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Determination of properties of transversely isotropic lamina using micromechanics approach

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ABSTRACT

Composites are finding increased use in structural applications, in particular for aerospace and automotive purposes. Fiber reinforced composite possess high strength and stiffness. Some of these materials perform equally well or better than many traditional metallic materials. In addition, fatigue strength-to-weight ratios as well as fatigue damage tolerance of many composite laminates are excellent.

To analyze metallic structures, properties of metals are easily available, but for composite structures properties of composite material are not readily available. Composite material is nothing but a laminate made from number of different lamina, and the properties of laminate depends on properties of lamina.

The material properties of composite are required for carrying out stress analysis and fatigue analysis which in turn predicts the life of component. Objective of present work is to study the behavior of composite materials. This investigation deals with lamina composed of polymer matrix and carbon fibers. The aim of this study is to determine following properties.

- Elastic properties, thermal properties and strength properties of transversely isotropic lamina by all methods of Micromechanics.
- Properties of orthotropic lamina using Method of Cells.
- Verifying the results predicted by Method of Cells with the other micromechanics methods like Composite Cylinder Assemblages (CCA) method, Rule of Mixture, Halpin-Tsai, Chamis method and Zing-ming Huang method.

Introduction

Micromechanics is the study of composite material behavior wherein the material is assumed homogeneous and the effects of the constituent materials are detected only as averaged apparent properties of the composite materials.

Methodology:

Fiber reinforced composites are often selected for weight-critical structural applications because of their high specific stiffness and strength. For determining the properties of transversely isotropic lamina following methods are used.

Mechanics of Material Approach (Rule of Mixture)

The relevant elastic properties are obtained as given below

a)Young's modulus of elasticity:

The first modulus of the composite material is determined, when subjected to loading along fiber direction as below,

$$E_l = E_f V_f + E_m V_m \qquad \text{---- (Eq.1)}$$

where,

E_l=Young's modulus of elasticity of lamina in longitudinal direction.

E_f=Young's modulus of fiber.

E_m=Young's modulus of matrix.

V_f=Fiber volume fraction

V_m=Matrix volume fraction

The apparent Young's modulus of the composite material in the direction transverse to the fiber, with assumption the same transverse stress is assumed to be applied to both the fiber and mat

$$E_t = \frac{E_f E_m}{V_m E_f + V_f E_m} \qquad \text{---- (Eq.2)}$$

Where,

Et=Young's modulus of lamina in transverse direction.

b) Poisson's Ratio in L-T plane:

The major Poisson's ratio is obtained by the same approach that used in analysis of E1

$$\nu_{lt} = \nu_f V_f + \nu_m V_m \qquad \text{---- (Eq.3)}$$

Where,

ν_{12} = Poisson's ratio of lamina in one two plane

ν_{ltf} = Fiber longitudinal Poisson's ratio

ν_m = Matrix Poisson's ratio

V_m = Matrix Volume fraction

V_f = Fiber Volume fraction

c) Shear modulus in L-T plane:

c) The Inplane shear modulus is determined as

$$G_{lt} = \frac{G_m G_f}{V_m G_f + V_f G_m} \qquad \text{---- (Eq.4)}$$

where,

G_{lt}=Shear modulus of lamina in L-T plane

G_m=Shear modulus of matrix

G_f=Shear modulus of fiber

V_f=Fiber volume fraction

V_m=Matrix volume fraction

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Halpin-Tsai Method

This is an interpolation method which is an approximate representation of more complicated micromechanics results. The relevant elastic properties are obtained as given below are presented by Jones [1], the expressions for axial Young;s modulus (E1) and axial Poisson's ratio are generally accepted results of rule of mixture. The Halpin-Tsai equations are equally applicable to fiber, ribbon, or particulate composite:

a)Young's modulus of elasticity:

The first modulus of the composite material is determined in the fiber direction when subjected to loading along fiber direction.

$$E_l = E_f V_f + E_m V_m \quad \text{---- (Eq.5)}$$

Young's modulus of elasticity:

