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Effect of thickness of the porous material on the peristaltic pumping when the inclined channel walls are provided with non-erodible porous lining

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ABSTRACT

In practical problems involving flow past a porous lining, it is necessary to involve directly the thickness of the porous lining to have an increase in the mass flow rate. The peristaltic pumping of a Newtonian fluid in an inclined channel lined with porous material is investigated under long wavelength and low Reynolds number assumptions. The velocity distribution, the volume flow rate, the pressure rise and the frictional force are obtained. The effect of thickness of porous lining on the peristaltic pumping is discussed.

Introduction

In recent years considerable interest has been shown in the study of peristaltic transport through and past porous media because of its important applications in biomechanics and medicine. Several investigations on peristaltic pumping through flexible impermeable walls are made based on the no-slip condition at the impermeable wall. Jaffrin et al. (1969), Ramachandra Rao and Usha (1995), Brasseur et al. (1987), Srivastava and Srivastava (1995) Vajravelu et al. (2005), Kodandapani and srinivas (2008), Srinivas and Gayatri (2009) and others studied fluid mechanics of peristaltic pumping considering the boundaries of the ducts as impermeable. Mishra and Ghosh (1997) proposed a mathematical model to study the blood flow taking the channel bounded by permeable walls. It is well known that peristaltic transport also takes place in small blood vessels. The tissue region in the blood vessels is modeled as porous medium by many researchers (Gopalan, 1981). In view of this Mishra (2004) studied the peristaltic pumping of a Newtonian fluid in a channel with a porous peripheral layer. In all the above investigations the thickness of the permeable bed has not considered in the analysis.

When the flow takes over a permeable bed the usual no slip condition is not valid at the permeable surface. Beavers and Joseph (1967), Saffman (1971), Ochoa – Tapia and Whitaker (1995) proposed different slip conditions at the permeable walls. Mishra and Ramachandra Rao (2004) discussed the peristaltic pumping through a porous tube assuming Saffman slip condition at the permeable boundary. Channabasappa and Ranganna (1976) studied the flow of viscous stratified fluid of variable viscosity past a porous bed.

The gastrointestinal tract is surrounded by a number a number of muscle layer having smooth muscles. One of the important smooth muscle layers in gastrointestinal tract are submucosa and a layer of epithelial cells and these are responsible for absorption of nutrients and water in the intestine. These layers consist of many folds and there are pores throughout the tight junctions of them. In view of this the study of peristaltic transport with porous peripheral layer is important. In practical problems involving flow past a porous lining it is necessary to involve directly the thickness of the porous lining to have an increase in the mass flow rate. Motivated by this the peristaltic pumping of a Newtonian fluid in an inclined channel lined with porous material is investigated under long wavelength and low Reynolds number assumptions. The velocity distribution, the volume flow rate, the pressure rise and the frictional force are obtained. The effect of thickness of porous lining on the peristaltic pumping is discussed

Mathematical formulation and solution

Consider the peristaltic pumping of a viscous fluid in an inclined channel of angle β and of half-width 'a'. The channel is bounded by flexible walls which are lined with non-erodible porous material of thickness h^1. A longitudinal train of progressive sinusoidal waves takes place on the upper and lower walls of the channel. For simplicity we restrict our discussion to the half width of the channel as shown in Figure.1.

The wall deformation is given by

$$H(X,t) = a + b \sin\frac{2\pi}{\lambda}(x - ct) \tag{1}$$

where b is the amplitude, λ is the wavelength and c is the wave speed.

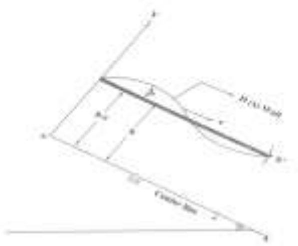

Figure 1: Physical M

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Equations of motion

Under the assumptions that the channel length is an integral multiple of the wavelength λ and the pressure difference across the ends of the channel is a constant, the flow becomes steady in the wave frame (x, y) moving with velocity c away from the fixed (laboratory) frame (X, Y). The transformation between these two frames is given by

$$x = X - ct;\ y = Y;\quad u(x, y) = U(X - ct, Y) - c;$$
$$v(x,y) = V(X - ct, Y)\ p^1(x) = P^1(X,\ t) \tag{2}$$

Where U and V are velocity components in the laboratory frame u, v are velocity components in the wave frame and p^1, P^1 are pressures in wave and fixed frame of references respectively. In many physiological situations it is proved experimentally that the Reynolds number of the flow is very small. So, we assume that the flow is inertia-free. Further, we assume that the wavelength is infinite.

