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# Distribution feeder capacitor allocation and sizing under uncertain load

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**ABSTRACT**

*This paper presents a novel method of allocation of capacitor size for a distribution feeder with uncertain load. The interval mathematics is used to perform the power flow solution of radial distribution system. The simulation results have been obtained using the INTLAB Toolbox in MATLAB environment. The standard 15-bus, 10-bus and the 33-bus radial distribution systems which are used by the researchers have been analyzed to demonstrate the effectiveness of the proposed approach. The developed program gives the range of the capacitor ratings in kVAr which could be selected for compensation, corresponding to the min and max limits of the voltages at each bus. For a range of values of capacitor ratings, which when installed at a particular bus results in the range of reduction in cost of capacitor and energy loss reduction. This information is very useful for a distribution system planner.*



## Introduction

Energy management through reactive power compensation on distribution systems has. Recently merged as a topic of current research interest [1]-[4]. Reactive power flow in a distribution system produces losses and results in increased rating for the system components. Shunt capacitors are usually installed to reduce these power losses, increase the released thermal capacities of the lines and transformers and improve the system voltage profile. However, the data employed in the reactive power compensation analysis is usually derived from many sources with varying degrees of accuracy. Accounting for such uncertainties is necessary to produce realistic results which utilities can employ to make informed decisions regarding reactive power compensation in their distribution systems. Uncertainties can be looked upon as a condition in which the possibility of errors exists as a result of having Jess than total information about the surrounding environment. They are beyond the utility's foreknowledge or control In a distribution system, the reactive load is always varying and it is not a realistic proposition to determine capacitor sizes and locations based on an average of the reactive loads as even this number is subject to change as the load varies. In addition, many of the reactive power compensation techniques involves the optimization of a cost function which require parameters such as the cost of the capacitors, the cost of energy and the cost of the peak power savings to which only an estimation (single-point) without exact certainty can be obtained [3]. Consequently, the validity of the results generated is questionable. Interval mathematics provides a powerful tool for the implementation and extension of the "unknown but bounded" concept [5]-[7]. Using interval analysis, there is no need for many simulations runs 8S the total variation of the solution considers the simultaneous variations of all inputs in a single run. In this form of mathematics, interval numbers are used instead of single point numbers. This paper presents an interval method coupled with a heuristic technique for maximizing the cost saving; by placing optimal capacitors at proper locations in interval format. Uncertainties in the parameters are integrated into the analysis, as interval numbers, to allocate, sequentially, the capacitors according to the upper limit of the maximum interval saving outcome. Once locations are identified, the standard capacitor size, at a selected location, is determined through the optimization of the cost saving function. The method offers utilities with alternatives for selecting the standard capacitor sizes to be used and the associated costs to be saved. To overcome the difficulty of· conservative bounds, a procedure is devised in order to produce sharp bounds of the interval outcomes and consequently enhances the decision making process. The proposed method is tested on a nine-bus distribution feeder and encouraging results are reported.

## Interval Mathematics in INTLAB

Interval Mathematics considers a set of methods for handling intervals that approximate uncertain data. These methods are based on the definition of both interval arithmetic and optimal scalar product. The maximal accuracy principle guarantees (by means of the outward rounding) the automatic control of errors in numerical computation. A *real interval* $X$ is a nonempty subset of real numbers R, $X = [x1, x2] = \{x \in \mathrm{R} \mid x1 \leq x \leq x2\}$, where $x1$ is the *infimum* and $x2$ is the *supremum*. The set of real intervals is denoted by IR. An interval $X = [x1; x2] \in \mathrm{IR}$ may not be representable on a machine if $x1$ and $x2$ are not machine numbers. In order to obtain a rounded interval $\tilde{X}$ such that $X \in \tilde{X}$ (i.e., $\tilde{X}$ is an approximation of $X$), $x1$ and $x2$ must be rounded downward and upward, respectively, which is called *outward rounding*. The *midpoint*, the *diameter* and the *radius* of an interval $X$ are given, respectively, by $mid(X) = \check{X} = 1/2\ (x1 + x2)$, $diam(X) = x2 - x1$ and $rad(X) = ½*diam(X)$. $X$ can be also denoted by $X = <mid(X), rad(X)>$. Interval arithmetic operations are defined such that the interval result encloses all possible real results, which guarantees the reliability of interval methods.

