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$D^{\approx\alpha\delta}$ -sets and associated separation axioms in topological spaces

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ABSTRACT

The notion of $\alpha\delta$--closed set was introduced and studied by R. Devi, V. Kokilavani and P. Basker [2]. In this paper, we introduce the concept of a $D^{\approx\alpha\delta}$ -sets and studied the associated separation axioms.

Introduction

Closedness are basic concept for the study and investigation in general topological spaces. This concept has been generalized and studied by many authors from different points of views. In particular, Njastad [6] and Velicko [7] introduced α -open sets and δ -closed sets respectively. R.Devi et al.[1] introduced α -generalized closed (briefly αg -closed) sets. More recently R.Devi, V.Kokilavani and P.Basker. [2] has introduced and studied the notion of $\alpha\delta$ -closed sets which is implied by that of δ -closed sets. In this paper we define a $D^{\approx\alpha\delta}$ -sets and studied the associated separation axioms.

Preliminaries

Throughout this present paper, spaces *X* and *Y* always mean topological spaces. Let *X* be a topological space and *A*, a subset of *X*. The closure of *A* and the interior of *A* are denoted by $Cl(A)$ and $\text{int}(A)$, respectively. A subset *A* is said to be regular open (resp. regular closed) if $A=\text{int}(cl(A))$ (resp. $A=cl(\text{int}(A))$, The δ -interior [7] of a subset *A* of *X* is the union of all regular open sets of *X* contained in *A* and is denoted by $\text{int}_{\delta}(A)$. The subset *A* is called δ -open [7] if $A=\text{int}_{\delta}(A)$, *i.e.*, a set is δ -open if it is the union of regular open sets. The complement of a δ -open set is called δ -closed. Alternatively, a set $A\subset(X,\tau)$ is called δ -closed [7] if $A=cl_{\delta}(A)$, where $cl_{\delta}(A)=\{x/x\in U\in\tau\Rightarrow\text{int}(cl(A))\cap A\neq\varphi\}$. The family of all δ -open (resp. δ -closed) sets in X is denoted by $\delta O(X)$ (resp. $\delta C(X)$). A subset A of X is called α -open [6] if $A\subset\text{int}(cl(\text{int}(A)))$ and the complement of a α -open are called α -closed. The intersection of all α -closed sets containing *A* is called the α -closure of *A* and is denoted by $\alpha cl(A)$, Dually, α -interior of *A* is defined to be the union of all α -open sets contained in *A* and is denoted by $\alpha\,\text{int}(A)$.

We recall the following definition used in sequel.

Definition 2.1. A subset *A* of a space *X* is said to be

(a) An α -generalized closed [1] (αg -closed) set if $\alpha cl(A)\subseteq U$ whenever $A\subseteq U$ and U is α -open in (X,τ).

(b) A $\alpha\delta$ -closed set [2] if $cl_{\delta}(A)\subseteq U$ whenever $A\subseteq U$ and U is αg -open in (X,τ).

(c) A point $x\in A$ is said to be $\alpha\delta$ -Interior point of A if A is a $\alpha\delta$ -nbhd of x. The set of all $\alpha\delta$ -Interior point of A is called the $\alpha\delta$ -Interior of A and is denoted by $\alpha\delta_{\text{int}}(A)$. [4]

(d) For a subset *A* of (X,τ), we define the $\alpha\delta$ -closure of *A* as follows $\alpha\delta_{cl}(A)=\cap\{F: F \text{ is } \alpha\delta-\text{closed in } X, A\subset X\}$. [4]

$D^{\approx\alpha\delta}$ -sets and associated separation axioms

Definition 3.1. A subset *A* of a topological space *X* is called a $D^{\approx\alpha\delta}$ -set if there are two $U,V\in\alpha\delta O(X,\tau)$ such that $U\neq X$ and $A=U/V$.

It is true that every $\alpha\delta$ -open set *U* different from X is a $D^{\approx\alpha\delta}$ -set if *A*=*U* and $V=\varphi$.

