



Multi-response optimization of Nd:YAG laser welding using Taguchi method based Grey relational analysis

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ABSTRACT

This paper presents a hybrid optimization approach for the determination of the optimum laser welding process parameters which minimize the weld bead width and maximize weld bead depth together in pulsed Nd:YAG laser welding of thin plates of SS 304 having thickness 2.9 mm. An exhaustive experimental study has been conducted with various process parameters like, pulse width, pulse height, stand-off distance, frequency, welding speed, gas flow angle and gas flow rate. Each of the parameters has three levels. Thus, an orthogonal array L18 has been adopted to accommodate all the above factors at their respective levels. The weld bead width and the weld bead depth are considered as process performances. A multi-response optimization has been carried out by using grey relational analysis (GRA). Also the significant process parameters have been found out for the above process optimization by performing an ANOVA.

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Introduction

In recent years, the use of laser beam welding (LBW) has steadily increased due to continuous demand for precision and accuracy in metal fabrication industry as well as in joining miniature electronic components. LBW has been a versatile process and is capable of welding carbon steels, high-strength low-alloy (HSLA) steels, stainless steel and materials having high strength to weight ratio like, aluminum, titanium and their alloys. It is frequently used in high volume applications, such as in the automotive industry. Due to high cooling rates, cracking is a concern while welding high-carbon steels. The speed of welding is proportional to the amount of power supplied but also depends on the type and thickness of the work pieces. The laser beam provides a concentrated heat source, allowing for narrow, deep welds and high welding rates. These advantages come from its high power density, which make the laser welding one of the keyhole welding processes [1]. The other advantages include high productivity, less residual stress and hence, a low distortion, a low heat affected zone, deep penetration, repeatability, and ease of automation. A robot assisted laser beam welding results in extremely high quality welds. A typical laser welding equipment consists of a laser beam generator, beam-directing optics to transfer the beam to the work and focus it to the needed spot size and power density. For most industrial laser welding applications in which keyhole (deep penetration) welding is required, a laser beam of several kW is focused onto the material surface with a focus spot diameter of approximately 0.1 mm or larger. This results in the power density in a range of 10^6 to 10^7 W/cm² at the beam focus [2]. However, welding quality is strongly characterized by the weld bead geometry. The weld bead geometry plays an important role in determining the mechanical properties of the welded joints. Therefore, the selection of the welding process parameters is very essential for obtaining optimal weld bead geometry [3]. Design of experiment (DOE) and statistical techniques are widely used to optimize

process parameters. Many researches were conducted to identify the optimal process input parameters. The most important input laser welding parameters which would control the welding quality outputs are laser power, welding speed and focus position [4–6]. However, the present paper attempts to determine the optimum process parametric combination for a simple good weld bead. A good weld bead is characterized by a bead geometry having minimum bead width and maximum bead depth (or height). Hence, optimization of this kind of problem falls under the category of multiple objective optimization problems. A host of methods are available under design of experiments to solve these problems. These approaches may be divided mainly into two groups, one group is related to Response Surface Methodology (RSM) and the other group is linked up with Taguchi's Robust Design Methodology. The first group may again be divided into three categories, viz., (a) approach based on overlapping of contour plots (b) approach based on utility functions (also called desirability functions) and (c) dual response system methodology. The second group may also be divided into two categories, as (a) approach based on Taguchi's quadratic loss function and (b) approach based on 'signal to noise ratio' (S/N ratio). However, presently many researchers have employed a hybrid approach, viz., Taguchi Method based Grey Relational Analysis (GRA) effectively and proven its usefulness in various applications [7–11] with regard to multi-response optimization problems. However, in the earlier researches the process parameters considered at a time were a few. In this study an attempt has been made to conduct an extensive study to find out the influences of as many as seven process parameters together and to determine the optimum parametric combination for having an optimum weld bead geometry. Taguchi Method-based GRA has been employed to directly integrate two laser welding quality characteristics, i.e., weld bead width and weld bead depth, and to conduct analysis thereafter.

