Available online at www.elixirpublishers.com (Elixir International Journal)

Statistics

Elixir Statistics 48 (2012) 9420-9423

ISSN: 2229-712X

A fuzzy queuing theoretic approach to child care quality and regulations

W. Ritha[1] and Sreelekha Menon.B[2]

[1]Department of Mathematics, Holy Cross College,Tiruchirappalli- 620002.

[2]Department of Mathematics, SCMS School of Engineering and Technology, Angamaly, Kerala -683 582.

ARTICLE INFO

Article history:

Received: 12 May 2012;
Received in revised form: 15 June 2012;
Accepted: 10 July 2012;

Keywords

Fuzzy risk analysis,
Generalized fuzzy numbers Function principle,
Graded Mean Integration Representation.

ABSTRACT

Child care has become an essential component of life in our society .Quality child care can make a significant difference in child's development .Here we consider some regulations in favour of lower child staff ratio ,higher educational standards for care givers and smaller group size . In this paper we present a methods for fuzzy risk analysis based on, child- staff ratio group size and care giver ability. We obtain the expected number of children in queue needing attention at any time by using both function principle and Graded Mean Integration method. Furthermore we calculate the proportion of time that a child would spend engaged by using both function principle and Graded Mean Integration method. The model presented in this paper can be applied to other educational services also

Introduction

The increase in female labor has resulted in the increased use of non-relative child care settings chosen namely Day care. The parents of the child usually find it difficult to assess the quality of child care. For many years researchers have been examining the aspect of child care that have a positive influence on children's development .Research data has confirmed that the child care provider is one of the most important element in quality child care .

Good staffing ratios are an essential ingredient in quality child care settings. Galinsky and__Phillips (1988) [8] recommended a ratio of at least one child care provider for every three to four infants and a child care provider for every four to six children under three. The study also found that centers with low child-staff ratios were seen as providing higher quality care. A quality environment is well planned and invites children to learn and grow. A key aspect of providing a good environment for children is the safety of the settings

Quality child care programs can provide a wealth of comprehensive services which contribute to the overall welfare of the children and their families. It has the capability of promoting trust, autonomy, and a true sense of happiness and well being in children. It should be given a high priority in our nation and be supported by all in our society.

Public intervention in day care markets might be justified either using arguments commonly made for the public provision or using arguments regarding imperfect information's .State regulators, recognizing both the informations and public good problems have imposed various minimum quality standards on day care providers. The proper role of government is over sight and regulation has emerged as an important issue in the national debate about child care . little is known about the effectiveness of these regulations .There is little research for example ,that provides evidence on the functional relationship between marginal changes in child staff ratios or group size and some measure of child care quality .Regulations may specify the number of mandated annual inspections ,maximum group size limits ,training requirements for care givers ,health and safety rules and minimum staff-child ratio.

In this paper we construct a fuzzy queuing model of the child care environment and we will use it to find the effects of regulations on quality of child care. Through this model we can find a relation between the three major regulated variables namely child –staff ratio ,group size and care giver qualifications . The task of handling risk analysis plays an important role . Schmucker (1984) [18] _presented a method for fuzzy risk analysis based on fuzzy number arithmetic operations .It is obvious that many facts can affect the result of a similarity measure –for example – the shapes of fuzzy numbers ,the position of fuzzy numbers , the area of fuzzy numbers etc. Hsieh and Chen (1999) [11] presented a similarity measure between fuzzy numbers using Graded Mean Integration Representation method .