The apparent Young's modulus of the composite material in the direction transverse to the fiber, is as given below

$$E_t = \frac{(1+ \xi \times \eta \times V_f) \times E_m}{1-\eta \times V_f} \quad \text{---- (Eq.6)}$$

Where,

$$\eta = \frac{(E_{tf} / E_m) - 1}{(E_{tf} / E_m) + \xi}$$

Constant $\xi =$ Measure of fiber reinforcement of composite material

$$\xi = 1 + 40 V_f^{10}$$

b) Poisson's Ratio in L-T plane:

The major Poisson's ratio is obtained by the same approach that used in analysis of E1

$$\nu_{lt} = \nu_f \mathrm{V}_f + \nu_m \mathrm{V}_m \quad \text{---- (Eq.7)}$$

c) Shear modulus in L-T plane:

The in plane shear modulus is determined as

$$G_{lt} = \frac{(1 + \xi \times \eta_2 \times V_f) \times G_m}{1 - \eta \times V_f} \quad \text{---- (Eq.8)}$$

Where

$$\eta_2 = \frac{(G_f / G_m) - 1}{(G_f / G_m) + \xi}$$

Composite Cylinder Assemblages (CCA)

Assumptions made in CCA are as follows:

1) All fibers have same radii.
2) Perfect bond between fiber and matrix.
3) Neglect the matrix between cylinder and assuming axisymmetric loading.
4) Fibers are linear elastic transversely isotropic material, and matrix is isotropic material.
5) Fiber properties like Young's modulus, shear modulus, Poisson's ratio, coefficient of thermal expansion and density in longitudinal and transverse direction are: -- $E_{lf}, E_{tf}, G_{ltf}, \nu_{ltf}, \nu_{ttf}, \alpha_{lf}, \alpha_{tf}, \rho_f$

Matrix Properties: -- $E_m, \nu_m, \alpha_m, \rho_m$

Fiber Volume Fraction: --V_f

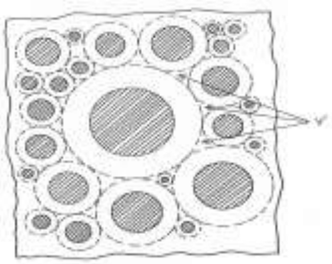

Figure .1Composite Cylinder Assemblage [2]

a)Matrix volume fraction:

$$V_m = 1 - V_f$$

b)Fiber transverse Poisson's ratio:

$$\nu_{ttf} = \frac{E_{tf}}{2G_{ttf}} - 1$$

c) Matrix bulk modulus:

$$K_{bm} = \frac{E_m}{3 \times (1 - 2\nu_m)}$$

d)Fiber transverse bulk modulus:

$$K_{tbf} = \frac{E_{tf}}{3(1 - 2\nu_{ttf})}$$

e) Matrix shear modulus:

$$G_m = \frac{E_m}{2(1 + \nu_m)}$$

f)Young's modulus of elasticity of lamina :

The first modulus of the composite material is determined in the fiber direction when subjected to loading along fiber directionis as below.

$$E_l = E_m V_m + E_{lf} V_f + \frac{4(\nu_{ltf} - \nu_m)^2 V_m V_f}{\left(\frac{V_m}{K_{tbf}} + \frac{V_f}{K_{bm}} + \frac{1}{G_m} \right)} \quad \text{---- (Eq.9)}$$

Young's modulus of elasticity

The apparent Young's modulus of the composite material in the direction transverse to the fiber, with assumption the same transverse stress is assumed to be applied to both the fiber and matrix.

$$E_t = \frac{4K_{tb} G_{t(+)}}{K_{tb} + m \times G_{t(+)}} \quad \text{---- (Eq.10)}$$

g) Shear modulus of lamina (indicates upper bonds):

$$G_{t(+)} = G^m \frac{[1 + \alpha(V_f)^3](\rho + \beta_1 V_f) - 3V_f (V_m)^2 \beta_1^2}{[1 + \alpha(V_f)^3](\rho - V_f) - 3V_f (V_m)^2 \beta_1^2}$$

h) Transverse bulk modulus of lamina:

$$K_{tb} = \frac{K_{bm}(K_{tbf} + G_m)V_m + K_{tbf}(K_{bm} + G_m)V_f}{(K_{tbf} + G_m)V_m + (K_{bm} + G_m)V_f} \quad \text{----(Eq.11)}$$