An Overview of Taguchi Method Based GRA

Optimization of process parameters is the key step in the Taguchi method to obtain high quality at low cost. The optimal process parameters are determined not only to improve quality, but also to be least sensitive to the variation of environmental conditions and other noise factors. Basically, classical process parameter design is complex and difficult to use. A large number of experiments have to be carried out when the number of process parameter increases. To solve this problem, the Taguchi method uses a special class of orthogonal arrays in the design of experiments to study the entire process parameter space, but with a small number of experiments. Taguchi recommends use of the loss function to measure the performance characteristic deviating from the desired value. In Taguchi's robust design, a loss function is defined to calculate the deviation between the experimental value and the desired value. The value of the loss function is further transformed into a signal-to-noise (S/N) ratio. Usually, there are three categories of the performance characteristic in the analysis of the S/N ratio, that is, the lower-the-better, higher-the-better, and nominal-the-better. The S/N ratio for each level of process parameter is computed based on the S/N analysis as per Taguchi's robust design methodology. Regardless of the category of the performance characteristic, the larger S/N ratio corresponds to a better performance characteristic. Therefore, the optimal level of a process parameter is the level with the highest S/N ratio of that process parameter. Thus, parametric optimization is done combining optimum levels of all the process parameters under study. This is true for the optimization of a single performance characteristic.

However, optimization of multiple performance characteristics is different from that of a single performance characteristic. The higher S/N ratio for one performance characteristic may correspond to a lower S/N ratio for another. Therefore, the overall evaluation of the S/N ratio is required for the optimization of multiple performance characteristics. The usual recommendation for the optimization of a process with multiple performance characteristics is left to the engineering judgment and verified by confirmation experiment [12]. Normally, the problem is tackled by using desirability function and/or weighting method. In weighting method, a suitable weighting factor is assigned to the normalized measure of a performance characteristic in percent indicating the importance or desirability of that performance characteristic for a particular application. Grey relational analysis (GRA) is such a method of optimization of multiple performance characteristics using weighting method.

The grey system theory proposed by Deng in 1982 [13] has been proven to be useful for dealing with poor, incomplete and uncertain information. The grey relational analysis is based on the grey system theory and can be used to solve complicated inter-relationships among multiple performance characteristics effectively. However, the first step of the grey relational analysis is the grey relational generation [8]. During this step, all the performance characteristics are normalized in the range between zero and one. Next, the grey relational coefficient is calculated from the normalized data to express the relationship between the desired and actual normalized performance values. Then, the grey relational grade is computed by assigning a suitable weighting factor (in percent) to the grey relational coefficient corresponding to each performance characteristic. Overall evaluation of the multiple performance characteristics is, thus, based on the grey relational grade. As a result, optimization of

the complicated multiple performance characteristics can be converted into optimization of a single grey relational grade. The optimal level of the process parameters is the level with the highest grey relational grade. Furthermore, a statistical analysis of variance (ANOVA) is performed to find which process parameters are statistically significant. With the grey relational analysis and statistical ANOVA, the optimal combination of the process parameters can be predicted. Finally, a confirmation experiment is conducted to verify the optimal process parameters obtained from the analysis.

Planning and Designing Experimental Study

The experiments were conducted on JK 650HP laser generator (GSI, UK) integrated with ABB IRB 1410 robot is shown in fig.1. This machine uses a Nd:YAG laser of wavelength (λ) = 1.06 μ m with nominal output power of 600 W at pulsed mode. An argon gas jet emerges from the side nozzle, which makes a fixed angle with the laser beam.

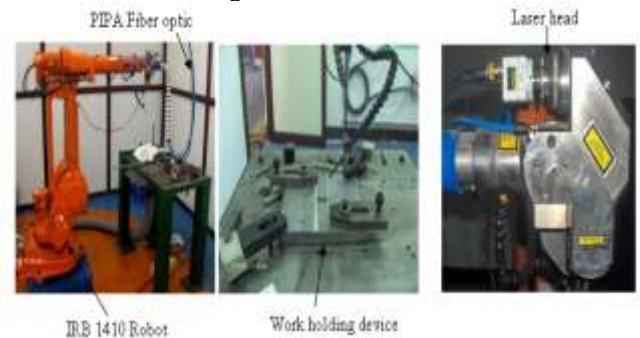


Fig. 1 JK650 laser generator and ABB Industrial Robot 1410 with welding head

The laser beam was focused using a 300 mm focal length lens. Stainless steel plates (SS 304) having 2.9 mm thickness were used as work piece material. The chemical composition of SS 304 is provided in Table 1.