Definition 3.2. A topological space (X,τ) is called

(a) $\alpha\delta^{D}_{**0}$ if for any distinct pair of points *x* and *y* of X there exists a $D^{\approx\alpha\delta}$ -set of X containing x but not y or a $D^{\approx\alpha\delta}$ -set of X containing y but not x .

(b) $\alpha\delta^{D}_{**1}$ if for any distinct pair of points *x* and *y* of X there exists a $D^{\approx\alpha\delta}$ -set of X containing x but not y and a $D^{\approx\alpha\delta}$ -set of X containing y but not x .

(c) $\alpha\delta^{D}_{**2}$ if for any distinct pair of points *x* and *y* of X there exists disjoint $D^{\approx\alpha\delta}$ -sets *G* and *E* of X containing x and y , respectively.

(d) $T_0^{\#\alpha\delta}$ if for any distinct pair of points in X , there is a $\alpha\delta$ -*open* set containing one of the points but not the other.

(e) $T_1^{\#\alpha\delta}$ [5] if each pair of distinct points *x* and *y* in X there exists a $\alpha\delta$ -*open* set U in *X* such that $x\in U$ and $y\notin U$ and a $\alpha\delta$ -*open* set V in X such that $y\in V$ and $x\notin V$.

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(f) $T_2^{\#\alpha\delta}$[5] if for each pair of distinct points *x* and *y* in X there exist $\alpha\delta$-*open* sets U and V such that $U \cap V = \varphi$ and $x \in U$, $y \in V$.

Remark 3.3. Obviously, we have:

(a) If (X,τ) is $T_k^{\#\alpha\delta}$, then it is $T_{k-1}^{\#\alpha\delta}$,*k=1,2.*

(b) If (X,τ) is $T_k^{\#\alpha\delta}$, then it is $\alpha\delta_{**k}^{:D}$, *k=0,1,2.*

(c) If (X,τ) is $\alpha\delta_{**k}^{:D}$, then it is $\alpha\delta_{**k-1}^{:D}$,*k=1,2.*

Theorem 3.4. For a topological space (X,τ) the following statements are true:

(a) (X,τ) is $\alpha\delta_{**0}^{:D}$ if and only if it is $T_0^{\#\alpha\delta}$.

(b) (X,τ) is $\alpha\delta_{**1}^{:D}$ if and only if it is $\alpha\delta_{**2}^{:D}$.

Proof. (a) The sufficiency is Remark 3.3 (b). To prove *necessity*. Let (X,τ) be $\alpha\delta_{**0}^{:D}$ Then for each distinct pair *x, y* in *X*, at least one of *x, y* say *x*, belongs to a $D^{\approx\alpha\delta}$-set *G* but $y \notin G$. Let $G = U_1 \setminus U_2$ where $U_1 \neq X$ and $U_1, U_2 \in \alpha\delta O(X,\tau)$.Then $x \in U_1$, and for $y \notin G$ we have two cases: (i) $y \notin U_1$ (ii) $y \in U_1$ and $y \in U_2$.

In case *(i)* , U_1 contains *x* but does not contain *y;*

In case *(ii),* U_2 contains y but does not contain x. Hence X is $T_0^{\#\alpha\delta}$.

(b) *Sufficiency.* Remark 3.3(c).

Necessity. Suppose X $\alpha\delta_{**1}^{:D}$. Then for each distinct pair $x, y \in X$, we have $D^{\approx\alpha\delta}$-sets G_1, G_2 such that $x \in G_1,\ y \notin G_1,$ and $y \in G_2,\ x \notin G_2,$. Let $G_1 = U_1 \setminus U_2$ and $G_2 = U_3 \setminus U_4$. From $x \notin G_2,$ we have either $x \notin U_3$ or $x \in U_3$ and $x \in U_4$. We discuss the two cases separately.

(a) $x \notin U_3$. From $y \notin G_1,$ we have two subcases:

(i) $y \notin U_1$. From $x \in U_1 \setminus U_2$ we have $x \in U_1 \setminus (U_2 \cup U_3)$ and from $y \in U_3 \setminus U_4$ we have $y \in U_3 \setminus (U_1 \cup U_4)$. It is easy to see that $U_1 \setminus (U_2 \cup U_3) \cap U_3 \setminus (U_1 \cup U_4) = \varphi$.

