave it will grow up and terminate into a shock wave at a critical distance x_c as shown by curve a. Thus behavior of wave at first frozen Mach wave is more or less similar to that for flow of a non-equilibrium as discussed by Takigami and Hiasimoto[7]. If $(\partial v/\partial x)_c$ is positive and has magnitude less than (u_0/ξ_0), it decreases as illustrated by curve c and d for case $u_0/2\xi_0<(\partial v/\partial x)_c<u_0/\xi_0$ and $0<(\partial v/\partial x)_c<u_0/2\xi_0$ respectively. For case $(\partial v/\partial x)_c=u_0/\xi_0$, it remains constant and is independent of x as shown by curve b .If $(\partial v/\partial x)_c$ is negative and less than (u_0/ξ_0) wave will decay out as illustrated by curves e,f,g which gives no shock formation condition a contradiction to the result that all compressive wave terminates into a shock-wave. In section four using perturbation technique we have obtained a set of boundary conditions at the shock which are homogeneous for first and second approximation but to ensure the consistency of second order boundary conditions appropriate relations are obtained.

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