A large numbers of independent parameters control the laser welding process. Some preliminary experiments were conducted in order to study the average influence of various parameters on the process performances. Thus, seven parameters such as, laser pulse width, laser pulse height, frequency, stand-off distance, gas flow angle, gas flow rate and welding speed were considered as the control parameters. Each of the parameters has three levels. An orthogonal array L18 has been adopted to accommodate all the above factors at their respective levels. The summary of the experimental conditions is given in Table 2. The weld bead width and weld bead height have been considered as the process performances. Each test piece was measured five times and an average value has been taken as a more accurate reading.

Analysis and Discussion on the Experimental Results

In this section, the optimization procedure using Taguchi method-based GRA is discussed and implemented for the laser welding problem.

Experimental results of the orthogonal array L18

As per L18 experimental layout, eighteen experimental runs of the orthogonal array have been performed and the welding bead width and welding bead height have been measured for each specimen in SEM (scanning electron microscope – HITACHI S-3000N). A few specimens are shown in Fig. 2. The experimental results along with the experimental layout have been displayed in Table 3. Next, the GRA is used to integrate two quality characteristics into a single output grade through a series of processes.

The Implemented Procedures of the GRA in this Study

In order to synthesize laser welding parameters and their levels, an orthogonal array L18 is adopted in this study. The welding depths and the weld bead widths are normalized within the range of zero to one (0–1), which is called grey relational generating. According to Taguchi method, the weld bead depths are converted by using the higher-the-better type quality characteristic, and weld bead widths are converted using the lower-the-better type quality characteristic. Further, the grey relational coefficient that describes the relation between the ideal and actual normalized values is calculated. Then, using a weighting method the grey relational grade of each experiment is computed. From the grey relational grades of all experiments, the optimal welding combination with the highest grade is determined. The effect of each welding parameter is found from the responses of the grey relational grade. Finally, combining the best level of each parameter that has the highest grade is used to obtain an optimal welding combination. Furthermore, the analysis of variance (ANOVA) is applied to determine the effect of each welding parameter on the welding quality characteristics. The robustness of the optimal welding combination that produces the best welding quality characteristics was tested by performing confirmation experiments.

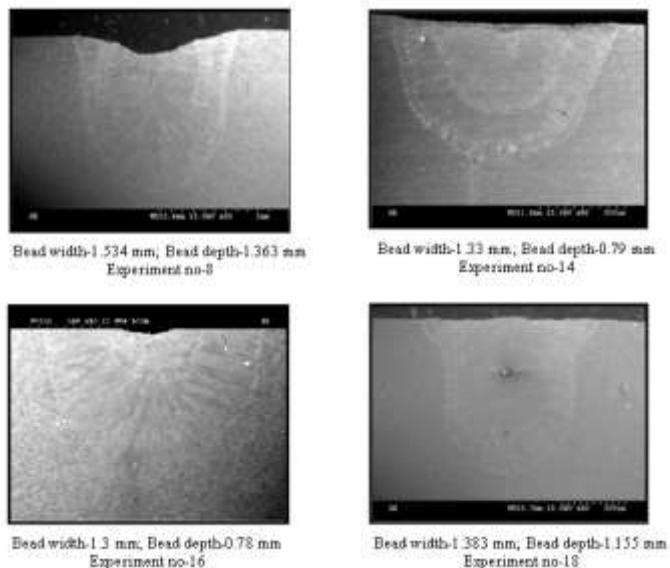


Fig. 2 The weld bead shape of a few experimental runs

From the above discussions, the implemented procedures to optimize the welding of a SS 304 strip by using GRA based on the Taguchi method are described as follows [9]:

1. Identifying the laser welding parameters and performance characteristics to be evaluated.
2. Determining the levels of the welding parameters.
3. Choosing an appropriate orthogonal array based on the welding parameters and their levels, and performing experiments with the orthogonal array to obtain welding quality characteristics.
4. Normalizing the experimental results of quality characteristics.
5. Calculating the grey relational, the grey relational coefficient, and the grey relational grade.
6. Analyzing the experimental results using the grey relational grade and statistical ANOVA.
7. Selecting the optimum levels of welding parameters.

