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Construction of group acceptance sampling plan indexed through indifference quality level and inverse rayleigh distribution

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ABSTRACT

Acceptance sampling plans developed relating to groups of items on testers are called the group acceptance sampling plan (GASP). In this scheme, a sample of items is distributed into different groups and a lot of product is rejected if more than a specified number of failures are recorded in any group. A group acceptance sampling schedule can be used to save the time and cost in inspection as compared with classical acceptance sampling. In this paper, a procedure for constructing a group acceptance sampling plan (GASP) indexed through indifference quality level (IQL) and Inverse Rayleigh distribution is proposed. Suitable tables and examples are also provided for easy selection of the plans.

Introduction

In life testing more than one characteristic is tested in the experiment simultaneously and in these situations experimenters cannot use the ordinary acceptance sampling plan to test the items as it consumes more time and cost .Experimenters can treat items as a group and group acceptance sampling plans are applied. In this scheme, a sample of items is distributed into different groups for inspection and a lot is rejected if more than a specified number of failures are recorded in any group. Aslam et.al (2009) constructed group acceptance sampling plan using binomial distribution with Pareto distribution of the second kind in life testing. Radhakrishna Rao (1977) suggested weighted Binomial Distribution can be used in the construction of sampling plans. Radhakrishnan and Alagirisamy (2011a,b,c) constructed Group Acceptance sampling plan using weighted binomial distribution as the base line distributions, Radhakrishnan and Mohanapriya (2008) suggested the weighted Poisson distribution can be used in the construction of sampling plans, Hamaker (1950) advocated the use of indifference quality level (IQL) and the relative slope of the OC curve at (IQL,0.5) to compare acceptance sampling plans and was of the opinion that any acceptance sampling plan can be found by matching the IQL and the relative slop of the OC curve at the point of control. The IQL can be used to obtain matching group acceptance sampling plans to enable comparison of the sampling plans with their matched group acceptance sampling plans.

In this Paper a group acceptance sampling plan is constructed by assuming the probability of acceptance of a lot as 0.50, the proportion defective corresponding to this probability of acceptance in the OC (Operating Characteristic) curve is termed as Indifference Quality Level using weighted Binomial distribution as the base line distribution with Inverse Rayleigh distribution in life testing.

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Glossary of symbols

p -Quality of submitted lots

α- producer's risk

β-consumer's risk

g-Group size

r- Sample size in each group

c - Acceptance number

a-Termination ratio

γ_I- -Scale parameter

μ_0 - Specified mean life

Pa (p) -Probability of acceptance of the lot quality p

n- Sample size

Design of Group Acceptance Sampling Plans

Weighted binomial Distribution

The probability mass function of the weighted binomial distribution (WBD) is given by

$P(X: n, p, \alpha) = x^k p(X, n, p, \alpha) / \Sigma x^k p(X, n, p, \alpha)$, $X = 0, 1, 2\ldots\ldots\ldots n$, $k \geq 0$

The probability mass function of the weighted binomial distribution for k = 1 is given by

$$L(P) = \binom{r-1}{i-1} p^{i-1}(1-p)^{r-i} \quad i = 1, 2, 3\ldots\ldots\ldots r \quad \ldots\ldots(1)$$

The Inverse Rayleigh Distribution

Suppose that the lifetime of a product follows the inverse Rayleigh distribution. It is convenient to determine the termination time t_0 as a multiple of the specified life μ_0 that is

$$\mathbf{F(t) = exp\left(\frac{-\lambda_I^2}{t^2}\right)} \qquad \mathbf{t > 0} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$t_0 = a\mu_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.1)$$

The scale parameter of the inverse Rayleigh distribution is

$$\lambda_I = \frac{\mu}{2\sqrt{\pi}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Using (2.1) and (3) the cdf of inverse Rayleigh distribution (2) can be written as

$$p = \exp\left(\frac{-\mu^2}{\pi a^2 \mu_0^2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

(Or)

$$p = \exp\left(\frac{-1}{\pi a^2}\left(\frac{\mu}{\mu_0}\right)^2\right) \ldots\ldots\ldots\ldots\ldots(5)$$

Conditions for the application of GASP with WBD in the product control

- Production is continuous, so that results of past, present and future lots are broadly indicative of a continuing process.
- Lots submitted may be sequentially.
- Inspection is by attributes, with the lot quality defined as the proportion defective.
- Items are to be submitted for inspection in groups.