Constant (m)

$$m = 1 + \frac{4K_{tb}\nu_{ltf}}{E_1}$$

Constant β_1

$$\beta_1 = \frac{1}{3 - 4\nu_m}$$

Constant β_2

$$\beta_2 = \frac{1}{3 - 4\nu_f}$$

Constant α

$$\alpha = \frac{\beta_1 - \gamma\beta_2}{1 + \gamma\beta_2}$$

Constant ρ

$$\rho = \frac{\gamma + \beta_1}{\gamma - 1}$$

Constant γ

$$\gamma = \frac{G_{ttf}}{G_m}$$

i)Axial Poisson's ratio:

The major Poisson's ratio is obtained by the same approach that used in analysis of E1

$$v_{12} = v_m V_m + v_{ltf} V_f + \frac{\left(v_{ltf} - v_m\right)\left(\frac{1}{K_{bm}} - \frac{1}{K_{tbf}}\right) V_m V_f}{\left(\frac{V_m}{K_{tbf}} + \frac{V_f}{K_{bm}} + \frac{1}{G_m}\right)} \quad \text{---- (Eq.12)}$$

j)Transverse Poisson's ratio:

$$v_{23} = \frac{K_{tb} - mG_{t(-)}}{K_{tb} + mG_{t(+)}} \quad \text{---- (Eq.13)}$$

Where,

$G_{t(-)}$ = Lower transverse shear modulus

$$G_{t(-)} = G_m \left[1 + \frac{V_f}{\left(\frac{1}{\gamma - 1} + \frac{V_m}{1 + \beta_1}\right)}\right]$$

k) Axial shear modulus of lamina:

The in plane shear modulus is determined as

$$G_{12} = G_m \frac{G_m V_m + G_{ltf}(1 + V_f)}{G_m(1 + V_f) + G_{ltf} V_m} \quad \text{---- (Eq.14)}$$

l) Transverse shear modulus of lamina:

The 2-3 plane shear modulus is determined as

$$G_{23} = G_m \frac{[1 + \alpha (V_f)^3](\rho + \beta_1 V_f) - 3V_f (V_m)^2 \beta_1^2}{[1 + \alpha (V_f)^3](\rho - V_f) - 3V_f (V_m)^2 \beta_1^2}$$

m) Axial thermal expansion coefficient of lamina:

$$\alpha_1 = \overline{\alpha_l} + \frac{\alpha_{lf} - \alpha_m}{\left(\frac{1}{K_{tbf}} - \frac{1}{K_{bm}}\right)}\left(\frac{3(1 - 2v_{12})}{E_1} - \overline{\frac{1}{K_l}}\right) \quad \text{---- (Eq.15)}$$

Where,

Constant $\overline{\alpha_l}$

$$\overline{\alpha_l} = \alpha_m V_m + \alpha_{lf} V_f$$

Constant $\overline{\frac{1}{K_l}}$

$$\overline{\frac{1}{K_l}} = \frac{V_m}{K_{bm}} + \frac{V_f}{K_{tbf}}$$

n)Transverse thermal expansion coefficient:

$$\alpha_2 = \bar{\alpha} + \frac{\alpha_{tf} - \alpha_m}{\left(\frac{1}{K_{tbf}} - \frac{1}{K_{bm}}\right)}\left(\frac{3}{2K_{tb}} - \frac{3(1 - 2v_{23})v_{23}}{E_1} - (\overline{\frac{1}{K_t}})\right) \quad \text{---- (Eq.16)}$$