8. Performing confirmation experiments to ensure the robustness of the optimal welding combination with the best welding quality characteristics.

Grey relational generating

In Grey Relational Analysis (GRA), the first step is to perform grey relational generating (GRG) process, i.e., normalization of the original data of a quality characteristic is done within a range of 0–1. The GRG is defined as the deviation between the experimental value and the ideal value. Three general types of the grey relational generating calculation include the higher-the-better, the lower-the-better, and the nominal-the-better. As a high welding bead depth is required in a good welding, the ‘higher-the-better type’ normalizes the welding bead depth (W_H). The normalized values of the welding bead depth (X_{ij}) can be computed by Eq. 1,

$$X_{ij} = \frac{Y_{ij} - \min Y_j}{\max Y_j - \min Y_j}, \quad i = 1 - 18, j = 1 \text{ for } W_H \quad (1)$$

Where Y_{ij} is the j -th welding quality characteristic of the i -th experiment of an orthogonal array for weld bead height, $\max Y_j$ and $\min Y_j$ are the maximum and minimum values of the j -th welding quality characteristic as weld bead height ($j = 1$). On the contrary, the ‘lower-the-better type’ is used to normalize the welding bead widths. Thus, the normalized values of widths X_{ij} are calculated using Eq. 2,

$$X_{ij} = \frac{\max Y_j - Y_{ij}}{\max Y_j - \min Y_j}, \quad i = 1 - 18, j = 2 \text{ for } W_W \quad (2)$$

Where Y_{ij} is the j -th welding quality characteristic of the i -th experiment of an orthogonal array for weld bead width, $\max Y_j$ and $\min Y_j$ are the maximum and minimum values of the j -th welding quality characteristic as weld bead width ($j = 2$). Table 4 shows the normalized values of two welding quality characteristics for SS 304 material. From Table 4, it is observed that a good welding with least weld bead width and highest weld bead depth is converted to the largest normalized value, ideally equal to one. By contrast, the shallowest welding depth and widest weld bead width is converted to the lowest normalized value approaching or equal to zero.

Grey relational coefficients

The grey relational coefficient (ξ_{ij}) is defined as the relation between the ideal and actual normalized value. The grey relational coefficient for SS 304 can be calculated by Eq. 3,

$$\xi_{ij} = \frac{\Delta_{\min} + \zeta \times \Delta_{\max}}{\Delta_{ij} + \zeta \times \Delta_{\max}}, \quad i = 1 - 18; j = 1, 2 \quad (3)$$

Where,

$$\Delta_{\min} = \min_{ij} |x_j - x_{ij}|, \quad \Delta_{\max} = \max_{ij} |x_j - x_{ij}|, \quad \Delta_{ij} = |x_j - x_{ij}|, \quad x_j$$

is an ideal normalized value for the j -th welding quality characteristic of SS 304. ‘ ζ ’ is the distinguishing or identification coefficient, and its value lie between zero to one. ‘ ζ ’ has been assumed 0.5 in this paper.

Calculation of grey relational grade

Next, the grey relational coefficients of two welding quality characteristics of SS304 materials are directly integrated to determine a single grey relational grade by utilizing a weighting

method. The grey relational grade (γ_i) for the i th experiment can be defined in Eq. 4,

$$\gamma_i = \frac{1}{2} \sum_{j=1}^6 W_j \xi_{ij} \quad , \quad i = 1 - 18 \quad (4)$$

Where, W_j is the weighting factors of j -th welding quality characteristic for SS304 material. In general, a complete welding is required at first. The width of a welding bead is generally less than the weld depth. The importance of laser welding quality characteristics in decreasing order is the weld bead depth, the weld bead width. In this paper, the weighting factor for both the weld bead depth and weld bead width are taken as 50%. Next, the grey relational grade (Y_i) for each experiment of the orthogonal array has been obtained and as shown in Table 5. From Table 5, it is seen that a complete welding state has a higher grade than an incomplete welding state, and a deeper welding depth has a higher grade among the complete welding states. In addition, the lower weld bead width resulted in a lower grade in the same welding state.

