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Justifications of weighted composition operators of relations between different Bergman spaces of bounded symmetric domains

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ABSTRACT

The verifications of boundedness and compactness of the weighted composition operators of relations between different Bergman spaces of bounded symmetric domains which are Hilbert are characterized by using Carleson measure. As an application, we study the relations of multipliers between different Bergman spaces

Introduction

In this paper we follow the same Literature and methods of XIAOFEN LV and XIAOMIN TANG [15] with a little change.

Let Ω be the bounded symmetric domain in C^n with Bergman Kernel $K(z,w)$. We assume that Ω is in its Harish-Chandra realization and the volume measure dv of Ω is normalized so that $K(z,0)=K(0,w)=1$ for all z and w in Ω. Define the Bergman Matrix of Ω to be

$$G_z=(g_{ij}(z))=\frac{1}{2}\left(\frac{\partial^2}{\partial z_i\partial \bar{z}_j}\log K(z,z)\right),\quad 1\le i,j\le n.$$

For $\gamma:[0,1]\to\Omega$, a piecewise smooth C^1 curve, we set

$$l(\gamma)=\int_0^1\left(\sum_{i,j=1}^n g_{ij}(\gamma(t))\gamma_i'(t)\overline{\gamma_j'(t)}\right)^{\frac{1}{2}}dt.$$

The Bergman distance function on Ω is defined as

$$d(z,w)=\inf\{l(\gamma):\gamma:[0,1]\to\Omega,\quad \gamma(0)=z,\quad \gamma(1)=w\}.$$

For $z\in\Omega$ and $r>0$, introduce the ball $E(z,r)=\{w\in\Omega: d(z,w)<r\}$, and denote $|E(z,r)|$ the normalized volume of $E(z,r)$, that is,

$$|E(z,r)|=\int_{E(z,r)}dv(w). \qquad (1)$$

Let $H(\Omega)$ be the family of all holomorphic functions on Ω, set $dv_\alpha(z)=K(z,z)^{1-\alpha}dv(z)$, where $\alpha>\dfrac{N-1}{N}$, N is the genus of Ω, then dv_α is a finite measure on Ω see, [1,16]. For $0<p<\infty$, the weighted Bergman space $A_a^p(\Omega,dv_\alpha)$ is the space of all functions $f\in H(\Omega)$ for which

$$\|f\|_{p,\alpha}=\left(\int_\Omega |f(z)|^p dv_\alpha(z)\right)^{\frac{1}{p}}<\infty.$$

For any $z\in\Omega$, set $k_z(w)=\dfrac{k(w,z)}{\sqrt{k(z,z)}}$, $w\in\Omega$,

Then k_z is the normalized reproducing kernel for $A_a^2(\Omega,dv)$, and k_z^α is a unit vector of $A_a^2(\Omega,dv_\alpha)$. For any $r>0$, there exists some $C>0$ such that for all $z\in\Omega$ and $w\in E(z,r)$,

$$C^{-1}\le|E(z,r)||k_z(w)|^2\le C. \qquad (2)$$

Taking $w=z$, we get

$$C^{-1}\le|E(z,r)||k(z,z)|\le C,\ z\in\Omega. \qquad (3)$$

Furthermore, for any fixed $r>0$, $s>0$, $R>0$, there exists some $C>0$ (depending only on r, s, R) such that

$$C^{-1}\le\frac{|E(z,r)|}{|E(w,s)|}\le C,$$

For every $z,w\in\Omega$ with $d(z,w)\le R$ for proofs see [3].