The elementary operations are defined by $X * Y = \{x*y \mid x \in X, y \in Y\}$, *for* $* \in \{-,+,\times,\div\}$ *and for*

$X = [x1, x2]$ *and* $Y = [y1, y2] \in IR$ *they are given by*

$X+Y = [x1+y1, x2+y2]$

$X-Y = [x1-y2, x2-y1]$

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*X×Y = [minρ, maxρ] with ρ = {x1y1, x1y2, x2y1, x2y2}*
*X÷Y = X × [$y_2^{-1}$, $y_1^{-1}$] if 0 ∉*

The Matlab toolbox INTLAB uses the routine set round to control the rounding mode (see IEEE standard for floating point arithmetic). Real intervals may be stored by either infimum and supremum or midpoint and radius. INTLAB enables basic interval operations to be performed on real and complex interval scalars, vectors and matrices.

Implementation of Interval Power flow, Capacitor allocation and sizing

Incorporation of Interval Load model into Power flow solution of a Radial Distribution System is as follows. The interval arithmetic technique for the power flow solution with the load variation limits as [$P_L$ $P_U$] and [$Q_L$ $Q_U$] respectively.

$$[I_L(k), I_U(k)] = \frac{[P_L(k), P_U(k)] - [Q_L(k), Q_U(k)]}{[V_L(k)^*, V_U(k)^*]}$$

Where I(k) is the node currents and P(k), Q(k) represents the active and reactive power load at $k^{th}$ node. The branch current equation is

$$[B_j] = [A_{ji}][I_k]$$

Where $[A_{ji}]$ is the incidence matrix which connects the bus with the branches. The elements of incidence matrix are $A_{ji} = 1$ if a Branch j is connected to the Bus i, $A_{ji} = 0$ otherwise. In order to account for uncertainties associated with the capacitors sizing and location problem, the maximum cost saving analysis is followed [4]. The input parameters' uncertainties, in interval format, are integrated into the equations as follows

$$P = \sum_{j=1}^{n} B_j^2 R_j \qquad (1)$$

Where $P$ is the total active power loss for a distribution system with n branches, $B_i$ and $R_i$ are the current magnitude and resistance, respectively of branch j. The branch current can be obtained from the load flow solution. This current has two components; active $B_a$ and reactive $B_r$. Thus, the system losses can written as

$$P = \sum_{j=1}^{n} B_{aj}^2 R_j + \sum_{j=1}^{n} B_{rj}^2 R_j \qquad (2)$$

If a capacitor of current $B_{ck}$ is placed at a node *k*, the System losses are

$$P = \sum_{j=1}^{n} B_{aj}^2 R_j + \sum_{j=1}^{k} (B_{rj} + B_{ck})^2 R_j + \sum_{j=k+1}^{n} B_{rj}^2 R_j \qquad (3)$$

Subtracting equation (3) from equation (2), the loss reduction $\Delta P_k$ is

$$\Delta P_k = -2B_{ck} \sum_{j=1}^{k} B_{rj} R_j - B_{ck}^2 \sum_{j=1}^{k} R_j \qquad (4)$$

Assuming there is no significant change in the node voltage after setting the capacitor and using the cost function equation, the cost reduction can be defined as

$$\Delta S = K_p \Delta P + K_e \Delta E - k_{ck} Q_{ck} \qquad (5)$$

Where $K_p$ is the annual cost in $/KW and $K_{ck}$ is the annual cost in $/KVAr for the capacitor placed at node *k* both represented in interval format, $K_e$ is the interval annual cost of KWh losses in $/KWh with the energy losses, defined over a time period *T*, using(4) as

$$\Delta E_k = -2L_f T B_{ck} \sum_{j=1}^{k} B_{rj} R_j - T B_{ck}^2 \sum_{j=1}^{k} R_j \qquad (6)$$

