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A Tabu search algorithm for distribution network design as a set partitioning problem

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ABSTRACT

Set partitioning problems are known to be NP-hard, thus it requires massive amounts of times and efforts to solve them using linear programming and traditional algorithms. This study proposes to use a tabu algorithm for such problems. The proposed algorithm is applied to the distributing network design problem of a wholesale company. The orders of 420 customers from 24 cities in the region are currently satisfied from a central warehouse of the company. The proposed algorithm in this study aims to set up groups of customers based on their geographic locations with the goal of minimizing the total amount of traveling, thus cost and time throughout the network. Experimental results show that the proposed algorithm reduces the total traveling by a considerable amount of 10%. The possible scenarios for alignment of the customers are suggested at the end of the study as a way of implementing the findings of this research.

Introduction

Distribution systems can be defined as a network established by the units inside the firm and the agencies, wholesalers, retailers outside the firm in order to market the goods and services (Tek, 1997). Although designing distribution systems has been an important issue for the last few decades, it is relatively new to design a complete distribution system to optimality using quantitative techniques. Network configuration involves issues relating to plant, warehouse, and retail location. These are strategic decisions because they have a long-lasting effect on the firm. Designing a supply chain consists of configuring the network so as to satisfy customer demands while minimizing fixed costs for facility construction or leasing and technology acquisition, and variable costs for production, storage and transportation. Because of the complexity of the problem, it is often decomposed into several components treated separately (Simchi-Levi etal, 2000).

The research at hand involves such a problem of designing a distribution network of a wholesale distributor operating in a large geographic area, covering nearly one-fourth of the country geographically. Traditionally, the company holds extensive amount of inventory in its single warehouse which is located within the headquarters. The orders received from the retailers are grouped based on the routes assigned to each salesperson's area and single trucks are routed to deliver several orders sequentially. The routing is solely based on the experience of the sales department which basically uses traditionally accepted routes without extensive effort to optimize neither the travelling time nor the cost.

Recently, the company is involved in search for a professional guidance to improve the distribution system as a tool to reduce the lead time and increase the customer satisfaction with timely deliveries, resulting in increased level of efficiency. The efforts are focused on adding two more warehouses to satisfy the orders from various parts of the region which is fairly away from each other. Including the current one, the three warehouses reflects the idea of dividing the region into three groups based on their geographic position and covering each group of customers from a separate warehouse assigned to it. However, determining these groups requires considering the distances between each customer in order to optimize the total traveling distances. Having 420 customers scattered throughout a large region imposes a challenge to come up with an optimum solution without appropriate quantitative techniques, one of which is optimization algorithms. The number of customers in each city covered by the distribution network is listed in the table below:

Table 1. Customer Numbers by City

Code	City	# of Customers	Code	City	# of Customers
1	Elazığ	67	13	Iğdır	7
2	Adıyaman	22	14	Kars	19
3	Ağrı	12	15	Malatya	39
4	Ardahan	9	16	Maraş	16
5	Batman	6	17	Mardin	11
6	Bayburt	3	18	Muş	6
7	Bingöl	15	19	Siirt	2
8	Bitlis	10	20	Sivas	27
9	Diyarbakır	35	21	Şırnak	14
10	Erzincan	1	22	Tunceli	1
11	Erzurum	51	23	Urfa	7
12	Hakkari	2	24	Van	38

Dividing a number of elements into a number of groups can be approached as a set partitioning problem (SP). This type of problems has a large field of application including workforce planning, vehicle routing, and planning for the best performance of vehicles and machines. This research utilizes a tabu algorithm to solve the SP problem on hand. The application consists of grouping 420 customers located in 24 cities which have to be visited frequently by the salesmen and the delivery trucks of the

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company. The strategy of the company is to group the customers into three of which consisting even number of customers. Depending on the volume of orders of each customer and the frequency of orders, the groups suggested by the algorithm used in this research will be analyzed by the decision makers and necessary adjustments will be undertaken in future research. An optimal solution must be found to minimize the total traveling time throughout the group, thus minimizing the overall cost and time required. Optimal solution can be found by setting up a 0-1 integer linear programming model and solving it to optimality (Mitchell, 2003).