Given $\varphi,\psi\in H(\Omega)$ and $\varphi(\Omega)\subseteq\Omega$, the weighted composition operator $W_{\varphi,\psi}$ is defined as

$$w_{\varphi,\psi}f(z)=\psi(z)(f\circ\varphi)(z)=\psi(z)f(\varphi(z)),$$
$$f\in H(\Omega),\ z\in\Omega. \qquad (4)$$

It is obvious that $W_{\varphi,\psi}$ is a linear operator, which is closely related to the composition operator and multiplier. The behavior

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of these two operators have been studied extensively on various spaces of holomorphic function see [6,7,13]. It is natural to consider the boundedness and compactness of weighted composition operators on Bergman spaces. In the unit disk and the unit ball, the problem has been studied by many authors see[4,5,11,14]. In the bounded symmetric domain considered the composition operator C_φ on Bergman spaces in [10,11]. Recently, the boundedness and compactness of weighted composition operators on weighted Bergman spaces studied [12]. Relatively to these papers, our work is to obtain the sufficient and necessary conditions on φ, ψ such that the operator $W_{\varphi,\psi} : A_a^p(\Omega, dv_\alpha) \to A_a^q(\Omega, dv_\beta)$ is bounded (or compact) for all $0 < p,q < \infty$, $\alpha, \beta > \frac{N-1}{N}$, where $\varphi, \psi \in H(\Omega)$ which $\varphi(\Omega) \subseteq \Omega$.

This will extend the results in [12].

In what follows we always suppose $\varphi, \psi \in H(\Omega)\varphi(\Omega) \subseteq \Omega$ and $\alpha, \beta > \frac{N-1}{N}$. C will stand for a positive constant whose value may change from line to line but is independent of the functions in $H(\Omega)$. The expression $A \; ; \; B$ means $C^{-1}A \le B \le CA$.

2 Boundedness and compactness of $W_{\varphi,\psi}$

First, we collect a few preliminary results that will be needed later in this paper. We begin with a result on compact weighted composition operators. For any $0 < p,q < \infty$, the operator $W_{\varphi,\psi} : A_a^p(\Omega, dv_\alpha) \to A_a^q(\Omega, dv_\beta)$ is compact if and only if $\lim_{k\to\infty} \| w_{\varphi,\psi} f_k \|_{q,\beta} = 0$

For any norm bounded sequence $\{f_k\} \subseteq A_a^p(\Omega, dv_\alpha)$ that converges to 0 uniformly on any compact subsets of Ω.

Suppose $q > 0$, $\psi \in A_a^p(\Omega, dv_\beta)$, define the nonnegative measure $\mu_{\varphi,\psi,q,\beta}$ to be

$$\mu_{\varphi,\psi,q,\beta}(E) = \int_{\varphi^{-1}(E)} |\psi|^q dv_\beta,$$

here E is a measurable subset of Ω. Using Theorem C in p. 163 of [8], we have the following change of variables formula:

$$\int_\Omega g d\mu_{\varphi,\psi,q,\beta} = \int_\Omega |\psi|^q (g o \varphi) dv_\beta, \qquad (5)$$

where g is an arbitrary measurable positive function on Ω.

Suppose μ is a finite positive Borel measure on Ω. It is said to be a δ - Carleson measure if

$$\sup_{z\in\Omega} \frac{\mu(E(z,r))}{|E(z,r)|^\delta} < \infty, \text{ where}$$

$$\mu(E(z,r)) = \int_{E(z,r)} d\mu(w).$$

Moreover, if $\lim_{z\to\partial\Omega} \frac{\mu(E(z,r))}{|E(z,r)|^\delta} = 0$, then μ is called a vanishing a δ - Carleson measure. It is well-known that δ - Carleson measure plays an important role in weighted Bergman space. More precisely, the following result holds.

Lemma2.1. Let μ be a finite positive Borel measure on Ω, $0 < p \le q < \infty$. Then the following statements are equivalent:

(i) There exists some C such that for any $f \in A_a^p(\Omega, dv_\alpha)$ we have

$$\left(\int_\Omega |f(z)|^q d\mu(z)\right)^{\frac{1}{q}} \le \left(\int_\Omega |f(z)|^p dv_\alpha(z)\right)^{\frac{1}{p}}$$

(ii) μ is a $\frac{\alpha q}{p}$ - Carleson measure;

(iii) $\sup_{z\in\Omega} \int_\Omega |k_z(w)|^{\frac{2\alpha q}{p}} d\mu(w) < \infty$.