The effect of each level of each welding parameter on the welding quality characteristics is obtained by using the grey relational grades of eighteen experiments. For example, if r_{A1} is the effect of the control factor A at level 1, then it is calculated by an average value from the grey relational grades of experiment no. 1 to 6 given in Eq. (5),

$$r_{A1} = \frac{r_1 + r_2 + r_3 + r_4 + r_5 + r_6}{6} \quad (5)$$

The effect of each level of each parameter is calculated in a similar way, and the results are listed in Table 6 and shown in Fig. 3. The best level of each parameter that has the highest grey relational grade is compared with other levels, which indicates the most significant effect on the welding quality characteristics.

Analysis of variance (ANOVA)

The grey relational grades (γ_i) of Table 5 can be utilized to perform the ANOVA and to investigate the effect of each welding parameter on the welding quality characteristics. The calculations of related parameters of ANOVA are given in Eq. (6);

$$\bar{Y} = \frac{1}{18} \sum_{i=1}^{18} y_i \quad , \quad S_T = \sum_{i=1}^{18} (y_i - \bar{y})^2 \quad , \quad S_B = \sum_{i=1}^3 (y_{Aq} - \bar{y})^2 \quad ,$$

$$S_e = S_T - S_B - S_C - S_D - S_E - S_F - S_G - S_H \quad , \quad V_B = \frac{S_B}{f_B} \quad , \quad F_B = \frac{V_B}{V_e} \quad , \quad \sigma_B = \frac{S_B}{S_T} \times 100\% \quad (6)$$

where \bar{Y} is the mean of sums of y_i , S_T is the sum of squares of the variance between y_i and \bar{Y} , S_B is the sum of squares of the variance between each level of control factor B and \bar{Y} , S_C is the sum of squares of the variance between each level of control factor C and \bar{Y} , q is the q th level of control factor, S_e is the error between S_T and the sum of squares of the variance of all control factors, F_B is the degree of freedom of control factor B, V_B is the variance of control factor B, V_e is the variance of control factor e, F_B is the F-test of control factor B, and σ_B is the percentage contribution of control factor B. The calculations of other control factors follow in a similar way, and the analyzed results of ANOVA are given in Table 7.

Determining optimum factor settings

The best values of the various levels of laser welding process parameters are identified where the grey relational grades have been maximized. Based on the plot of grey relational grades as depicted in Fig. 3, the optimal combination of the parametric setting is found out as B3C2D1E1F1G3H1, i.e., laser pulse width at 6 ms, laser pulse height at 25%, frequency at 30 Hz, feed at 7 mm/s, stand-off distance at (-4mm), gas flow rate at 12L/min and gas flow angle at 30 degree.

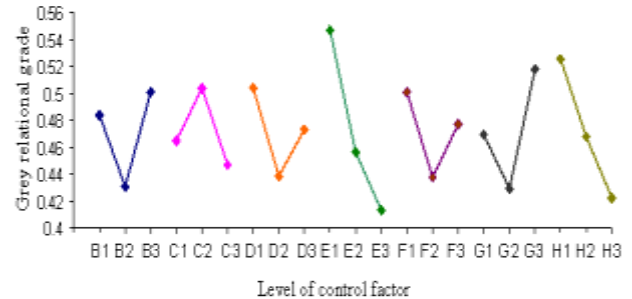


Fig. 3 Plot of grey relational grades

Confirmation experiments

To confirm the robustness of the foregoing parametric design, a confirmation experiment was performed with the factors along with their levels obtained in the optimal combination (B3C2D1E1F1G3H1). Thus, for the confirmation experiment laser pulse width (6 ms), laser pulse height (25%), frequency (30 Hz), feed (7 mm/s), stand-off distance(-4mm), gas flow rate (12 lit/min) and gas flow angle (30°) were used. It has been found that the weld bead depth has increased from 1.4 mm to 1.52 mm and weld width has reduced from 1.534 mm to 1.46 mm. The result shows that adoption of the grey-based Taguchi method leads to an improvement of the weld quality.