(i) $y \in U_1$ and $y \in U_2$. We have $x \in U_1 \setminus U_2$, $y \in U_2$. $(U_1 \setminus U_2) \cap U_2 = \varphi$.

(b) $x \in U_3$ and $x \in U_4$. We have $y \in U_3 \setminus U_4$, $x \in U_4$. $(U_3 \setminus U_4) \cap U_4 = \varphi$.

From the discussion above we know that the space *X* is $\alpha\delta_{**2}^{:D}$.

Theorem 3.5. If (X,τ) is $\alpha\delta_{**1}^{:D}$, then it is $T_0^{\#\alpha\delta}$.

Proof. The proof of this theorem follows by *Remark 3.3* and *Theorem 3.4.*

Theorem 3.6. A topological space (X,τ) is $T_0^{\#\alpha\delta}$ if and only if for each pair of distinct points *x, y* of X , $\alpha\delta_{cl}(\{x\}) \neq \alpha\delta_{cl}(\{y\})$.

Proof. *Sufficiency:* Suppose that $x, y \in X$, $x \neq y$ and $\alpha\delta_{cl}(\{x\}) \neq \alpha\delta_{cl}(\{y\})$.Let z is a point of X such that $z \in \alpha\delta_{cl}(\{x\})$ but $z \notin \alpha\delta_{cl}(\{y\})$. We claim that $x \notin \alpha\delta_{cl}(\{y\})$. For, if $x \in \alpha\delta_{cl}(\{y\})$ then $\alpha\delta_{cl}(\{x\}) \subset \alpha\delta_{cl}(\{y\})$. And this contradicts the fact that $z \notin \alpha\delta_{cl}(\{y\})$. Consequently *x* belongs to the $\alpha\delta$-open. Set $[\alpha\delta_{cl}(\{y\})]^c$ to which *y* does not belong.

Necessity: Let (X,τ) be a $T_0^{\#\alpha\delta}$-space and *x, y* be any two distinct points of X. There exists a $\alpha\delta$-open set *G* containing x or y, say x but not y. Then G^c is a $\alpha\delta$-closed set which does not contain x but contains y. Since $\alpha\delta_{cl}(\{y\})$ is the smallest $\alpha\delta$-closed set containing y (Theorem 2.11[3]), $\alpha\delta_{cl}(\{y\}) \subset G^c$, and so $x \notin \alpha\delta_{cl}(\{y\})$. Consequently $\alpha\delta_{cl}(\{x\}) \neq \alpha\delta_{cl}(\{y\})$.

Theorem 3.7 A topological space (X,τ) is $T_1^{\#\alpha\delta}$ if and only if the singletons are $\alpha\delta$-closed sets.

Proof. Suppose (X,τ) is $T_1^{\#\alpha\delta}$ and x be any point of X. Let $y \in \{x\}^c$. Then $x \neq y$ and so there exists a $\alpha\delta$-open set U_y such that $y \in U_y$ but $x \notin U_y$. Consequently $y \in U_y \subset \{x\}^c$ *i.e.,* $\{x\}^c = \Upsilon\{U_y \setminus y \in \{x\}^c\}$ which is $\alpha\delta$-open.

Conversely. Suppose $\{p\}$ is $\alpha\delta$-closed for every $p \in X$. Let $x, y \in X$ with $x \neq y$. Now $x \neq y$ implies $y \in \{x\}^c$. Hence $\{x\}^c$ is a $\alpha\delta$-open set containing y but not containing *x*. Similarly $\{x\}^c$ is a $\alpha\delta$-open set containing x but not containing y. Accordingly X is a $T_1^{\#\alpha\delta}$-space.

Definition 3.8. A point $x \in X$ which has only X as the $\alpha\delta$-neighborhood is called a $\alpha\delta$-neat point.