Proof. This easily follows from Theorem2.1 in [9], just taking $\eta = \frac{q}{p}$.

Lemma2.2. Let μ be a finite positive Borel Measure on Ω, $0 < q < p < \infty$. Then the following statements are equivalent:

(i)There exists some C such that for any $f \in A_a^p(\Omega, dv_\alpha)$, we have

$$\left(\int_\Omega |f(z)|^q d\mu(z)\right)^{\frac{1}{q}} \le C\left(\int_\Omega |f(z)|^p dv_\alpha(z)\right)^{\frac{1}{p}}$$

(ii) $\frac{\mu(E(z,r))}{|E(z,r)|^\alpha} \in A^s(\Omega, dv_\alpha)$,

where $\frac{1}{s} + \frac{q}{p} = 1, A^s(\Omega, dv_\alpha)$ is the usual Lebesque space.

Our main justifications results are the following three theorems.

Theorem2.3. Suppose $0 \le \varepsilon < 2$. Then the following statements are equivalent:

(i) $w_{\varphi,\psi} : A_a^{2-\varepsilon}(\Omega, dv_\alpha) \to A_a^2(\Omega, dv_\beta)$ is bounded;

(ii) $\mu_{\varphi,\psi,2,\beta}$ is a $\frac{2\alpha}{2-\varepsilon}$ - Carleson measure;

(iii) $\sup_{z\in\Omega} \int_\Omega |k_z(w)|^{\frac{4\alpha}{2-\varepsilon}} d\mu_{\varphi,\psi,2,\beta}(w) < \infty$.

Proof. (i) $\Rightarrow$ (iii). For $z \in \Omega$, set

$$g_z(w) = (k_z(w))^{\frac{2\alpha}{2-\varepsilon}}, w \in \Omega. \qquad (6)$$

Then $g_z \in H(\Omega)$ and $\|g_z\|_{2-\varepsilon,\alpha} \le C$. By the boundedness of $w_{\varphi,\psi}$, (4) and(5) we have

$$\int_\Omega |k_z(w)|^{\frac{4\alpha}{2-\varepsilon}} d\mu_{\varphi,\psi,2,\beta}(w) = \int_\Omega |g_z(w)|^2 d\mu_{\varphi,\psi,2,\beta}(w)$$
$$= \int_\Omega |\psi(w)|^2 |(g_z o \varphi)(w)|^2 dv_\beta(w)$$
$$= \| w_{\varphi,\psi} g_z \|_{2,\beta}^2$$
$$\le C.$$

Hence, $\sup_{z\in\Omega}\int_\Omega |k_z(w)|^{\frac{4\alpha}{2-\varepsilon}} d\mu_{\varphi,\psi,2,\beta}(w) < \infty$.

(iii) $\Rightarrow$(ii). The condition(iii) implies

$$\frac{\mu_{\varphi,\psi,2,\beta}(E(z,r))}{|E(z,r)|^{\frac{2\alpha}{2-\varepsilon}}} = \frac{1}{|E(z,r)|^{\frac{2\alpha}{2-\varepsilon}}}\int_{E(z,r)} d\mu_{\varphi,\psi,2,\beta}(w)$$

$$; \int_{E(z,r)} |k_z(w)|^{\frac{4\alpha}{2-\varepsilon}} d\mu_{\varphi,\psi,2,\beta}(w)$$

$$\le \int_\Omega |k_z(w)|^{\frac{4\alpha}{2-\varepsilon}} d\mu_{\varphi,\psi,2,\beta}(w).$$

Thus $\mu_{\varphi,\psi,2,\beta}$ is a $\frac{2\alpha}{2-\varepsilon}$- Carleson measure.