The rest of the paper is organized as follows .In section 2 we discuss the membership function of trapezoidal fuzzy numbers and represent the trapezoidal fuzzy number by Graded Mean Integration Representation method . Also to calculate the fuzzy average we define the mean of trapezoidal fuzzy numbers . In section 3 we discuss some arithmetical operations of fuzzy numbers by Chen's function principle (1985)[3] for simplifying calculations .In section 4 we will discuss a fuzzy queuing theoretic approach to child care . In section we will provide two numerical examples and section 6 will be conclusion

Section 2: Membership Function And Mean Of Trapezoidal Fuzzy Numbers

The fuzzy numbers were introduced by Zadeh [23] and arise in decision making , control theory , fuzzy system and approximate reasoning problems .A fuzzy number $\tilde{M}$ is a convex normalized fuzzy set of $\tilde{M}$of the real line $\mathcal{R}$ such that

i. There exist exactly one $x_0 \in \mathcal{R}$ with $\mu_{\tilde{M}}(x_0)$ = w (x_0 is called the mean value of $\tilde{M}$)

Tele:
E-mail addresses: ritha_prakash@yahoo.co.in

ii. $\mu_{\tilde{M}}(x)$ is piecewise continuous ,where $\mu_{\tilde{M}}(x)$ is the membership function

Now a days this definition is modified very often .For the sake of computational efficiency and ease of data acquision ,the trapezoidal fuzzy numbers are used .

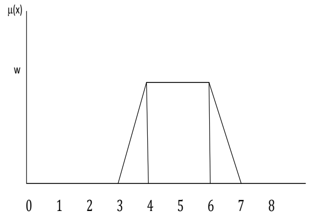


Figure 1

Fig 1 shows such a fuzzy set which could be called" approximately 5 " and which would normally be defined as the quadruple { 3,4,6,7} .Strictly speakins it is a fuzzy interval .however it is not a random variable . A random variable is defined in terms of theory of probability ,which has evolved from theory of measurement . A random variable is an objective ndatum whereas a fuzzy number is a subjective datum . It is a value not a measure .

Suppose $\tilde{M}$ is a trapezoidal fuzzy number as shown in figure 1 . Chen (1999)[4] represented a generalized trapezoidal fuzzy number $\tilde{M}$ as $\tilde{M}$ = (a,b,c,d :w) where a,b,c,d are real values and 0 < w $\le$ 1 . If w=1 , then the generalized fuzzy number , denoted as $\tilde{M}$ =(a,b,c,d) is a normal fuzzy number as shown in fig 1. The membership function $\mu_{\tilde{M}}$ of the fuzzy number $\tilde{M}$ satisfies the following conditions

1) $\mu_{\tilde{M}}(x)$ is a continuous mapping from $\mathcal{R}$ to [0,1].

2) $\mu_{\tilde{M}}(x) = 0 ; -\infty < x \le a$

3) $\mu_{\tilde{M}}(x)$ = L(x) is strictly increasing on [a,b]

4) $\mu_{\tilde{M}}(x) = w ; b \le x \le c$

5) $\mu_{\tilde{M}}(x)$ = R(x) is strictly decreasinf on [c,d]

6) $\mu_{\tilde{M}}(x) = 0 ; d \le x < \infty$

Chen et al introduced Graded Mean Integration Representation method based on the integral value of graded mean h-level of a generalized fuzzy number for defuzzifing generalized fuzzy numbers . In Graded Mean Integration Representation method L^{-1} and R^{-1} are the inverse functions of L and R respectively and the graded h-level value of generalized (a,b,c,d;w) is $\frac{h[L^{-1}(h)+R^{-1}(h)]}{2}$ as shown in fig 2 . Then the Graded Mean Integration Representation of $\tilde{M}$ is P($\tilde{M}$) with grade w where P($\tilde{M}$) = $\frac{\int_0^w h\frac{[L^{-1}(h)+R^{-1}(h)]}{2}dh}{\int_0^w h\,dh}$ with 0< h $\le$w and 0< w $\le$ 1.

Through out this paper only normal trapezoidal fuzzy numbers are considered .