Using the non-dimensional quantities,

$$\bar{u} = \frac{u}{c};\quad \bar{x} = \frac{x}{\lambda};\quad \bar{y} = \frac{y}{a};\quad \bar{h} = \frac{h}{a};\quad \in = \frac{h^1}{a}$$
$$\bar{p} = \frac{pa^2}{\lambda\mu c};\quad \bar{q} = \frac{q}{ac};\ \bar{t} = \frac{ct}{\lambda};\quad Da = \frac{k}{a^2}\ \phi = \frac{b}{a};\ \Psi = \frac{\psi}{ac}$$

The non-dimensional form of equations governing the motion (dropping the bars) becomes

$$0 = -\frac{\partial p^1}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \eta \sin\beta \tag{3}$$
$$0 = -\frac{\partial p^1}{\partial y} - \eta_1 \cos\beta \tag{4}$$

where $\eta = \frac{a_1^2 g}{\upsilon c}$ and $\eta_1 = \frac{a_1^3 g}{\upsilon c\lambda}$, g is the acceleration due to gravity.

Let $P^1 = P(x) - \eta_1 \cos\beta$

Then equation (3) becomes

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \eta \sin\beta \tag{5}$$

The non-dimensional boundary conditions are

$$\frac{\partial u}{\partial y} = 0 \quad \text{at } y = 0 \tag{6}$$
$$u = -\frac{\sqrt{Da}}{\alpha}\frac{\partial u}{\partial y} - 1 \quad \text{at} \quad y = h - \in \tag{7}$$

Solution

Solving equation (5) with the boundary conditions (6) and (7) we get the velocity as

$$u = (P - \eta \sin\beta)\left[\frac{y^2}{2} - \frac{\sqrt{Da}}{\alpha}(h - \in) - \frac{(h - \in)^2}{2}\right] - 1 \tag{8}$$

Integrating the equation (8) and using the condition $\psi = 0$ at y = 0, we get

$$\psi = (P - \eta \sin\beta)\left[\frac{y^3}{6} - \frac{\sqrt{Da}}{\alpha}(h - \in)y - \frac{(h - \in)^2}{2}y\right] - y \tag{9}$$

The volume flux 'q' through each cross section in the wave frame is given by

$$q = \int_0^{h-\in} u\, dy$$
$$= (P - \eta \sin\beta)\left[-\frac{(h - \in)^3}{3} - \frac{\sqrt{Da}}{\alpha}(h - \in)^2\right] - (h - \in) \tag{10}$$

The instantaneous volume flow rate $Q(x, t)$ in the laboratory frame between the centre line and the wall is

$$Q(X, t) = \int_0^{H-\in} U(X, Y, t)\, dY$$
$$= (P - \eta \sin\beta)\left[-\frac{(h - \in)^3}{3} - \frac{\sqrt{Da}}{\alpha}(h - \in)^2\right] \tag{11}$$

From equation (10), we have

$$\frac{dp}{dx} = \frac{-3(q + h - \in)}{(h - \in)^3 + \frac{3\sqrt{Da}}{\alpha}(h - \in)^2} + \eta \sin\beta \tag{12}$$

Averaging equation (11) over one period yields the time mean flow (time –averaged flow rate) $\bar{Q}$ as

$$\bar{Q} = \frac{1}{T}\int_0^T Q\ dt$$
$$= q + 1 \tag{13}$$

The pumping characteristics

Integrating the equation (12) with respect to x over one wavelength, we get the pressure rise (drop) over one cycle of the wave as

$$\Delta p = \int_0^1 \frac{-3(\bar{Q} - 1 + h - \in)}{(h - \in)^3 + \frac{3\sqrt{Da}}{\alpha}(h - \in)^2} dx + \eta \sin\beta \tag{14}$$

The pressure rise required to produce zero average flow rate is denoted by ΔP_0 . Hence ΔP_0 is

$$\Delta p_0 = \int_0^1 \frac{-3(h - \in - 1)}{(h - \in)^3 + \frac{3\sqrt{Da}}{\alpha}(h - \in)^2} dx + \eta \sin\beta \tag{15}$$

It is observed that as $Da \to 0$, $\beta = 0$ and $\in \to 0$, equations (9), (10) and (14) reduce to the corresponding result of Jaffrin and Shapiro (1971) for the peristaltic transport of the Newtonian fluid in a channel.

The dimensionless frictional force F at the wall across one wavelength in the inclined channel is given by

$$F = \int_0^1 h\left(-\frac{dp}{dx}\right)dx$$
$$F = \int_0^1 \frac{-3h(\bar{Q} - 1 + h - \in)}{(h - \in)^3 + \frac{3\sqrt{Da}}{\alpha}(h - \in)^2} dx + \eta \sin\beta \tag{16}$$

Discussions of the Results:

From equation (14), we have calculated the pressure difference as a function of $\bar{Q}$ for different values of ϵ, for fixed $\varphi = 0.6, Da = 0.01, \beta = \frac{\pi}{4}$ $and\ \alpha = 0.001$ is shown in figures (2) to (4). It is observed that for a given flux $\bar{Q}$ the pressure difference ΔP increases with increasing ϵ. Further it is observed that the effect of the porous lining on the walls of the channel is to increase the pressure rise in the channel. We also observe that the increase in the inclination of the angle $\beta\left(0 \leq \beta \leq \frac{\pi}{2}\right)$, will give rise to an increase in the pressure difference ΔP.