Conclusions

In this study, Taguchi method-based grey relational analysis has been used to determine the optimum laser welding parameters for butt welding of SS 304 strips with consideration of multiple quality characteristics. The application of this methodology directly integrates the multiple quality characteristics (i.e., welding bead height and welding bead depth, in this study) into a single performance characteristic called grey relational grade. The grade obtained for each experiment can immediately reflect the actual welding results in terms of state and quality of the weld. Thus, an optimal combination of laser welding parameters has been found out as well as the effect of each individual welding parameter has been obtained. From the results of ANOVA the contributions of welding parameters on the welding quality characteristics in decreasing order are the welding speed, gas flow angle, gas flow rate, pulse width, pulse height, stand of distance and frequency. The total contributions of the seven welding parameters are more than 95%. This ensures that these seven parameters are the main factors in determining the welding quality. Among these experiments, experiment no 1 and 11 have no weld condition as the energy is less than 3J. Experiment no 12 has less weld bead depth with lowest grade and experiment no 8 has highest weld bead depth with highest grade. By using GRA approach, the weighting factor for multiple quality characteristics can directly be adjusted for quality requirements as desired.

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References

- Zhang YM, Kovacevic R, Li L. Characterization and realtime measurement of geometrical appearance of the weld pool. *Int J Mach Tools Manuf* 1996;36(7):799–816.
- Dawes C. *Laser welding*. New York (NY): Abington Publishing; 1992.
- Tarng YS, Yang WH. Optimisation of the weld bead geometry in gas Tungsten Arc welding by the Taguchi Method. *J Adv Manuf Technol* 1998;14:549–54.
- Pan LK, Wang CC, Hsiao YC, Ho KC. Optimization of Nd: YAG laser welding onto magnesium alloy via Taguchi analysis. *Opt Laser Technol* 2005;37(1):33–42.
- Lee H-K, Han H-S, Son K-J, Hong S-B. Optimization of Nd: YAG laser welding parameters for sealing small titanium tube ends. *Mater Sci Eng A* 2006;415(1–2):149–55.
- Tarng YS, Juang SC, Chang CH. The use of grey-based Taguchi methods to determine submerged arc welding process parameters in hard facing. *J Mater Process Technol* 2002;128:1–6.
- C. L. Lin, Use of the Taguchi Method and Grey Relational Analysis to Optimize Turning Operations with Multiple

Performance Characteristics, Materials And Manufacturing Processes, Vol. 19, No. 2, pp. 209–220, 2004.

- Ulas Caydas, Use of relational analysis to determine optimum laser cutting parameters with multi performance characteristics, *Optics & laser technology* 40 (2008) 987-994.
- Lin JL, Lin CL. The use of the orthogonal array with grey relational analysis to optimize the electrical discharge machining process with multiple performance characteristics. *International Journal of Machine Tools & Manufacture* 2002; 42:237–44.
- Ming-Jong Tsai, The use of grey relational analysis to determin laser cutting parameters for QFN packages with multiple performance characteristics. *Optics & Laser Technology* 41 (2009) 914-921.
- Raghavendra Rao, Vinod Yadava. Multi-objective optimization of Nd:YAG laser cutting of thin superalloy sheet using grey relational analysis with entropy measurement, *Optics & Laser Technology* 41 (2009) 922–930.
- Phadke M.S. Phadke, *Quality Engineering using Robust Design*, Prentice Hall, Englewood Cliffs, New Jersey, 1989.
- J. Deng, *Introduction to grey theory*, *J. Grey Syst.* 1 (1) (1989) 1–24.

Table 1 Nominal chemical composition of 304 SS

C	Si	Mn	P	S	Cr	Ni	N	Mo	Fe
0.08	0.75	2.0	0.045	0.03	18.01	7.99	0.098	0.26	Balance.

Table 2 Process variables with their levels

Variables	Code	Unit	Level 1	Level 2	Level 3
Pulse width	B	ms	2	4	6
Frequency	C	Hz	20	30	40
Pulse Height	D	%	20	25	30
Welding Speed	E	mm/s	7	10	13
Stand of Distance	F	mm	-4	-2	0
Gas flow rate	G	lit/min	4	8	12
Flow Angle	H	degree	30	45	60