Theorem 3.9. For a $T_0^{\#\alpha\delta}$ topological space (X,τ) the following are equivalent:

(a) (X,τ) is $\alpha\delta_{**1}^{:D}$

(b) (X,τ) has no $\alpha\delta$-neat point.

Proof. $(a) \Rightarrow (b)$. Since (X,τ) is $\alpha\delta_{**1}^{:D}$, so each point x of X is contained in a $D^{\approx\alpha\delta}$-set $O = U \setminus V$ and thus in *U.* By definition $U \neq X$. This implies that x is not a $\alpha\delta$-neat point.

$(b) \Rightarrow (a)$. If X is $T_0^{\#\alpha\delta}$, then for each distinct pair of points $x, y \in X$, atleast one of them, x (say) has a $\alpha\delta$-neighborhood U containing x and not y .Thus U which is different from X is a $D^{\approx\alpha\delta}$-set. If X has no $\alpha\delta$-neat point, then y is not a $\alpha\delta$-neat point. This means that there exists a $\alpha\delta$-neighborhood *V* of y such that $V \neq X$. Thus $y \in (V \setminus U)$ but not x and $V \setminus U$ is a $D^{\approx\alpha\delta}$-set. Hence X is $\alpha\delta_{**1}^{:D}$.

Remark 3.10. It is clear that a $T_0^{\#\alpha\delta}$ topological space (X,τ) is not $\alpha\delta_{**1}^{:D}$ if and only if there is a unique $\alpha\delta$-neat point in X . It is unique because if x and y are both $\alpha\delta$-neat point in X ; then at least one of them say x has a $\alpha\delta$-neighborhood U containing x but not y . But this is a contradiction since $U \neq X$.

Definition 3.11. A topological space (X,τ) is $\alpha\delta$-symmetric if for x and y in X; $x \in \alpha\delta_{cl}(\{y\})$ implies $y \in \alpha\delta_{cl}(\{x\})$.

Definition 3.12. A subset A of a topological space (X,τ) is called a $\alpha\delta^{\#\alpha\delta}$-closed set (briefly. $\alpha\delta^{\#\alpha\delta}$-closed -set) if $\alpha\delta_{cl}(A) \in U$ whenever $A \subset U$ and U is $\alpha\delta$-open in (X,τ).

Lemma 3.13. Every $\alpha\delta$-closed set is $\alpha\delta^{\#\alpha\delta}$-closed -set.

Theorem 3.14. A topological space (X,τ) is $\alpha\delta$-symmetric if and only iff $\{x\}$ is $\alpha\delta^{\#\alpha\delta}$-closed for each $x \in X$.

Proof. Assume that $x \in \alpha\delta_{cl}(\{y\})$ but $y \notin \alpha\delta_{cl}(\{x\})$. This means that the complement of $\alpha\delta_{cl}(\{x\})$ contains y. Therefore the set $\{y\}$ is a subset of the complement of $\alpha\delta_{cl}(\{x\})$. This implies that $\alpha\delta_{cl}(\{y\})$ is a subset of the complement of $\alpha\delta_{cl}(\{x\})$. Now the complement of $\alpha\delta_{cl}(\{x\})$ contains x which is a contradiction.

Conversely, suppose that $\{x\} \subset E \in \alpha\delta O(X,\tau)$, but $\alpha\delta_{cl}(\{x\})$ is not a subset of E. This means that $\alpha\delta_{cl}(\{x\})$ and the complement of E are not disjoint. Let y belongs to their intersection. Now we have $x \in \alpha\delta_{cl}(\{y\})$ which is a subset of the complement of E and $x \notin E$. But this is a contradiction.

Corollary 3.15. If a topological space (X,τ) is a $T_1^{\#\alpha\delta}$-space, then it is $\alpha\delta$-symmetric.

Proof. In a $T_1^{\#\alpha\delta}$-space singleton sets are $\alpha\delta$-closed (Theorem 3.7) and therefore $\alpha\delta^{\#\alpha\delta}$-closed (Lemma 3.13). By Theorem 3.14, the space is $\alpha\delta$-symmetric.