The operating procedure of Group Acceptance sampling plan

The producer's risk and the consumer's risk are represented by α and β respectively in the development of acceptance sampling plans. The producer's risk is defined as the probability of rejecting a good lot. The consumer's risk is the probability of accepting a bad lot. We are interested in constructing a GASP to ensure that the mean life μ is greater than a specified mean life μ_0.

The following scheme is used to develop the GASP:

- Determine the group size g and select a Sample n=gr items from a lot randomly and allocate r items to each group for the life test.
- Determine the acceptance number c for every group and specify the termination time of the life test t_0 and
- Implement the life test based on the g groups of items si-multaneously. Accept the lots if at most c failed items are found in every group by the termination time.
- Truncate the life test and reject the lot if more than c failures are found in any group.

The parameters in the GASP to be determined are the group size g, the sample size in each group r, the acceptance number c and the termination ratio t/μ_0. If r = l, the GASP reduces to the ordinary acceptance sampling plan. As the decision is either to accept or to reject the lot on the basis of a sample selected from a big lot, the weighed binomial distribution can also be used to develop the GASPs. The lot acceptance probability for the proposed plan is given by

$$L(p) = \left[\sum_{i=1}^{c} \binom{r-1}{i-1} p^{i-1}(1-p)^{r-i}\right]^g \quad \ldots\ldots\ldots\ldots\ldots(6)$$

Where p = function of cumulative distribution function given in (2).

It would be convenient to take the termination time as a multiple of the specified number *a*, i.e., $t_0 = a\mu_0$. Therefore p can be derived from (2) as

$$p = \exp\left(\frac{-1}{\pi a^2}\left(\frac{\mu}{\mu_0}\right)^2\right)$$

The minimal group size (g) and minimal ratio (μ/μ_0) of the true mean life to the specified mean life can be determined using the equations (6) and (7), for a specified value of α and β

$$L(p) \le \beta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

$$L(p) \ge 1-\alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

When consumer risk (β) and producers risk (α) are specified**.**

Construction of GASP plans Indexed through IQL

By fixing the probability of acceptance of the lot, Pa(p) as 0.50 with weighted Binomial Distribution as the basic distribution and from equation (1), the values of the IQL are obtained for the various combinations of r ,a, β' and 'c' using a Excel package and are presented in Table 1. The parameters of the group acceptance sampling plan, r, g and c are recorded for various combinations of IQL.

Construction of Tables

The minimum group size (g) is obtained using equations (4), (5), (6) & (7) for various values of β, α, termination ratio (a) with r =4, various values of a and c = 2 for a specified value of r = 4, c =2, α = 0.5, λ = 2 and with the help of Excel package the results are calculated and presented in table1, table2 and table3.

Table1: Minimal Group Sizes for GASP for r =4, c = *2*

a / β	0.7	0.8	1.0	1.2	1.5	2.0
0.01	14	10	6	4	3	2
0.05	9	6	4	3	2	2
0.10	7	5	3	2	2	1
0.25	4	3	2	1	1	1

Table2: Minimum ratio of true average life to specified life for the producer's risk of σ=0.5

a / β	0.7	0.8	1.0	1.2	1.5	2.0
0.01	1.31	1.50	1.95	2.44	3.18	4.35
0.05	1.28	1.45	1.85	2.28	2.92	3.92
0.10	1.25	1.40	1.77	2.16	2.73	3.62
0.25	1.17	1.29	1.55	1.84	2.26	2.91

Construction of GASP

Example1

For a specified β = 0.05, a = 0.8, r = 4, c =2 the value of g can be obtained from table1 as 6. The relevant GASP is n=24,g=6,r=4 and manager's increase the true mean 1.45 times for the specified life.

Explanation

If the consumer fixes β = 0.05 (5 non-confirmative at of 100) the manufacturer has to select 24 items from the lot and allocate them into 6 groups of 4 items each and allow for life test till the time t_0 and the life test is stopped when more than 2 failed items are recorded during the time and the lot is rejected, otherwise the lot is accepted. If the lot is rejected inform the management for corrective action.

Conclusion

In this paper a new procedure for the construction of GASP indexed through Indifference quality level (IQL) using weighted binomial distribution as the base line distribution with Inverse Rayleigh distribution in life testing is presented. The procedure outlined in this paper can be used for other plans also. Tables are also presented for the easy selection of groups and mean ratio by the engineers and additional tables can also be developed depending on the choice of a, α, β, λ and c.

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