Where $L_f$ is the interval load factor and $Q_{ck}$ is the capacitor size at node k and equals

$$Q_{ck} = B_{ck} V_k \qquad (7)$$

Substituting for equation (4), (6-7) in equation (5) we have

$$\Delta S = -2K_p' B_{ck} \sum_{j=1}^{k} B_{rj} R_j - K_p'' B_{ck}^2 \sum_{j=1}^{k} R_j \qquad (8)$$

Where

$$K_p' = K_p + K_e T L_f + \frac{K_{ck} V_k}{2\sum_{j=1}^{k} B_{rj} R_j} \qquad \textbf{(9)}$$

$$K_p'' = K_p + K_e T$$

The value of $B_{ck}$ that maximizes the cost reduction is obtained by $\frac{\partial \Delta S}{\partial B_{ck}} = 0$

From equation (9) the interval $B_{ck}$ is

$$B_{ck} = -\frac{K_p' \sum_{j=1}^{k} B_{rj} R_j}{K_p'' \sum_{j=1}^{k} R_j} \qquad (11)$$

Substituting from equation (11) into (8) and (4) we obtain the interval maximum net saving and the corresponding interval loss reduction as

$$\Delta S_{max} = \frac{(K_p')^2 \left(\sum_{j=1}^{k} B_{rj} R_j\right)^2}{K_p'' \sum_{j=1}^{k} R_j} \qquad (12)$$

$$\Delta P_{max} = \left(2 - \frac{K_p'}{K_p''}\right) \frac{K_p' \left(\sum_{j=1}^{k} B_{rj} R_j\right)^2}{K_p'' \sum_{j=1}^{k} R_j} \qquad (13)$$

Using equation (7) and (11-13) we can calculate the size of the capacitor used at a certain node k that maximizes the total system cost reduction and the corresponding loss reduction in interval format.

Algorithm for capacitor placement node identification

The algorithm for identifying the capacitor placement nodes best suitable for capacitor location is:

1. Read the given line and load data of radial distribution network.
2. Perform the power flows and calculate the base case total active power loss.
3. By compensating the reactive power injections (Qc) at each node (except source node) in all the phases, run the power flow solution and calculate the active power losses in each case.
4. Calculate the power loss reduction and power loss indices using the PLI eqn. (6)
5. Select the capacitor placement nodes whose PLI > Tolerance obtained by experimentation
6. Stop.

$$PLI[i] = \frac{(LR[i] - LRmin[i])}{(LRmax[i] - LRmin[i])}$$

**Uncertainties in capacitor sizing and placement problem**

Inspection of equations (7-13) reveals that it is likely that values for $K_p$, $K_e$, $Kck$, and $L_f$ cannot be obtained with absolutecertainty. For instance, $K_p$ and $K_e$, the costs for the peak power and energy losses respectively can be calculated in many ways but it is probably known that there is an upper and lower bound for these costs which can be attributed with more certainty/than a single-point value for each cost. Likewise, for the reactive load factor $L_f$ range of values can also be determined. Thus by using interval mathematics, the uncertainties associated with the capacitor allocation technique could be more effectively understood if these input parameters were treated as interval numbers whose ranges contain the uncertainties in those parameters, The resulting computations, carried out entirely in interval form, would then literally carry the uncertainties associated with the data through the analysis. Likewise, the final outcome in interval Conn would contain all possible solutions due to the variations in input parameters.