Mathematical Model for Set Partitioning Problem

The set partitioning problem arises when each set element must appear in exactly one subset, and the constraints in this problem are equality constraints. The set partitioning problems can be formulated as follow (Van Krieken etal, 2003):

$$Min\ Z = \sum_{j=1}^{n} C_j X_j \qquad (1)$$

subject to:

$$\sum_{j=1}^{n} a_{ij} X_j = 1 \qquad i = 1, \dots, m \qquad (2)$$

$$X_j \in \{0,1\} \qquad j = 1, \dots, n \qquad (3)$$

C_j in the objection function is the cost of j^{th} subset, thus $C_j \geq 0$.

$X_j = 1$, if the j^{th} subset is one of the subsets to be chosen for the optimal solution; otherwise, $X_j = 0$.

$a_{ij} \in \{0,1\}$. a_{ij} is a 0-1 matrix of mxn dimension. $a_{ij} = 1$, if the i^{th} unit is in the j^{th} subset; otherwise, $a_{ij} = 0$.

m , is the number of units to be divided into subsets.

n , is the number of subsets.

$Eq.(2)$ ensures that each unit is placed in only one subset.

Mitchell models the realignment problem as a *k*-way equipartition problem, where the nodes of a graph are divided into *k* equally sized sets in order to minimize an appropriate objective function. In this model, the teams are regarded as the vertices of a graph and an edge between two teams has weight equal to the distances between the two corresponding cities. The teams are partitioned into eight divisions, each division containing four teams. The total intra-divisional travel distance is the sum of the edge weights of all edges that have both endpoints in the same division. The objective is to minimize this sum. The branch-and-cut algorithm is utilized to solve the problem. This constructs an integer programming formulation of the problem. It then solves a linear programming relaxation of the problem. The solution to the relaxation can be used to generate feasible integer solutions (Mitchell, 2003).

A Tabu Search Algorithm to Solve SP

Due to the complexity of a great variety of combinatorial optimization problems and insufficiency of the traditional algorithms in solving these problems, new techniques are required to solve such problems. The main drawback of algorithms such as deterministic exchange procedures is their inability to continue the search upon becoming trapped in a local optimum. This invites consideration of recent techniques for guiding known heuristics to overcome local optimality. Tabu search is proven to be one of the most effective algorithms in this sense.

Tabu search is a metaheuristic, a master strategy that forces a local heuristic search procedure to explore the solution space beyond local optimal (Glover and Laguna, 1997). Current applications of tabu search span a wide area of application such as resource planning, telecommunications, VLSI design, financial analysis, scheduling, space planning, energy distribution, molecular engineering, logistics, pattern classification, flexible manufacturing, waste management, mineral exploration, biomedical analysis, environmental conservation and others. A distinguishing feature of tabu search is embodied in its exploitation of adaptive forms of memory, which equips it to penetrate complexities that often confound alternative approaches.

This basic tabu search mechanism is often so effective that it suffices for the problem under investigation. Unfortunately, the basic tabu search approach fails for some problems and the implementation of more advanced, readily available concepts is required.

Xu and Kelly (1999) use a complex tabu search algorithm that is able to produce high quality solutions for the basic vehicle routing problem with capacity constraints only. The algorithm uses a mixture of the two neighborhood moves, the ejection chain based move and the swap move that exchanges two customers between routes. Paraskevopoulos et al. (2008) use a two-phase approach. In the first phase several initial solutions are produced using a semi-parallel construction heuristic for different combinations of parameter settings, followed by a route elimination procedure to reduce the number of vehicle routes and improve the vehicle's utilization using an enhanced ejection-chain heuristic, and finally, a subset of high quality solutions is selected for further improvement. In the second phase, a reactive variable neighborhood tabu search method is used to minimize the total distribution cost. Tabu search explores the solution space by moving, at each iteration, from a solution *x* to the best solution in the neighborhood $N(x, \sigma)$.

Dammeyer and Voß (1993) consider a version of tabu search to prevent the search from endlessly cycling between the same solutions.