(ii) $\Rightarrow$(i). Suppose $f \in A_a^2(\Omega, dv_\alpha)$. Lemma2.1 and (ii) show

$$\|w_{\varphi,\psi} f\|_{2,\beta}^2 = \int_\Omega |\psi(w)|^2 |(f o\varphi)(w)|^2 dv_\beta(w)$$

$$= \int_\Omega |f(w)|^2 d\mu_{\varphi,\psi,2,\beta}(w)$$

$$\le C\int_\Omega |f(w)|^{2-\varepsilon} dv_\alpha(w).$$

Hence, $w_{\varphi,\psi}: A_a^{2-\varepsilon}(\Omega, dv_\alpha) \to A_a^2(\Omega, dv_\beta)$ is bounded. The proof is completed.

Theorem2.4. Suppose $0 \le \varepsilon < 2$. Then the following statements are equivalent:

(i) $w_{\varphi,\psi}: A_a^{2-\varepsilon}(\Omega, dv_\alpha) \to A_a^2(\Omega, dv_\beta)$ is compact;

(ii) $\mu_{\varphi,\psi,2,\beta}$ is a vanishing $\frac{2\alpha}{2-\varepsilon}$- Carleson measure;

(iii) $\lim_{z\in\partial\Omega}\int_\Omega |k_z(w)|^{\frac{4\alpha}{2-\varepsilon}} d\mu_{\varphi,\psi,2,\beta}(w) = 0$.

Proof. (i) $\Rightarrow$(iii). For $z \in \Omega$, define g_z as in (6). Then $g_z \in H(\Omega)$ and $g_z \to 0$ weakly in $A_a^{2-\varepsilon}(\Omega, dv_\alpha)$ as $z \to \partial\Omega$. By Lemma3.2 in [12] and equation(4), we get

$$\int_\Omega |k_z(w)|^{\frac{4\alpha}{2-\varepsilon}} d\mu_{\varphi,\psi,2,\beta}(w) = \int_\Omega |g_z(w)|^2 d\mu_{\varphi,\psi,2,\beta}(w)$$

$$= \int_\Omega |\psi(w)|^2 |(g_z o\varphi)(w)|^2 dv_\beta(w)$$

$$= \|w_{\varphi,\psi} g_z\|_{2,\beta}^2$$

$$\to 0 \quad (z \to \partial\Omega).$$

(iii)$\Rightarrow$(ii). This is similar to the proof of (iii)$\Rightarrow$(ii) in Theorem2.3. (ii)$\Rightarrow$(i). Notice that, for any $r > 0$, be Lemma5 in [2], we can choose a sequence $\{a_j\} \subseteq \Omega$ which $a_j \to \partial\Omega$ as $j \to \infty$ satisfying (i) $\Omega = \bigcup_{j=1}^{\infty} E(a_i, r)$; (ii)there is a positive integer N such that each point $z \in \Omega$ belongs to at most N of the sets $E(a_i, 2r)$. Then for any $\varepsilon > 0$, by (ii), we get

$$\frac{\mu_{\varphi,\psi,2,\beta}(E(a_j, r))}{|E(a_j, r)|^{\frac{4\alpha}{2-\varepsilon}}} < \delta, \tag{7}$$

If j is sufficiently large. Suppose $\{f_k\}$ is any norm bounded sequence in $A_a^{2-\varepsilon}(\Omega, dv_\alpha)$ and $f_k \to 0$ uniformly on each compact subsets of Ω. We claim $\lim_{k\to\infty}\|w_{\varphi,\psi} f_k\|_{2,\beta} = 0$.

In fact,

$$|f(z)|^{2-\varepsilon} \le \frac{C}{|E(z,r)|}\int_{E(z,r)} |f(w)|^{2-\varepsilon} dv(w)$$

$$= \frac{1}{|E(z,r)|}\int_{E(z,r)} |f(w)|^{2-\varepsilon} \frac{dv_\alpha(w)}{k(w,w)^{1-\alpha}}$$