Corollary 3.16. For a topological space (X,τ) the following are equivalent:

(a) (X,τ) is $\alpha\delta$-symmetric and $T_0^{\#\alpha\delta}$

(b) (X,τ) is $T_1^{\#\alpha\delta}$.

Proof. By Corollary 3.15 and Remark 3.3 it suffices to prove only $(a) \Rightarrow (b)$. Let, then $x \neq y$ and by $T_0^{\#\alpha\delta}$, we may assume that $x \in G_1 \subset \{y\}^c$ for some $G_1 \in \alpha\delta O(X,\tau)$. Then $x \notin \alpha\delta_{cl}(\{y\})$ and hence $y \notin \alpha\delta_{cl}(\{x\})$. There exists a $G_2 \in \alpha\delta O(X,\tau)$ such that $y \in G_2 \subset \{x\}^c$ and (X,τ) is a $T_1^{\#\alpha\delta}$-space.

Theorem 3.17. For a $\alpha\delta$-symmetric topological space (X,τ) the following are equivalent:

(a) (X,τ) is $T_0^{\#\alpha\delta}$;

(b) (X,τ) is $\alpha\delta_{**1}^{D}$;

(c) (X,τ) is $T_1^{\#\alpha\delta}$.

Proof. $(a) \Rightarrow (c)$: Corollary 3.16.

$(c) \Rightarrow (b) \Rightarrow (a)$: Remark 3.3 and Theorems 3.4 and 3.5.

Definition 3.18. A function $f:(X,\tau) \to (Y,\sigma)$ is said to be $\alpha\delta$-irresolute if for each $x \in X$ and each $\alpha\delta$-open set V containing $f(x)$, there is a $\alpha\delta$-open set U in X containing x such that $f(U) \subset V$.

Lemma 3.19. A function $f:(X,\tau) \to (Y,\sigma)$ is $\alpha\delta$-irresolute if and only if the inverse image of each $\alpha\delta$-open set is $\alpha\delta$-open.

Definition 3.20. Let (X,τ) be a topological space, $x \in X$ and $\{x_s, s \in S\}$ a net of X. We say that the net $\{x_s, s \in S\}$ $\alpha\delta$-converges to x if for each $\alpha\delta$-open set U containing x there exists an element $s_0 \in S$ such that $s \geq s_0$ implies $x_s \in U$.

Definition 3.21. A filterbase F is called $\alpha\delta$-convergent to a point x in X, if for any $\alpha\delta$-open set U of X containing x, there exists B in F such that B is a subset of U.

Theorem 3.22. For a function $f:(X,\tau) \to (Y,\sigma)$, the following statements are equivalent:

(a) f is $\alpha\delta$-continuous;

(b) For each $x \in X$ and each filterbase F which $\alpha\delta$-converges to x, $f(F)$ $\alpha\delta$-converges to $f(x)$.

(c) For each $x \in X$ and each net $\{x_s, s \in S\}$ in X which $\alpha\delta$-converges to x, we have that the net $\{f(x_s), s \in S\}$ of Y $\alpha\delta$-converges to $f(x) \in Y$.

Proof. Obvious.

Definition 3.23. A function $f:(X,\tau) \to (Y,\sigma)$ is said to be $D^{\approx\alpha\delta}$-Irresolute if for each $x \in X$ and each $D^{\approx\alpha\delta}$-set V containing $f(x)$, there is a $D^{\approx\alpha\delta}$-set U in X containing x such that $f(U) \subset V$.

Lemma 3.24. A function $f:(X,\tau) \to (Y,\sigma)$ is $D^{\approx\alpha\delta}$-Irresolute if and only if the inverse image of each $D^{\approx\alpha\delta}$-set is $D^{\approx\alpha\delta}$-set.

Definition 3.25. Let (X,τ) be a topological space, $x \in X$ and $\{x_s, s \in S\}$ be a net of X. We say that the net $\{x_s, s \in S\}$ $D^{\approx\alpha\delta}$-converges to x if for each $D^{\approx\alpha\delta}$-set U containing x there exists an element $s_0 \in S$ such that $s \geq s_0$ implies $x_s \in U$.