For example :- let the membership function of $\tilde{M}$ be

$$\mu_{\tilde{M}}(x) = \begin{cases} \frac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \frac{x-d}{c-d}, & c \le x \le d \\ 0, & \text{otherwise} \end{cases}$$

$L(x) = \frac{x-a}{b-a}$, $a \le x \le b$

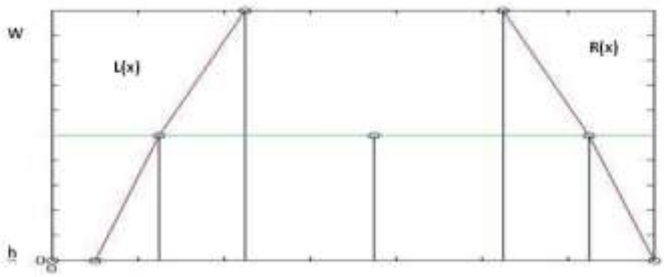
$L^{-1}(h) = a + h(b-a)$, $0 \le h \le 1$

$R(x) = \frac{x-d}{c-d}$, $c \le x \le d$

$R^{-1}(h) = d + h(c-d)$, $0 \le h \le 1$

Thus we get the graded mean representation as

$$P(\tilde{M}) = \frac{\int_0^1 \frac{h[a+d+(b-a-d+c)h]}{2}}{\int_0^1 h\,dh} = \frac{a+2b+2c+d}{6}$$



a $L^{-1}(h)$ b $\frac{h(L-1(h)+R-1(h))}{2}$ c $R^{-1}(h)$ d

Figure 2

The graded k-preference h level value of a generalized fuzzy number

$\tilde{M}$ = (a,b,c,d :w) is given by

$$P_k(\tilde{M}) = \frac{\int_0^W h[kL^{-1}(h)+(1-k)R^{-1}(h)]dh}{\int_0^W h\,dh}$$,where 0 < h $\le$ w ; 0 < w $\le$ 1 and

$0 \le k \le 1$.

Thus the graded k-preference integration of $\tilde{M}$ is

$$P_k(\tilde{M}) = \frac{k(a+2b)+(1-k)(2c+d)}{3}$$

Section B1 : The Fuzzy Arithmetical Operations Under Function Principle

Chen proposed function principle to be as the fuzzy arithmetical operations by membership function of fuzzy numnbers .Here we describe some fuzzy arithmetical operations under function principle as follows .

Suppose $\tilde{X}$ = (x_1, x_2, x_3, x_4) and $\tilde{Y}$ = (y_1, y_2, y_3, y_4) are two trapezoidal fuzzy numbers.

1. The addition of $\tilde{X}$ and $\tilde{Y}$ is

$\tilde{X} \oplus \tilde{Y} = (x_1 + y_1, x_2 + y_2, x_3 + y_3, x_4 + y_4)$, where $x_1, x_2, x_3, x_4, y_1,$ y_2, y_3, y_4 are real numbers .

2. The multiplication of $\tilde{X}$ and $\tilde{Y}$ is

$\tilde{X} \otimes \tilde{Y} = (t_1, t_2, t_3, t_4)$ where $T = (x_1 y_1, x_1 y_4, x_4 y_1, x_4 y_4)$

$T_1 = (x_2 y_2, x_2 y_3, x_3 y_2, x_3 y_3)$, $t_1 = \min T$, $t_2 = \min T_1$, $t_3 = \max T_1$,
$t_4 = \max T$

If $x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4$ are positive real numbers then,

$\tilde{X} \otimes \tilde{Y} = (x_1 y_1, x_2 y_2, x_3 y_3, x_4 y_4)$

3. $-\tilde{Y} = (-y_4, -y_3, -y_2, -y_1)$ Then the subtraction of $\tilde{X}$ and $\tilde{Y}$ is

$\tilde{X} \ominus \tilde{Y} = (x_1 - y_4, x_2 - y_3, x_3 - y_2, x_4 - y_1)$ where $x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4$ are real numbers .

4 . $\frac{1}{\tilde{Y}} = \tilde{Y}^{-1} = \left(\frac{1}{y_4}, \frac{1}{y_3}, \frac{1}{y_2}, \frac{1}{y_1}\right)$ where , y_1, y_2, y_3, y_4 are all positive real numbers .