From(3) we have

$$|f(z)|^{2-\varepsilon} ; \frac{1}{|E(z,r)|}\int_{E(z,r)} |f(w)|^{2-\varepsilon} |E(w,r)|^{1-\alpha} dv_\alpha(w)$$

$$; \frac{1}{|E(z,r)|^\alpha}\int_{E(z,r)} |f(w)|^{2-\varepsilon} dv_\alpha(w),$$

the first inequality follows from Lemma7 in [2]. Then

$$\sup\left\{|f(z)|^p : z \in E(a,r)\right\}$$

$$\le \sup\left\{\frac{C}{|E(z,r)|^\alpha}\int_{E(z,r)} |f(w)|^{2-\varepsilon} dv_\alpha(w) : z \in E(a,r)\right\}$$

$$\le \frac{C}{|E(z,r)|^\alpha}\int_{E(a,2r)} |f(w)|^{2-\varepsilon} dv_\alpha(w).$$

Thus, from(4) we have

$$\|w_{\varphi,\psi} f_k\|_{2,\beta}^2 = \int_\Omega |\psi(z)|^2 |(f_k o\varphi)(z)|^2 dv_\beta(z)$$

$$= \int_\Omega |f_k(z)|^2 d\mu_{\varphi,\psi,2,\beta}(z)$$

$$\le \sum_{j=1}^{\infty}\int_{E(a_j,r)} |f_k(z)|^2 d\mu_{\varphi,\psi,2,\beta}(z)$$

$$\le \sum_{j=1}^{\infty}\frac{\mu_{\varphi,\psi,2,\beta}(E(a_j,r))}{|E(a_j,r)|^\alpha}\sup\left\{|f_k(z)|^2 : z \in E(a_j,r)\right\}$$

$$\le \sum_{j=1}^{\infty}\frac{\mu_{\varphi,\psi,2,\beta}(E(a_j,r))}{|E(a_j,r)|^{\frac{2\alpha}{2-\varepsilon}}}\left(\int_{E(a_j,2r)} |f_k(z)|^{2-\varepsilon} dv_\alpha(z)\right)^{\frac{2}{2-\varepsilon}}$$

$$= \left(\sum_{j=1}^{J_0} + \sum_{j=J_0}^{\infty}\right)\frac{\mu_{\varphi,\psi,2,\beta}(E(a_j,r))}{|E(a_j,r)|^{\frac{2\alpha}{2-\varepsilon}}}\left(\int_{E(a_j,2r)} |f_k(z)|^{2-\varepsilon} dv_\alpha(z)\right)^{\frac{2}{2-\varepsilon}}$$

$$= I_1 + I_2.$$

On the one hand, when $1 \le j \le J_0$, $E(a_j, 2r)$ is a compact subset of Ω, if k is sufficiently large. Then $I_1 \le C\delta^2$.

On the other hand, (7) yields

$$I_2 \le CN\delta\left(\int_\Omega |f_k(z)|^{2-\varepsilon} dv_\alpha(z)\right)^{\frac{2}{2-\varepsilon}} \le CN\delta \|f_k\|^2_{2-\varepsilon,\alpha} \le C\delta \cdot$$

Therefore $\lim_{k\to\infty} \|w_{\varphi,\psi} f_k\|_{2,\beta} = 0$. The proof is completed.

Theorem2.5. Suppose $0 < \varepsilon < 2$.Then the following statements are equivalent:

(i) $w_{\varphi,\psi} : A^2_a(\Omega, dv_\alpha) \to A^{2-\varepsilon}_a(\Omega, dv_\beta)$ is bounded;

(ii) $w_{\varphi,\psi} : A^2_a(\Omega, dv_\alpha) \to A^{2-\varepsilon}_a(\Omega, dv_\beta)$ is compact;

(iii) $\frac{\mu_{\varphi,\psi,2-\varepsilon,\beta}(E(z,r))}{|E(z,r)|^\alpha} \in A^s(\Omega, dv_\alpha)$, where $s = \frac{2}{\varepsilon}$.

Proof. The implication(ii) $\Rightarrow$ (i)is trivial.