In this research, we explore ways to make use of improvement heuristics to build solutions that are competitive or better than those obtained by the more complicated algorithms. The heuristics utilized in this research are easy to comprehend and work fast, also flexible enough to handle practical constraints.

TS Solution Procedure

The company works with 420 customers in its region regularly. These customers are located in 24 cities which are assigned a number to be represented by during the solution procedures. The company aims to establish two more warehouses in addition to the one in its central location. Thus, resulting in three groups of customers each served by a warehouse assigned to it. Naturally, each group consists of 8 cities. The salesmen regularly visit the customers in their region sequentially during the business trips in certain time periods. The same scenario goes for the delivery trucks as well. The trucks loaded with orders of several customers distribute the goods in a sequential manner. Thus, the smaller the distances between the customers, the smaller the total cost throughout the network. Though many arrangements of the groups can be utilized historically, one possibility is to choose the divisions so as to minimize the total travel distances. This objective has the benefit of placing nearby cities in the same group, thus resulting in smaller traveling and costs essentially. If this objective is used by the company, the computational results presented in this research suggest that the company can cut the sum of traveling costs by 10%. The current classification utilized by the company is determined based on experience and intuition as listed below:

Table 2. Current Group Scheme

Code	Group 1	Code	Group 2	Code	Group 3
1	Elazığ	9	Bitlis	17	Muş
2	Malatya	10	Bingöl	18	Batman
3	Maraş	11	Van	19	Hakkari
4	Sivas	12	Ağrı	20	Şırnak
5	Bayburt	13	Ardahan	21	Urfa
6	Erzurum	14	Iğdır	22	Diyarbakır
7	Erzincan	15	Kars	23	Adıyaman
8	Tunceli	16	Siirt	24	Mardin

Tabu search (TS) algorithm applied in this study on the other hand, searches for the best solution to group the 24 cities into three in a way to minimize the total distance travelled throughout the network.

TS explores the solution space by moving at each iteration from a solution *x* to the best available solution in the neighborhood $N(x,\sigma)$. The process of changing neighborhoods with increasing cardinality, in case of no improvements, corresponds to a diversification of the search. To avoid cycling, solutions possessing some attributes of recently explored ones are assigned the status of *tabu*. That is achieved using what is called short-term memory. Tabu moves are represented by attributes which are stored in a *tabu list*. The best move is chosen as the highest evaluation move in the neighborhood of the current solution in terms of objective function and tabu restrictions.

The tabu list is used to prevent the search from revisiting solutions that are visited before and to guide the search process into unvisited regions of the solution space. The neighborhood of the current solution is restricted to the solutions that do not belong to the tabu list. At each iteration the best solution of $N(x,\sigma)$ that is not in the tabu list, is chosen as the new current solution. This solution is then added to the tabu list and the oldest solution in the current tabu list is removed. The tabu status can be overridden when predetermined conditions are met which is known as aspiration criterion which can be used either to improve current best solution or provide diversification by guiding the search process into a different subset of solutions. The procedure iterates until a termination criterion is met.

The outline of the TS applied is provided below:

Initialization

Initialize the tabu list

$BestSolution = initial\ solution$, $iteration = \mathbf{0}$

Do While iteration ≤ maxiteration

Apply *shift and swap* moves

Find $x' \in Nx, \forall x$ subject to tabu and aspiration criteria

Choose the first improving x'

Update Tabu List

If $x' < BestSolution$ then

$BestSolution = x'$

Endif

iteration = iteration + 1

End While

$Z^* = BestSolution$

An mxm $(24x24)$ matrix generated to represent the distances between the teams. In this A matrix, $A[k][l]$ is the distance between the k^{th} and l^{th} cities. The distances are obtained from the General Directorate of Highways of Turkey.

Each solution visited by the algorithm is represented by a $\mathbf{3x8}$ matrix *G*, where 3 is the number of subsets, and 8 is the number of cities in each subset. The subsets used by the company are used as the initial solution for the TS algorithm.