If $x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4$ are all non-zero positive real numbers , then the division of $\tilde{X}$ and $\tilde{Y}$ is

$\tilde{X} \oslash \tilde{Y} = \left(\frac{x_1}{y_4}, \frac{x_2}{y_3}, \frac{x_3}{y_2}, \frac{x_4}{y_1}\right)$

5. Let $\alpha \in R$, then

i. $\alpha \geq 0$, $\alpha \otimes \tilde{X} = (\alpha x_1, \alpha x_2, \alpha x_3, \alpha x_4)$

ii. $\alpha < 0$, $\alpha \otimes \tilde{X} = (\alpha x_4, \alpha x_3, \alpha x_2, \alpha x_1)$

Section 4 : A Fuzzy Queuing Approach To Child Care

Here we will discuss about a fuzzy queuing theoretic approach to child care .A fuzzy queuing model can be presented for examining the relationship between the key regulatory variables and the expected portion of time that a child is engaged ″on –task ″ in some productive activity without waiting for the assistance of a care giver . We also explain how the technique may be used by child care provider to determine the effects of altering the child staff ratio . The fuzzy queuing model can also be used for assessing staffing needs . It also provides a direct measure of the degree and intensity of the individual interactions between a care giver and a child . The caregiver –child interactions is one of the most important measure of quality to researchers in recent years .

In child care fuzzy queuing model the arrivals are the number of children needing assistance which is a fuzzy number ; the service process is the time taken by the caregiver to interact with each child who needs assistance which is also a fuzzy number – both of which follow some probability distribution. The arrival process and the the service process can be characterized by Erlang Distribution .The notation used for arrival process and service process are respectively λ and μ . The exponential assumption for interarrival times and service times is equivalent to assuming that the arrival rate and service rate are random variables that follow a Poisson distribution .The mean interarrival time is the reciprocal of the mean breakdown rate while the mean service time is the reciprocal of the mean service rate . The probability that a child who needs attention will have to wait in a queue for assistance depends on how often children are likely to need help ,the number of children in the group and on the number of caregivers .

For a group consisting of C children and a single caregiver ,the expected number of children in queue needing attention (queue length) at any point of time is

$L_q = C\Theta(1\Theta P_0)\otimes(1\oplus\mu\oslash\lambda)$ [12] , where P_0 is the probability that no child is in need of attention . The portion of the time that a child would spent engaged or ″on – task ″ is

OTT = $1\Theta(L_q \oslash C)$ [12]

Section 5 : Numerical Example 1

Consider a child care environment with one caregiver and 2 infants . Let the arrival rate be (2•1 ,2•2,2•3,2•4) which is a fuzzy number with fuzzy service rate (1•1 , 1•2 ,1 •3 , 1•4) and the fuzzy probability that no child is in need of attention be (0•1 , 0•2 , 0•3 , 0•4) . Using function principle we try to calculate the average queue length and the portion of time that a child is ″on – task ″.

Here C=2 ,μ =(1•1 , 1•2 ,1 •3 , 1•4) ,λ =(2•1 ,2•2,2•3,2•4) and P_0= (0•1 , 0•2 , 0•3 , 0•4)

$L_q = C\Theta(1\Theta P_0)\otimes(1\oplus\mu\oslash\lambda)$

= 2 Θ (0•6 , 0•7, 0•8 , 0•9) ⊗ (1•45833, 1•52173 ,1 •59091 , 1•66667)

= (0•499997 , 0•727272 , 0•934789 , 1•25002)

Let L = (0•5 , 0•7 , 0•9 , 1)

OTT $= 1\Theta(L_q \oslash C)$

= 1Θ (0•2499985, 0•363636 , 0•4673945 , 0•62501)

= (0•37949 , 0•5326055 , 0 • 636364 ,0•7500015)

Let OTT = (0•3 , 0•5 , 0•6 , 0•7)

The graded k-preference value of L and OTT can be calculated using the formula

$P_k(\tilde{M}) = \frac{k(a+2b)+(1-k)(2c+d)}{3}$ Where k = 0, 0•1 ,0•2 , …………….0•9, 1