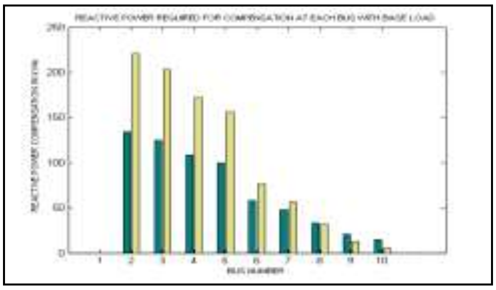



**Fig.1: 10-bus radial distribution system which requires the optimal reactive rating of the capacitor to be placed at each bus with the lower load (base) and the upper load (150% of base load).**

**Algorithm for capacitor placement and sizing in INTLAB (Interval Laboratory)**

The implementation of the proposed optimal capacitor sizing and placement technique in interval mathematics is performed in the Matlab environment with INTLAB Toolbox.

The step of the algorithm is as follows:

Step-1: Run the power flow program for the original uncompensated feeder to calculate the voltages and

current at each bus using the Forward-Backward Sweep method.

Step-2: Assume an initial value for the single point estimate capacitor cost $Kck$ as the average cost for all available standards for the studied feeder.

Step-3: Let the input parameters ($K_p'$, $K_e$, $Kck$, and $L_f$ ) as an interval numbers with a realistic tolerance of ±5% of their single point estimates.

Step-4: Select a bus and apply (7), (11-13) to compute the interval capacitor size, the interval current capacitor, the interval maximum saving and the corresponding interval loss reduction respectively. Repeat this step forall buses in the feeder, except the source bus.

Step-5: Identify the candidate bus that has the highest interval cost saving

Step-6: Once a bus is identified as a candidate bus determine all the standard capacitor sizes lying within the interval capacitor size at this bus. In case no standard size lies within the interval, then the one

nearest to the interval is selected (i.e, the closest standard size to both the lower upper bounds of the interval). These procedures are applied at anyone candidate bus selected.

Step-7: Perform the load flow calculations, for every single standard capacitor selected earlier, to ensure that no voltage violation takes place. If there is a voltage violation for one or more standard capacitor sizes, eliminate them from further consideration. If all the capacitor sizes result in voltage violation, then go to step 5 to select the next candidate bus.

Step-8: If there is no voltage violation, set the standard capacitor size, among the series of standard sizes in this interval, that provides the highest cost saving at this bus and take the corresponding exact capacitor cost value $Kck$

Step-9: Repeat steps 4-8 to get the next capacitor bus and hence the sequence of buses to be compensated until it is found that there is no significant cost saving can be achieved by further capacitor placement.

**Fig.2**

The complex current flows in each section of the feeder reveal the large change in the imaginary part of the current than the real part with the placement of the capacitor at the appropriate buses which reduces the net real power losses in the system. The lower complex current that is flowing due to the base load and the upper complex current due to 150% of base load in each branch of the 9-section feeder is shown in fig.2. From the fig.2 it is clear that the current circles overlap in case of lower load interval also the area of the circles are more than that of the upper load interval. This graphical representation of the range of variation of the real and reactive current helps the system planner clearly identify the relay settings and coordination for proper protection of the feeders and the sub-sections of the distribution system. It is also observed form the fig.2 that the range of variation in the upper complex current is small when compared to that of the lower complex current.

Also the changes due to R/X ratio of the feeder, which leads to changes in the real and reactive power, voltage and currents flows of the distribution system, can be observed. Since, the distribution system analysis is done considering the complex variables. The complex voltage (circles with 1% radius in p.u) with and without the capacitor placed at the appropriate buses which helps in improvement of the real part of the voltage than the imaginary part.

Fig.3 depicts the lower and upper complex voltages at each bus for the base and 150% of the base loads respectively. This graphical representation of the range of variation of the real and reactive voltage helps the system planner clearly identify the relay settings and coordination for proper protection of the feeders and the sub-sections of the distribution system. It is also observed form the fig.3 that the range of variation in the upper complex voltage is small when compared to that of the lower complex voltage. This range of complex voltages variation from lower to upper limits (the area of circles) is less when a capacitor is installed at one or two optimum locations in the considered 10-bus distribution system.