Table 3 L18 orthogonal array with coded value of levels and experimental results

Expt. No.	Control Factors								Weld bead width (mm)	Weld bead height (mm)
	A	B	C	D	E	F	G	H		
1	1	1	1	1	1	1	1	1	No weld	
2	1	1	2	2	2	2	2	2	1.228	0.322
3	1	1	3	3	3	3	3	3	1.12	0.358
4	1	2	1	1	2	2	3	3	1.083	0.272
5	1	2	2	2	3	3	1	1	1.142	0.52
6	1	2	3	3	1	1	2	2	1.428	0.855
7	1	3	1	2	1	3	2	3	1.108	0.756
8	1	3	2	3	2	1	3	1	1.534	1.363
9	1	3	3	1	3	2	1	2	1.25	0.618
10	2	1	1	3	3	2	2	1	1.098	0.218
11	2	1	2	1	1	3	3	2	No weld	
12	2	1	3	2	2	1	1	3	1.316	0.331
13	2	2	1	2	3	1	3	2	1.094	0.33
14	2	2	2	3	1	2	1	3	1.33	0.792
15	2	2	3	1	2	3	2	1	0.832	0.342
16	2	3	1	3	2	3	1	2	1.3	0.788
17	2	3	2	1	3	1	2	3	1.253	0.573
18	2	3	3	2	1	2	3	1	1.383	1.155

Table 4 Data processing of the experimental result

Experiment no	Grey relational generating		Grey relational coefficient	
	Weld width	Weld depth	Weld width	Weld depth
Ideal sequence	1	1	1	1
1	1.0000	0.0000	1.0000	0.3333
2	0.1995	0.2362	0.3845	0.3956
3	0.2699	0.2627	0.4065	0.4041
4	0.2940	0.1996	0.4146	0.3845
5	0.2555	0.3815	0.4018	0.4470
6	0.0691	0.6273	0.3494	0.5729
7	0.2777	0.5547	0.4091	0.5289
8	0.0000	1.0000	0.3333	1.0000
9	0.1851	0.4534	0.3803	0.4777
10	0.2842	0.1599	0.4113	0.3731
11	1.0000	0.0000	1.0000	0.3333
12	0.1421	0.2428	0.3682	0.3977
13	0.2868	0.2421	0.4121	0.3975
14	0.1330	0.5811	0.3658	0.5441
15	0.4576	0.2509	0.4797	0.4003
16	0.1525	0.5781	0.3711	0.5424
17	0.1832	0.4204	0.3797	0.4631
18	0.0984	0.8474	0.3567	0.7662

Table 5 Grey Relational grades and its order

Experiment no.	Grey relational Grade	Order
1	0.6667	-
2	0.3901	17
3	0.4053	13
4	0.3995	15
5	0.4244	11
6	0.4612	6
7	0.4690	5
8	0.6667	1
9	0.4290	10
10	0.3922	16
11	0.6667	-
12	0.3830	18
13	0.4048	14
14	0.4549	8
15	0.4400	9
16	0.4567	7
17	0.4214	12
18	0.5615	4

Table 6 Response table for Grey Relational grades of each control parameter

Grey Relational grade						
Factor		Level 1	Level 2	Level 3	Delta	Rank
B	(Pulse width (ms))	0.48397	0.430811	0.500709	0.0699	4
C	(Frequency (Hz))	0.46482	0.504025	0.446645	0.0574	7
D	(Pulse Height (%))	0.50388	0.438782	0.472829	0.0651	5
E	(W. speed (mm/s))	0.54665	0.455990	0.412852	0.1338	1
F	(SOD (MM))	0.50062	0.437863	0.477008	0.0628	6
G	(GFR (Lit/Min))	0.46912	0.428969	0.517404	0.0884	3
H	(Flow Angle(Degree))	0.52523	0.468075	0.422189	0.1030	2

Table 7 Results of the ANOVA

Factor	Sum of Squares(S)	Degree of freedom(f)	Mean squares(V)	F-Value(F)	Prob>F	Contribution (%)
B	0.0160	2	0.00799	8.846239	0.0552	9.672
C	0.0103	2	0.00516	5.711195	0.0949	6.244
D	0.0127	2	0.00636	7.041181	0.0736	7.698
E	0.0560	2	0.02798	30.97218	0.0099	33.863
F	0.0121	2	0.00603	6.672786	0.0786	7.296
G	0.0235	2	0.01176	13.02171	0.0332	14.237
H	0.0320	2	0.01599	17.69815	0.0218	19.350
Error(e)	0.0027	3	0.00090	-	-	1.640
Total(T)	0.1653	17				100.000