The graded Mean Representation of L and OTT can be calculated using the formula

$P(\tilde{M}) = \frac{a+2b+2c+d}{6}$

The calculated values are given in the following table

k	$P_k(\tilde{L})$	$P(\tilde{L})$	$P_k(\widetilde{OTT})$	$P(\widetilde{OTT})$
0	0•93333	0•78333	0•63333	0•53333
0•1	0•90333	0•78333	0•61333	0•53333
0•2	0•87333	0•78333	0•59333	0•53333
0•3	0•84333	0•78333	0•57333	0•53333
0•4	0•81333	0•78333	0•55333	0•53333
0•5	0•78333	0•78333	0•53333	0•53333
0•6	0•75333	0•78333	0•51333	0•53333
0•7	0•72333	0•78333	0•49333	0•53333
0•8	0•69333	0•78333	0•47333	0•53333
0•9	0•66333	0•78333	0•45333	0•53333
1	0•63333	0•78333	0•43333	0•53333

Numerical Example 2

Consider another child caregiver with one care giver and 4 infants . In this environment let the arrival rate and service rate are fuzzy numbers respectively as λ = (2•5 , 2•6 , 2•7 , 2•8) and μ = (1•5 , 1•6 , 1•7 , 1•8) . Let P_0 = (0•01 , 0•02 , 0•03 , 0•04)be also a fuzzy number . We are trying to calculate the average queue length and the portion of the time a child is ″on – task ″using function principle .

Here C=4 ,μ =(1•5 , 1•6 ,1 •7 , 1•8) ,λ =(2•5 ,2•6,2•7,2•8) and

P_0 = (0•01 , 0•02 , 0•03 , 0•04)

L_q = C⊖(1⊖P_0)⊗(1⊕μ∅λ)

= 4⊖ (0•96 , 0•97, 0•98 , 0•99) ⊗ (0•5357, 0•5926 ,0•6538 , 0•7200)

= (3•2872 , 3•3593 , 3•4252 , 3•4858)

OTT = 1⊖(L_q ∅C)

= 1⊖ (0•8218, 0•83983 , 0•8563 , 0•87145)

= (0•12855 0•1437 , 0•16017 , 0•1782)

The graded k-preference value of L and OTT can be calculated using the formula

$$P_k\left(\tilde{M}\right) = \frac{k(a+2b)+(1-k)(2c+d)}{3}$$ Where k = 0, 0•1 ,0•2 , …………….0•9, 1

The graded Mean Representation of L and OTT can be calculated using the formula

$$P(\tilde{M}) = \frac{a+2b+2c+d}{6}$$

The calculated values are given in the following table

k	$P_k\left(\tilde{L}\right)$	$P(\tilde{L})$	$P_k\left(\widetilde{OTT}\right)$	$P(\widetilde{OTT})$
0	3•4454	3•39033	0•16618	0•152415
0•1	3•4343	3•39033	0•16343	0•152415
0•2	3•4234	3•39033	0•16067	0•152415
0•3	3•4124	3•39033	0•15792	0•152415
0•4	3•4013	3•39033	0•15517	0•152415
0•5	3•3903	3•39033	0•15242	0•152415
0•6	3•3793	3•39033	0•14966	0•152415
0•7	3•3683	3•39033	0•14691	0•152415
0•8	3•3573	3•39033	0•14416	0•152415
0•9	3•3463	3•39033	0•14140	0•152415
1	3•3353	3•39033	0•13865	0•152415

Conclusion

In this paper we developed a fuzzy queuing model which will clarify the importance of child staff ratio ; group size and caregivers ability in child care in order to understand better potential costs and benefits of regulating them . Here we used function principle to simplify the calculations of fuzzy average queue length and fuzzy proportion of time that a child would engaged in some activity. In the fuzzy environment, it may be possible and reasonable to discuss these models with k-preference fuzzy value . The model is also executable and useful in the real world .For a special case that all variables are set as real numbers, the result will be same as traditional non-fuzzy model .

References

1. Chen .S .H and C.H Hsieh ; Graded mean Integration Representation of Generaliuzed Fuzzy Numbers ‘ Proceedings of 6th Conference on Fuzzy Theory and its Application , Chinese Fuzzy System Association , taiwan PP 1-6 (1998) .