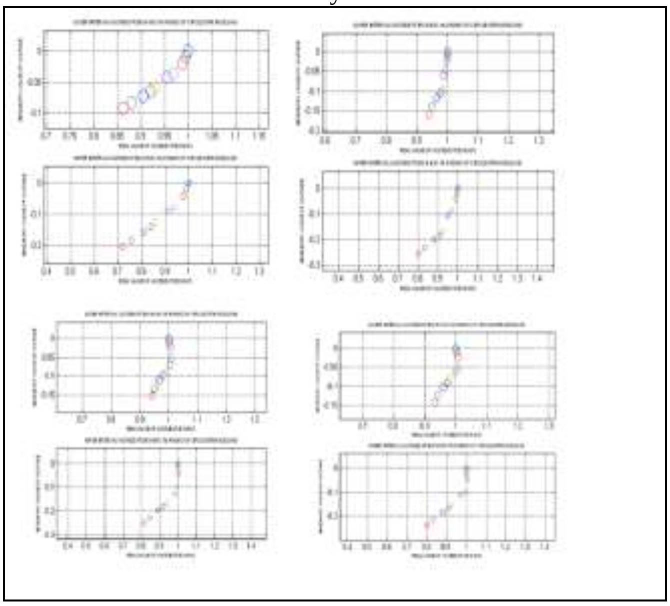


**Fig.3**

Without and with the placement capacitor the real and reactive power losses in the feeder sections helps the system planner judge appropriate bus where the capacitors are to be placed. For example in our 10-bus test system the real and reactive power losses in each line of the feeder without with capacitors placed at 6 and 10, 4 and 10 and 5 and 10 busses are shown in fig. above in clockwise direction.

Fig.4 depicts the lower and upper complex power losses at each bus for the base and 150% of the base loads respectively. This graphical representation of the range of variation of the real and reactive voltage helps the system planner clearly identify the relay settings and coordination for proper protection of the feeders and the sub-sections of the distribution system. It is also observed form the fig.4 that the range of variation in the upper complex voltage is small when compared to that of the lower complex voltage. This range of complex power losses.

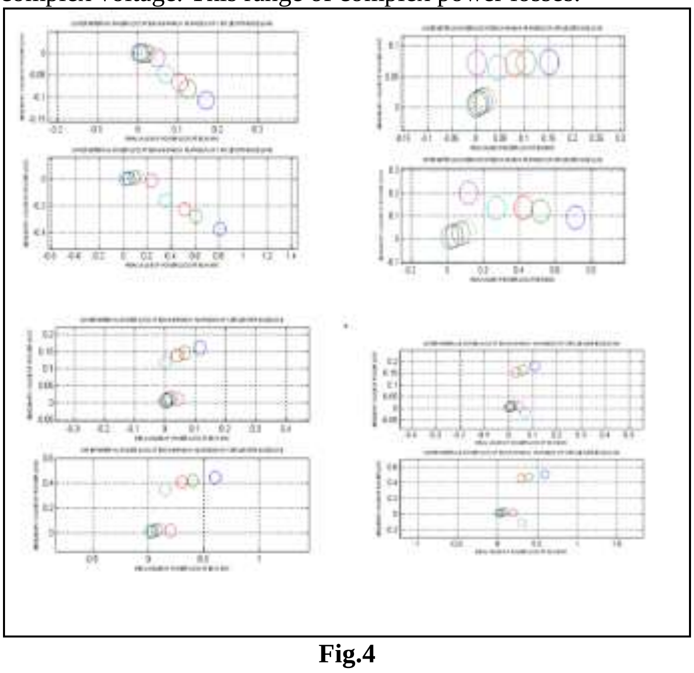
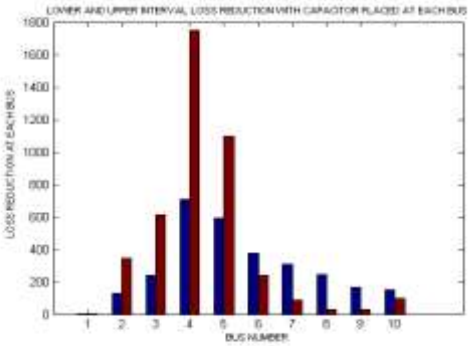
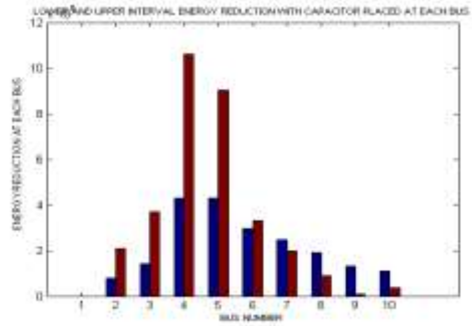