From equation (14), we have calculated the pressure difference as a function of $\bar{Q}$ for different values of Da and β for fixed $\varphi = 0.6, \alpha = 0.001 \text{ and } \eta = 2$ and is shown in figures (5), (6) and (7). It is observed that for a given flux $\bar{Q}$ the pressure rise depends on Da and it decreases with increasing Darcy number. For a given ΔP, the flux $\bar{Q}$ decreased with increasing Da. For free pumping there is no difference in flux for variation in Darcy number. From figures (5) to (7) we observe that the pump work against more pressure rise for a vertical channel when compared with a horizontal channel.

The variation of pressure rise with time averaged flow rate is calculated from equation (14) for different amplitude ratios and is shown in fig (8) for fixed $\alpha = 0.001, \eta = 1, \beta = \frac{\pi}{4} \text{ and } Da = 0.01.$ We observe that the larger the amplitude ratio, the greater the pressure rise against which the pump works. For a given ΔP, the flux $\bar{Q}$ depends φ and it increases with increase in φ finally, from equations (14), we have calculated the frictional force as a function of $\bar{Q}$ for a fixed $\varphi = 0.6$ and for different values of ϵ, Da and is depicted in figures (9) to (12). It is observed that the frictional force F has the opposite behaviour compared with pressure rise ΔP. It is observed that as $\beta = 0, Da \rightarrow 0$ and $\in \rightarrow 0$ the results are reduced to the Jaffrin and Shapiro (1971) for the peristaltic transport of the Newtonian fluid in a channel.

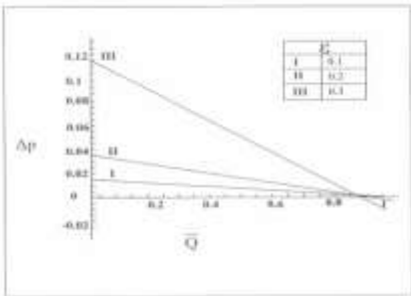

Fig 2: The variation of ΔP with $\bar{Q}$ for different values of ε with Da=0.01, $\beta = 0,\ \alpha = 0.01,\ \phi = 0.6,\ \eta = 2$

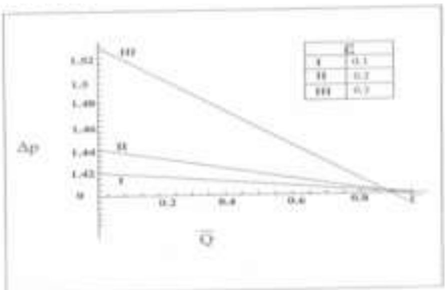

Fig 3: The variation of ΔP with $\bar{Q}$ for different values of ε with Da=0.01, $\beta = \frac{\pi}{4},\ \alpha = 0.01,\ \phi = 0.6,\ \eta = 2$

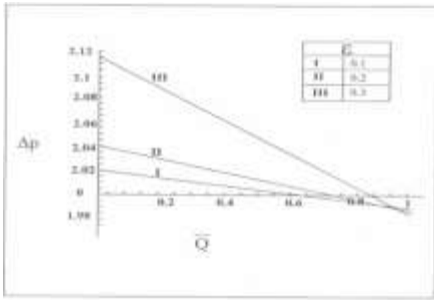

Fig 4: The variation of ΔP with $\bar{Q}$ for different values of ε with Da=0.01, $\beta = \frac{\pi}{2},\ \alpha = 0.01,\ \phi = 0.6,\ \eta = 2$

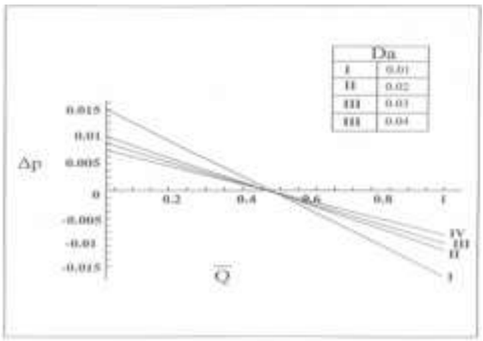

Fig 5: The variation of ΔP with $\bar{Q}$ for different values of Da with $\beta = 0, \varepsilon = 0.1,\ \alpha = 0.01,\ \phi = 0.6,\ \eta = 2$