2. Chen .S .H and C.H Hsieh ; Representation of Ranking ,Distance and Similarity of l-R type Fuzzy Number and Applications ; Australian Journal of intelligent Processing Systems ; Vol ;6 PP 217 – 229 (20000 .

3. Chen S.H (1985) ; operations on Fuzzy Numbers with Function Principle ; tamkang journal of Management Science 6 (10 13-25 .

4. Chen S.H (1999) ; Ranking Generalized Fuzzy Number with Graded Mean Integration ; ; in the proceedings of 8th International Fuzzy System Association World Congress Vol 2

5. Chih Hsun Hsieh ; Optimization of Fuzzy Inventory Models Under k- preference ; Tamsui oxford journal of Mathematical Science 19 (1) (2003) 79-97

6. Chih Hsun Hsien ; Optimization of Fuzzy Inventory Model Under Fuzzy Demand and Fuzzy Lead Time ; Tamsui Oxford Journal of Management Science ; Vol ;20 ,No;2 PP 21-36 .

7. D. Dubois , H.Prade ; optimization on Fuzzy Numbers ; International Journal of System Science 9 (19780 613 -626

8. Galinsky .E and Philips .D (1988) ; The Daycare debates Parents 63, 114-115

9. Gross .D and Harris C (1985) Fundamentals of Queuing Theory 2nd Edition Wiley New York .

10. Hsieh .C .H (2002) Optimization Of Fuzzy Production Inventory Models ; Information Sciecnce – An Internatioanl Journal 146 PP 29-40 .

11. Hsieh c.H and Chen S.H (19990 ; Similarity of Generalised Fuzzy Numbers with Mean Integration Representation ; in the proceedings of 8th International Fuzzy System Association World Congress Vol 2 PP 551-555 ;Taipei ,Taiwan , Republic of China .

12. James .J. Mulligan and Saul .D .Hoffman ; Day care Quality and Regulations : A queuing theoretic Approach ; Ecnomics of Education Review ; Elsivier Science Ltd (1998) Vol.17 No;1 PP1-13

13. Kaufmann A and M M Gupta ; ″ Introduction to Fuzzy Arithmetic Theory and Applications ″ ; Van Nostrand Reinhold (1991) .

14. National Research Council (1991) ″ Who Carea For American’s Children ″ national Research Council ; Washington DC .

15. P.S Kechagras and Basil .k .Papadopoulos ; Computational Method to Evaluate Fuzzy Arithmetic operations ; Science Direct – Applied Mathematics and Computations 185 (2007) 169-177. PP 899-902;Taipei ,Taiwan , Republic of China .

16. S.H Chen and C.H .Hsieh ; A new Method of Representing Generalized Fuzzy Number ; Tamsui Oxford Journal of management Sciences Vol 13-14 (1998) 133-143 .

17. S.H Chen and C.H .Hsieh ; Graded mean Integration Representation of Generaliuzed Fuzzy Numbers ; Journal of Chinese Fuzzy Systems Association ;Vol 5 No. 2 PP 1-7 (1999)

18. Schumucker .k.J (1984) Fuzzy Sets ,Natural Languages ,Computations and Risk Analysis ; m.d Computer Science Press

19. Shan Huo Chen , Chien –Chung Wang and Shu Man Chang ; Fuzzy Economic production Quality Model For items With imperfect Quality ; international jpournal of innovative Computing , Information and Control Vol 3 ,No;1 Feb 2007 .

20. Shih – Hua Wei ,Shyi-Ming Chen ; A New Approach for Fuzzy Risk Analysis Based on Similarity Measures of Generalised Fuzzy Numbers ; Science Direct – Expert System with Application 36 (2009) 589-598.

21. Taha .h .A ; operations Research ; Prentice Hall New Jersey USA 1997 .

22. Tasneem Chipty and Ann Dryden Witte ; Economic Effects of Quality Regulations in Daycare Industry ; NBER working paper 4953-Dec 1994 .

23. Zadeh .L .A ; Fuzzy Sets ; Information and Control Vol .8 PP 338-353 (1965).