Where,

Constant $\overline{\alpha_t}$

$$\overline{\alpha_t} = \alpha_m V_m + \alpha_{tf} V_f$$

Constant $\overline{\frac{1}{K_t}}$

$$\overline{\frac{1}{K_t}} = \frac{V_m}{K_{bm}} + \frac{V_f}{K_{tbf}}$$

Chamis Method

Chamis [4, 5] method is used to calculate lamina elastic properties. Fiber properties, matrix properties and fiber volume fraction are input to this method. The elastic properties can be found out as:

a) Calculation of matrix volume fraction:

$$V_m = 1 - V_f$$

Where,

V_m = Matrix Volume fraction

V_f = Fiber Volume fraction

b) Calculation of fiber transverse Poisson's ratio:

$$v_{ttf} = \frac{E_{tf}}{2G_{ttf}} - 1$$

c) Calculation of matrix shear modulus:

$$G_m = \frac{E_m}{2(1 + v_m)}$$

d) Calculation axial Young's modulus of Lamina:

$$E_1 = E_{lf} V_f + E_m V_m \quad \text{---- (Eq.17)}$$

e) Calculation of Poisson's ratio of Lamina in one two plane:

$$v_{12} = v_{ltf} V_f + v_m V_m \quad \text{---- (Eq.18)}$$

f)Calculation of Poisson's ratio of lamina in two three plane:

$$v_{23} = \frac{E_2}{2G_{23}} - 1 \quad \text{----(Eq.19)}$$

Where,

v_{23} = Poisson's ratio of lamina in two three plane

E_2 = Young's modulus of lamina in direction two

G_{23} = Shear modulus of lamina in two three plane

g)Calculation of transverse Young's modulus of lamina:

$$E_2 = \frac{E_m}{1 - \sqrt{V_f}\left(1 - E_m / E_{tf}\right)} \quad \text{---- (Eq.20)}$$

h) Calculation of shear modulus or modulus of rigidity of lamina in l-t plane:

$$G_{12} = \frac{G_m}{1 - \sqrt{V_f}\left(1 - G_m / G_{ltf}\right)} \quad \text{---- (Eq.21)}$$

i) Calculation of shear modulus or modulus of rigidity of lamina in t-t plane:

$$G_{23} = \frac{G_m}{1 - \sqrt{V_f}\left(1 - G_m / G_{ttf}\right)} \quad \text{----(Eq.22)}$$

J) Calculation of longitudinal tensile strength of lamina:

$$X_t = V_f X_{lf} \quad \text{---- (Eq.23)}$$

Where,

X_t = Tensile strength of lamina in direction one

V_f = Fiber volume fraction

X_{lf} = Fiber longitudinal tensile strength

k) Calculation of transverse tensile strength of lamina:

$$Y_t = \left[1 - \left(\sqrt{V_f} - V_f\right)\left(1 - E_m / E_{tf}\right)\right] X_{tm} \quad \text{---- (Eq.24)}$$

Where,

Y_t = Tensile strength of lamina in direction two

V_f = Fiber volume fraction

E_m = Matrix Young's modulus

E_{tf} = Fiber transverse Young's modulus

X_{tm} = Matrix tensile strength

l) Calculation of longitudinal compressive strength of lamina:

$$X_c = V_f X_{cf} \quad \text{---- (Eq.25)}$$

Where,

X_c = Compressive strength of lamina in direction one

V_f = Fiber volume fraction

X_{cf} = Compressive strength of fiber

m) Calculation of transverse compressive strength:

$$Y_c = \left[1 - \left(\sqrt{V_f} - V_f\right)\left(1 - \frac{E_m}{E_{tf}}\right)\right] X_{cm} \quad \text{---- (Eq.26)}$$

Where,

Y_c = Compressive strength of lamina in direction two

V_f = Fiber volume fraction

E_m = Matrix Young's modulus

E_{tf} = Fiber transverse Young's modulus

X_{cm} = Matrix compressive strength

n)Calculation of axial shear stress in one two plane:

$$S_{12} = \left[1 - \left(\sqrt{V_f} - V_f\right)\left(1 - \frac{G_m}{G_{ltf}}\right)\right] S_m \quad \text{---- (Eq.27)}$$