Another enhancement of the algorithms developed consists embedding the improvement method within a tabu search procedure aimed at searching the solution space beyond the local optima as well as providing a basis to exploit adaptive memory. The resulting algorithm implements a RAMP approach (Rego, 2004) by effectively integrating the cross-parametric relaxation with an adaptive memory. Although we have implemented a simplified RAMP approach in which adaptive memory is limited to the tabu search short-term memory component, we have determined significant improvements in the results provided by this procedure relative to its previous memory-less relaxation-based local search version. In order to prevent cycling in the search process, each performed move's attributes are added to a tabu list for a pre-defined number of iterations. Short-term memory is implemented through the use of a tabu list to prevent cycling in the search and the classical aspiration criterion that drops the tabu status of a move, if it improves the best solution found so far. If a tabu move results with an objective function value that is better than the best value found so far, the move is executed regardless of its tabu structure.

The improvement phase of the algorithm utilizes two types of moves on a given solution to reach another solution, with the goal of reducing the objective function value. If the new value is larger than the value of the current solution, the move is not executed and the current solution is kept. Otherwise, the move is executed and the solution is updated.

The first type of move, exchanges two cities in the same column of *G* matrix (shift move), whereas a second type of move exchanges two cities located in different columns of the *G* matrix (swap move). A solution visited during the improvement phase is given a tabu status for a number of iterations for the diversification purposes. An oscillation criterion is also set to overrule the tabu status. If a solution provides the best solution value found so far, the solution is visited regardless of its status.

The number of iterations for the improvement phase is proven to be greater than $\mathbf{2}xm$ which constitutes a lower bound for number of iterations, based on the experimental results.

Computational Experiments

The algorithm developed in this study is coded in C++ and run on a Pentium 1.73GHz processor, 0.99 GB of RAM.

The computational experience consists of 420 customers located in 24 cities served by the company. The tabu search algorithm utilized to minimize the total distance travelled by the delivery trucks during the year, uses the groups set up by the company as an initial solution. The distances between the cities are obtained from the General Directorate of Highways of Turkey. The algorithm has emerged its best solution after 48 iterations of improvement. The proposed solution by the algorithm is presented in the Table 3 as follow:

Table 3. The Groups Set by the Proposed Tabu Search Algorithm

Code	Group 1	Code	Group 2	Code	Group 3
1	Elazığ	9	Bitlis	17	Bingöl
2	Malatya	10	Muş	18	Batman
3	Bingöl	11	Van	19	Siirt
4	Sivas	12	Ağrı	20	Şırnak
5	Bayburt	13	Ardahan	21	Urfa
6	Erzurum	14	Iğdır	22	Diyarbakır
7	Erzincan	15	Kars	23	Adıyaman
8	Tunceli	16	Hakkari	24	Mardin

The traveling distances totaling up to 52,472 km (32,604 miles) according to the groups currently used by the company. The solution above found by the TS algorithm on the other hand, requires 47,568 km (29,557 miles) which presents a 10% reduction in the total traveling time, and the traveling costs consequently.

Conclusions

The search for more efficient and suitable algorithms to solve NP-hard problems continues due to the insufficiency of linear programming techniques and traditional algorithms for such problems. Heuristic approach which emerges in near-optimal solutions very quickly has drawn a great deal of attention. Tabu search algorithm is proven to be one of the most popular and effective metaheuristic methods.

This study develops a tabu search based algorithm to solve a set partitioning problem involving distribution network design of a wholesale company operating in a large region. The 24 cities covered by the company are divided into three subsets in a way to minimize the total distance travelled by the delivery trucks and the salesmen in the same manner. The experimental results show that the proposed algorithm finds a solution reducing the total travelled distances, hence the cost of travelling by 10% compared to the groups currently utilized by the company.

As promising as it sounds, the improvement efforts of the company strives for further achievements. The next direction is to work on the efficient routing of the trucks and salesmen. Thus, the future direction for this research will be applying optimization techniques for this problem.

Another parameter to be added into the problem is the customer intensity in each city. Naturally, the cities with higher number of customers requires larger warehouses and more frequent deliveries which in return might affect both the grouping scheme and routing of the trucks.

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