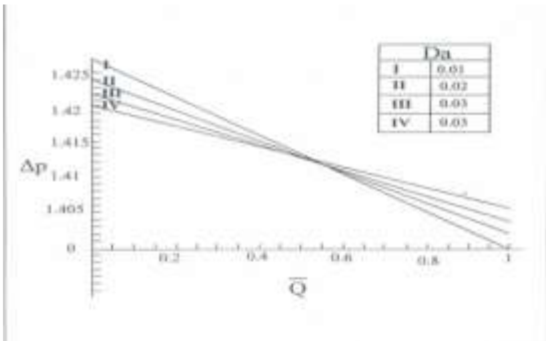

Fig 6: The variation of ΔP with $\bar{Q}$ for different values of with $\beta = \frac{\pi}{4}, \varepsilon = 0.1,\ \alpha = 0.01,\ \phi = 0.6,\ \eta = 2$

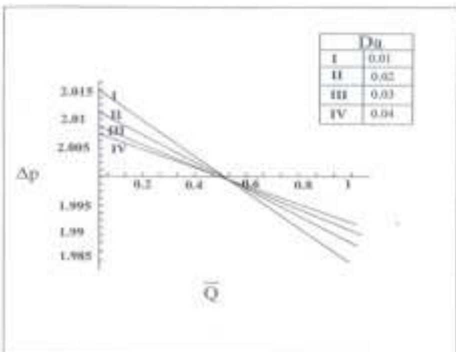

Fig 7: The variation of ΔP with $\bar{Q}$ for different values of Da with $\beta = \frac{\pi}{2}, \varepsilon = 0.1,\ \alpha = 0.01,\ \phi = 0.6,\ \eta = 2$

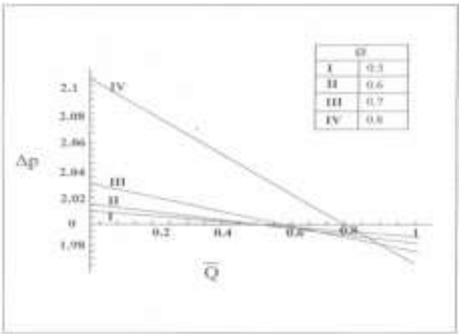

Fig 8: The variation of ΔP with $\overline{Q}$ for different values of ϕ with $\beta = \frac{\pi}{2}, \varepsilon = 0.1,\ \alpha = 0.01,\ Da = 0.01,\ \eta = 2$

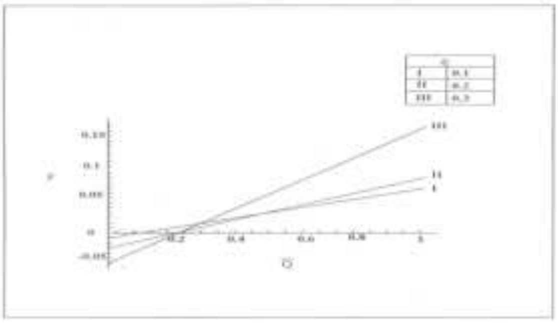

Fig 9: The variation of F with $\overline{Q}$ for different values of ε with $\beta = 0, \phi = 0.6,\ \alpha = 0.01,\ Da = 0.01,\ \eta = 2$

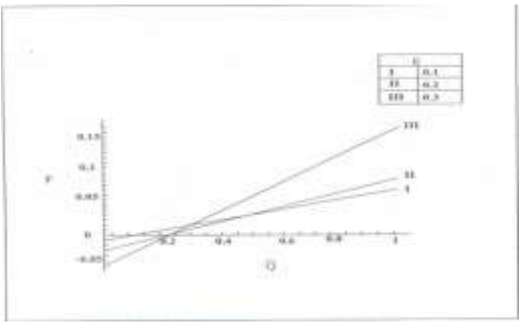

Fig10: The variation of F with $\overline{Q}$ for different values of ϕ with $\beta = \frac{\pi}{2}, \varepsilon = 0.1,\ \alpha = 0.01,\ Da = 0.01,\ \eta = 2$

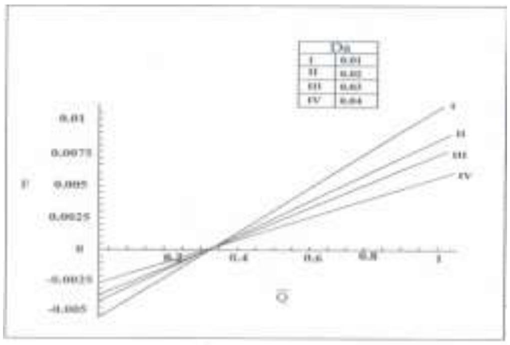

Fig11: The variation of F with $\overline{Q}$ for different values of Da with $\beta = 0, \varepsilon = 0.1,\ \alpha = 0.01,\ \phi = 0.6,\ \eta = 2$

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