Definition 3.26. A filterbase F is called $D^{\approx\alpha\delta}$-convergent to a point x in X, if for any $D^{\approx\alpha\delta}$-set U of X containing x, there exists B in F such that B is a subset of U.

Theorem 3.27. For a function $f:(X,\tau) \to (Y,\sigma)$, the following statements are equivalent:

(a) f is $D^{\approx\alpha\delta}$-Irresolute;

(b) For each $x \in X$ and each filterbase F which $D^{\approx\alpha\delta}$-converges to x, $f(F)$ $D^{\approx\alpha\delta}$-converges to $f(x)$.

(c) For each $x \in X$ and each net $\{x_s, s \in S\}$ in X which $D^{\approx\alpha\delta}$-converges to x, we have that the net $\{f(x_s), s \in S\}$ of Y $D^{\approx\alpha\delta}$-converges to $f(x) \in Y$.

Proof. Obvious.

Theorem 3.28. If $f:(X,\tau) \to (Y,\sigma)$ is a $\alpha\delta$-continuous surjective function and E is a $D^{\approx\alpha\delta}$-set in Y; then the inverse image of E is a $D^{\approx\alpha\delta}$-set in X.

Proof. Let E be a $D^{\approx\alpha\delta}$-set in . Then there are $\alpha\delta$-open sets U_1 and U_2 in Y such that $S = U_1 \setminus U_2$ and $U_1 \neq Y$. By the $\alpha\delta$-continuous of f, $f^{-1}(U_1)$ and $f^{-1}(U_2)$ are $\alpha\delta$-open in X. Since $U_1 \neq Y$, we have $f^{-1}(U_1) \neq X$. Hence $f^{-1}(E) = f^{-1}(U_1) \setminus f^{-1}(U_2)$ is a $D^{\approx\alpha\delta}$-set.

Theorem 3.29. If (Y,σ) is $\alpha\delta_{**1}^{D}$ and $f:(X,\tau) \to (Y,\sigma)$ is $\alpha\delta$-continuous and bijective, then (X,τ) is $\alpha\delta_{**1}^{D}$.

Proof. Suppose that Y is a $\alpha\delta_{**1}^{D}$ space. Let x and y be any pair of distinct points in X. Since f is injective and Y is $\alpha\delta_{**1}^{D}$, there exist $D^{\approx\alpha\delta}$-sets G_x and G_y of Y containing $f(x)$ and $f(y)$ respectively, such that $f(y) \neq G_x$ and $f(x) \neq G_y$. By Theorem 3.28, $f^{-1}(G_x)$ and $f^{-1}(G_y)$ are $D^{\approx\alpha\delta}$-sets in X containing x and y respectively. This implies that X is a $\alpha\delta_{**1}^{D}$ space.

Theorem 3.30. A topological space (X,τ) is $\alpha\delta_{**1}^{D}$ if and only if for each pair of distinct points $x, y \in X$, there exists a $\alpha\delta$-

continuous surjective function $f:(X,\tau)\to(Y,\sigma)$, where Y is a $\alpha\delta^{:D}_{**1}$ space such that $f(x)$ and $f(y)$ are distinct.

Proof. *Necessity*. For every pair of distinct points of X, it suffices to take the identity function on *X*.

Sufficiency. Let *x* and *y* be any pair of distinct points in X. By hypothesis, there exists a $\alpha\delta$-continuous, surjective function f of a space X onto a $\alpha\delta^{:D}_{**1}$ space *Y* such that $f(x)\neq f(y)$. Therefore, there exist disjoint $D^{\approx\alpha\delta}$-sets G_x and G_y in Y such that $f(x)\in G_x$ and $f(y)\in G_y$. Since f is $\alpha\delta$-continuous and surjective, by Theorem 3.28, $f^{-1}(G_x)$ and $f^{-1}(G_y)$ are disjoint $D^{\approx\alpha\delta}$-sets in X containing *x* and *y* respectively. Hence by Theorem 3.6, X is $\alpha\delta^{:D}_{**1}$ space.

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