Where,

S_{12} = Shear strength of lamina in one two plane

V_f = Fiber volume fraction

G_m = Matrix shear modulus

G_{ltf} = Fiber longitudinal shear modulus

S_m = Matrix shear strength

o) Calculation of transverse shear strength:

$$S_{23} = \left[\frac{1 - \sqrt{V_f}\left(1 - \frac{G_m}{G_{ttf}}\right)}{1 - V_f\left(1 - \frac{G_m}{G_{ttf}}\right)}\right] S_m \quad \text{---- (Eq.28)}$$

Where,

S_{23} = Shear strength of lamina in two three plane

V_f = Fiber volume fraction

G_m = Matrix shear modulus

G_{ttf} = Fiber transverse shear modulus

S_m = Matrix shear strength

p) Calculation of density of lamina:

$$\rho = \rho_f V_f + \rho_m V_m \quad \text{---- (Eq.29)}$$

Where,

ρ = Density of lamina

ρ_f = Fiber density

ρ_m = Matrix density

V_f = Fiber volume fraction

V_m = Matrix volume fraction

q) Calculation of coefficient of moisture expansion of lamina in direction one:

$$\beta_1 = \frac{(\beta_m E_m \rho)}{E_l \rho_m}$$

Where,

β_1 = Coefficient of moisture expansion of lamina in direction one

β_m = Coefficient of moisture expansion of matrix

E_m = Young's modulus of matrix

ρ = Density of lamina

E_l = Young's modulus of lamina in direction one

ρ_m = Density of matrix

r) Calculation of coefficient of moisture expansion of lamina in direction two:

$$\beta_2 = \frac{(1 + \nu_m)\beta^m \rho}{\rho^m} - \beta^m \nu_{12} \quad \text{---- (Eq.30)}$$

Where,

β_2 = Coefficient of moisture expansion of lamina in direction two

ν_m = Poisson's ratio of matrix

β^m = Coefficient of moisture expansion of matrix

ρ = Density of matrix

ρ^m = Density of matrix

ν_{12} = Poisson's ratio of lamina in one two plane

s)Calculation of coefficient of thermal expansion of lamina in direction one:

$$\alpha_1 = \frac{V_f \alpha_{lf} E_{lf} + V_m \alpha_m E_m}{E_1} \quad \text{---- (Eq.31)}$$

Where,

α_1 = Coefficient of thermal expansion of lamina in direction one

α_{lf} = Coefficient of thermal expansion of fiber in longitudinal direction

E_{lf} = Fiber Young's modulus in longitudinal direction

α_m = Coefficient of thermal expansion of matrix

E_m = Young's modulus of matrix

E_1 = Young's modulus of lamina in direction one

t) Calculation of coefficient of thermal expansion of lamina in direction two:

$$\alpha_2 = \alpha_{tf} V_f + \alpha_m V_m \left(1 + \frac{V_f \nu_m E_{lf}}{E_1}\right) \quad \text{---- (Eq.32)}$$

Where,

α_2 = Coefficient of thermal expansion of lamina in direction two

α_{tf} = Coefficient of thermal expansion of fiber in longitudinal direction

E_1 = Young's modulus of lamina in direction one

α_m = Coefficient of thermal expansion of matrix

ν_m = Poisson's ratio of matrix

Zheng-ming Huang Method

Zheng-ming Huang is a method which is used to calculate elastic properties of transversely isotropic lamina.