(i) $\Leftrightarrow$ (iii). The operator $w_{\varphi,\psi} : A^2_a(\Omega, dv_\alpha) \to A^{2-\varepsilon}_a(\Omega, dv_\beta)$ is bounded if and only if, there exists some C such that for any $f \in A^2_a(\Omega, dv_\alpha)$,

$$\|w_{\varphi,\psi} f\|^{2-\varepsilon}_{2-\varepsilon,\beta} = \int_\Omega |\psi(w)|^{2-\varepsilon} |(f o\varphi)(w)|^{2-\varepsilon} dv_\beta(w)$$

$$= \int_\Omega |f(w)|^{2-\varepsilon} d\mu_{\varphi,\psi,2-\varepsilon,\beta}(w)$$

$$\le C\int_\Omega |\psi(w)|^{2-\varepsilon} dv_\alpha(w). \quad (8)$$

By Lemma2.2, the necessary and sufficient condition for (8) is

$$\frac{\mu_{\varphi,\psi,2-\varepsilon,\beta}(E(z,r))}{|E(z,r)|^\alpha} \in A^s(\Omega, dv_\alpha), \quad \text{where } s = \frac{2}{\varepsilon}.$$

Now, we will show that (iii) $\Rightarrow$(ii). Suppose $\{f_k\}$ is any norm bounded sequence in $A^2_a(\Omega, dv_\alpha)$ and $f_k \to 0$ uniformly on each compact subsets of Ω. Since $\chi_{E(z,r)}(w) = \chi_{E(w,r)}(z)$, then

$$\|w_{\varphi,\psi} f_k\|^{2-\varepsilon}_{2-\varepsilon,\beta} = \int_\Omega |\psi(z)|^{2-\varepsilon} |(f_k o\varphi)(z)|^{2-\varepsilon} dv_\beta(z)$$

$$= \int_\Omega |f_k(z)|^{2-\varepsilon} d\mu_{\varphi,\psi,2-\varepsilon,\beta}(z)$$

From(1) we have

$$\|w_{\varphi,\psi} f_k\|^{2-\varepsilon}_{2-\varepsilon,\beta}$$

$$\le \int_\Omega \frac{1}{|E(z,r)|^\alpha} d\mu_{\varphi,\psi,2-\varepsilon,\beta}(z) \int_{E(z,r)} |f_k(w)|^{2-\varepsilon} dv_\alpha(w)$$

$$= \int_\Omega \frac{1}{|E(z,r)|^\alpha} d\mu_{\varphi,\psi,2-\varepsilon,\beta}(z) \int_\Omega \chi_{E(z,r)}(w) |f_k(w)|^{2-\varepsilon} dv_\alpha(w)$$

$$= \int_\Omega |f_k(w)|^{2-\varepsilon} dv_\alpha(w) \int_\Omega \frac{\chi_{E(w,r)}(z)}{|E(z,r)|^\alpha} d\mu_{\varphi,\psi,2-\varepsilon,\beta}(z)$$

$$= \int_\Omega |f_k(w)|^{2-\varepsilon} dv_\alpha(w) \int_{E(w,r)} \frac{d\mu_{\varphi,\psi,2-\varepsilon,\beta}(z)}{|E(z,r)|^\alpha}$$

$$; \int_\Omega |f_k(w)|^{2-\varepsilon} \frac{d\mu_{\varphi,\psi,2-\varepsilon,\beta}(E(w,r))}{|E(z,r)|^\alpha} dv_\alpha(w) = I \cdot$$

For any $\delta > 0$, (iii) implies, there exists some $r \in (0,1)$ such that

$$\int_{\Omega \setminus r\Omega} \left(\frac{\mu_{\varphi,\psi,2-\varepsilon,\beta}(E(w,r))}{|E(w,r)|^\alpha} \right)^s dv_\alpha(w) < \delta^{s(2-\varepsilon)}, \quad (9)$$