**Fig.4**




| Bus No. | Qc (kVAr) | Loss Reduction $\Delta P$ $(kW)$ | Cost Savings $\Delta S$ $(Rs.)$ |
|---|---|---|---|
| 2 | [1340.52, 2206.02] | [-130.5, -346.6] | [-799, -2077] |
| 3 | [1248.42, 2026.02] | [-238.6, -615.2] | [-1434, -3714] |
| 4 | [1084.17,1720.44,] | [-710.4, -1752.3] | [-4284, -10614] |
| 5 | [999.26, 1561.38] | [-592.3, -1097.5] | [-4307, -9020] |
| 6 | [581.58, 761.85] | [-375.1, -239.6] | [-2980, -3331] |
| 7 | [477.20, 565.11] | [-305.3, -89.3] | [-2472, -1989] |
| 8 | [334.88, 318.13] | [-245.8, -29.8] | [-1925, -893] |
| 9 | [209.31, 125.58] | [-166.3, 27.2] | [-1322, -99] |
| 10 | [146.51, 54.42] | [-147.7, 99] | [-1095, 373] |

Table indicates the optimal size of the single located capacitor (Qc), the maximum cost saving ($\Delta S$ ) and the loss reduction ($\Delta P$ ), for all the busses, as interval numbers. The negative sign in the loss reduction column indicate the loss reduction; whereas the positive sign indicates increase in losses. So, for the 150% load of the base load it is not advisable to place the capacitors at busses 9 and 10. It is recommended that the capacitors be placed at a bus which gives the highest loss reduction as well as the maximum cost savings of the capacitor. 4$^{th}$, 5$^{th}$ and 6$^{th}$ bus are the right choice for placing the capacitors for base load (lower interval) as well as the 150% of the base load (upper interval) where the maximum loss reduction as well as the maximum cost savings of the capacitors are maximum negative values.

Based on the available standard sizes of the capacitors, the one which gives the highest cost saving is chosen and placed at a bus which gives the maximum loss reduction.

Additional economic benefits may be realized using the above interval outcomes of the proposed technique. The interval analysis provides utilities with alternatives of using any available standard capacitor size, lying within the interval capacitor size outcome, together with the associated cost saving. The maximum cost saving, achieved by the selection of any of these standard sizes, would *certainly* have a lower limit which corresponds to the lower bound of the interval outcome. Prior knowledge of such information could be of significance in utility planning.

**Conclusions**

The capacitor sizing and placement problem is modeled using a combined heuristic and interval mathematics method. Use of interval mathematics enables' the integration of the effects of parameters Uncertainties into the analysis and eliminates the need for many simulation runs. While catering for Uncertainties, the method offers utilities with alternatives for selecting the standard capacitor sizes to be used and the associated costs to be saved. This enhances their ability to make informed decisions regarding installing capacitors for reactive power compensation in their distribution feeders. A procedure is devised in order to produce sharp bounds of the interval outcomes. Successful implementation of the method is described using a nine- bus test distribution feeder.

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