Fiber properties, matrix properties and fiber volume fraction are input to this method. The elastic properties can be found out as given by Zheng-ming Huang [6,11]:

a) Young's modulus of lamina in direction-1:

$$E_l = V_f E_{lf} + V_m E_m \quad \text{---- (Eq.33)}$$

b) Poisson's ratio of lamina in 1-2 plane:

$$\nu_{lt} = V_f \nu_{ltf} + V_m \nu_m \quad \text{---- (Eq.34)}$$

c) Young's modulus of lamina in direction – 2:

$$E_2 = \frac{(V_f + V_m a_{11})(V_f + V_m a_{22})}{(V_f + V_m a_{11})(V_f S_{22}^f + a_{22} V_m S_{22}^m) + V_f V_m (S_{21}^m - S_{21}^f) a_{12}} \quad \text{--- (Eq.35)}$$

Where,

Constants $a_{12}, a_{13}, a_{22}, a_{33}, a_{44}$ are calculated as shown below

$$a_{13}=a_{12}=\frac{\left(S_{12}^{f}-S_{12}^{m}\right)\times(a_{11}-a_{22})}{\left(S_{11}^{f}-S_{11}^{m}\right)}$$

$$a_{22}=a_{33}=a_{44}=0.5\times\left(1+\frac{E_m}{E_{tf}}\right)$$

Coefficients of stiffness matrix are calculated as shown below,

$$S_{22}^{f}=\frac{1}{E_{tf}}$$

$$S_{22}^{m}=\frac{1}{E_m}$$

$$S_{21}^{f}=\frac{1}{E_{tf}}$$

$$S_{21}^{m}=\frac{1}{E_m}$$

$$S_{12}^{f}=-\frac{v_{ltf}}{E_{lf}}$$

$$S_{12}^{m}=-\frac{v_m}{E_m}$$

$$S_{11}^{m}=\frac{1}{E_m}$$

d) Shear modulus of lamina :

$$G_{lt}=G_m\times\frac{\left(G_{ltf}+G_m\right)+V_f\left(G_{ltf}-G_m\right)}{\left(G_{ltf}+G_m\right)-V_f\left(G_{ltf}-G_m\right)} \qquad \text{---- (Eq.36)}$$

e) Shear modulus of lamina :

$$G_{tt} = \frac{0.5\left(V_f + V_m a_{22}\right)}{V_f\left(S_{22}^{f} - S_{23}^{f}\right) + V_m a_{22}\left(S_{22}^{m} - S_{23}^{m}\right)} \qquad \text{---- (Eq.37)}$$

Where,

$$a_{22}=a_{33}=a_{44}=0.5\times\left(1+\frac{E_m}{E_{tf}}\right)$$

$$S_{22}^{f}=\frac{1}{E_{tf}}$$

$$S_{22}^{m}=\frac{1}{E_m}$$

$$S_{23}^{f}=-\frac{v_{ttf}}{E_{tf}}$$

$$S_{23}^{m}=-\frac{v_m}{E_m}$$

Method of Cells (MOC)

The major contribution to the Method of Cells is by Aboudi [7, 8].

He has developed this theory to predict properties of lamina. The prediction of ultimate stresses of unidirectional fiber composites under complex loading system using micromechanics approach has been presented [9].

Aboudi and Pindera [10] extended this method to generate initial yield surfaces unidirectional and cross-ply metal matrix composites.

For Theoretical Formulation of Determination of Properties of Transversely Lamina a computer program in FORTRAN 77 is developed for determination of Elastic properties, thermal properties, and strength properties of lamina using micromechanics approach by method of cells, Halpin-Tsai method.

Results:

Results are represented in graphical form as follows:

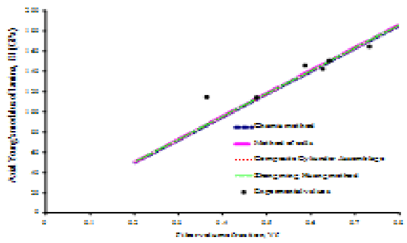

Figure 2: Axial Young's modulus Vs Fiber volume fraction

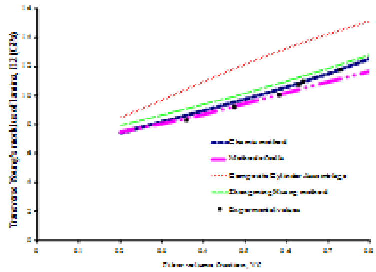

Figure 3: Modulus of elasticity (Transverse) Vs fiber volume fraction

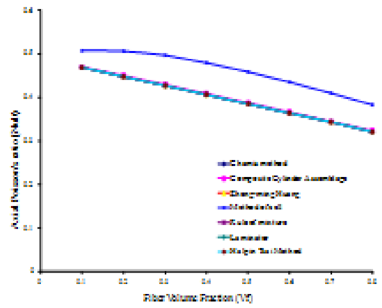

Figure 4: Poisson's ratio (Longitudinal) Vs fiber volume fraction

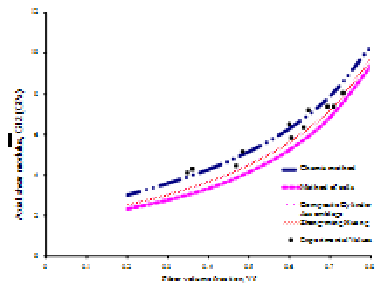

Figure 5: Shear modulus of lamina (longitudinal) Vs fiber volume fraction

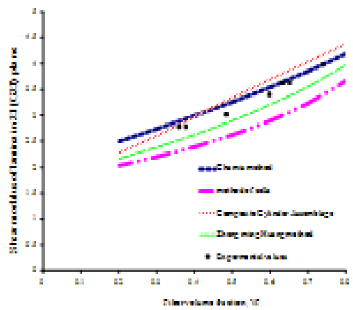

Figure 6: Shear modulus of lamina (Transverse) Vs fiber volume fraction

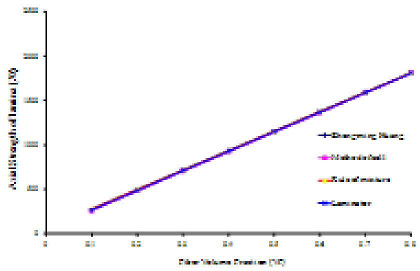

Figure 7: Axial strength Vs fiber volume fraction

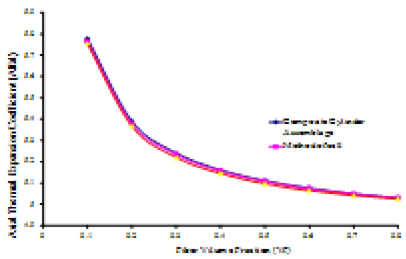

Figure 8: Coefficient of Thermal expansion (Axial) Vs fiber volume

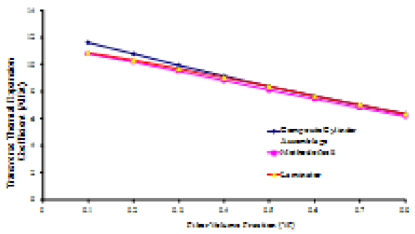

Figure 9: Coefficient of Thermal expansion (Transverse) Vs fiber volume fraction

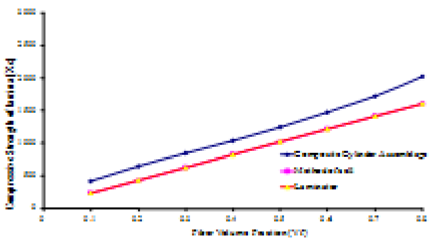

Figure 10: Compressive strength Vs fiber volume fraction

Conclusions

1) Axial Young's modulus, transverse Young's modulus, shear modulus and strength of lamina goes in increasing as the percentage of fiber volume fraction increases.

2) Poisson's ratio and coefficient of thermal expansion goes on decreasing as percentage of fiber volume fraction increases.

3) The results obtained using micromechanical model of Method of Cells are found upper bound for transverse properties.

4)This method of cells, composite cylinder assemblage method are useful for prediction of all properties of lamina like elastic, thermal and Strength properties where as other methods can not predict all properties.

5) Results obtained for all properties of lamina by analytical methods are in excellent agreement with experimental results and results by software package the Laminator.

6) Empirical expressions are developed to predict properties of orthotropic lamina by method of cells. Method of cells can be effectively applied for transversely isotropic as well as orthotropic lamina where as other methods used only for transversely isotropic lamina.

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