Where $r\Omega = \{rz : z \in \Omega\}$ is a compact subset of Ω. Hence, using the Holder inequality, we have

$$I = \left(\int_{\Omega\setminus r\Omega} + \int_\Omega\right) |f_k(w)|^{2-\varepsilon} \frac{\mu_{\varphi,\psi,2-\varepsilon,\beta}(E(w,r))}{|E(w,r)|^\alpha} dv_\alpha(w)$$

$$\le \left(\int_\Omega |f_k(w)|^2 dv_\alpha(w)\right)^{\frac{2-\varepsilon}{2}} \cdot \left(\int_{\Omega\setminus r\Omega} \left(\frac{\mu_{\varphi,\psi,2-\varepsilon,\beta}(E(w,r))}{|E(w,r)|^\alpha} \right)^s dv_\alpha(w) \right)^{\frac{1}{s}}$$

$$+ \left(\int_{r\Omega} |f_k(w)|^2 dv_\alpha(w)\right)^{\frac{2-\varepsilon}{2}} \cdot \left(\int_\Omega \left(\frac{\mu_{\varphi,\psi,2-\varepsilon,\beta}(E(w,r))}{|E(w,r)|^\alpha} \right)^s dv_\alpha(w) \right)^{\frac{1}{s}}$$

$$< C\delta^{(2-\varepsilon)},$$

If k is sufficiently large. The last inequality is obtained by (9) and the fact $f_k \to 0$ uniformly on $r\Omega$. Thus, $\lim_{k\to\infty} \|w_{\varphi,\psi} f_k\|_{2-\varepsilon,\beta} = 0$, which means that $w_{\varphi,\psi} : A^2_a(\Omega, dv_\alpha) \to A^{2-\varepsilon}_a(\Omega, dv_\beta)$ is compact.

Application

Let X and Y be two spaces of holomorphic function. We call ψ a pointwise multiplier from X to Y if $M_\psi f = \psi f \in Y$ for every $f \in X$. The collection of all pointwise multipliers from X to Y denoted by $M(X,Y)$. Setting $\varphi(z) = z$, the weighted composition operator $w_{\varphi,\psi}$ is just the multiplication operator M_ψ. The multiplication operators have been studied by many authors for example[7]. By the main results in §2, we can obtain the property of ψ.

Theorem3.1. Suppose $0 \le \varepsilon < 2$ or $\varepsilon > 2$. Then the following statements are equivalent:

(i) $\psi \in M\left(A^{2-\varepsilon}_a(\Omega, dv_\alpha), A^2_a(\Omega, dv_\beta)\right)$;

(ii) $\sup_{z\in\Omega} \frac{\int_{E(z,r)} |\psi(w)|^2 dv_\beta(w)}{|E(z,r)|^{\frac{2\alpha}{2-\varepsilon}}} < \infty$;

(iii) $\sup_{z\in\Omega} \int_\Omega |k_z(w)|^{\frac{4\alpha}{2-\varepsilon}} |\psi(w)|^2 dv_\beta(w) < \infty$.

Theorem 3.2. Suppose $0 < \varepsilon < 2$. Then $\psi \in M\left(A^2_a(\Omega, dv_\alpha), A^{2-\varepsilon}_a(\Omega, dv_\beta)\right)$ if and only if

$$\frac{\int_{E(z,r)} |\psi(w)|^{2-\varepsilon} dv_\beta(w)}{|E(z,r)|^\alpha} \in A^s(\Omega, dv_\alpha),$$

where

$\frac{1}{s}=\frac{\varepsilon}{2}$.

Set $\psi\equiv 1$. Then $w_{\varphi,\psi}$ is just the composition operator C_{φ}, and we can easily obtain the main theorems in, [9,10] by the results in §2. Furthermore, we can obtain the results as follows.

Theorem3.3. Let $0<\varepsilon<2$, and let C_{φ} be bounded on $A_{a}^{2-\varepsilon}(\Omega,dv_{\alpha})$. Then C_{φ} is bounded on $A_{a}^{2}(\Omega,dv_{\alpha})$.

Theorem3.4. Let $0<\varepsilon<2$, and let C_{φ} be compact on $A_{a}^{2-\varepsilon}(\Omega,dv_{\alpha})$. Then C_{φ} is compact on $A_{a}^{2}(\Omega,dv_{\alpha})$.

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