# Analysis of means for analyzing data from orthogonal array experiments

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## ABSTRACT

A step-by-step analysis of means (ANOM) procedure has been extended to analyze the data from orthogonal array experiments. The procedure is also illustrated with the help of a numerical example.

## Introduction

Analysis of Means (ANOM) introduced by Professor Ellis R Ott (1967) is a graphical procedure and an alternative to analysis of variance (ANOVA) to analyze the data from experimental designs with factors at fixed levels. The ANOM procedure has been extended by Schilling (1973) and introduced analysis of means for treatment effects (ANOME) to analyze the fixed effects in the crossed classifications, nested designs, balanced incomplete block designs etc. Subramani (1992) has applied the ANOME procedure to analyze the missing data from randomized block designs.

This procedure has been extended by Subramani (2008 a, b) to analyze missing data from Latin Square Designs, Graeco Latin Square Designs, Hyper Graeco Latin Squares Designs, Replicated Latin Squares Designs of Type I, Type II and Type III, Cross-over Designs without Residual Effects and F-Square Designs.

The fundamental idea is to get the estimates of the missing values, which may be obtained by using any one of the methods discussed by Subramani and Ponnuswamy (1989), Wilkinson (1958) and Yates (1933) and then the usual ANOME is carried out. The ANOME procedure for analyzing missing data from various experimental designs is also explained with the help of numerical examples.

However, the ANOME procedure has not been applied to analyze the data from orthogonal array experiments introduced by Taguchi ( ). Hence an attempt is made in this paper and a step-by-step ANOME procedure is given to analyze the data from orthogonal array experiments. The procedure is also illustrated with the help of a numerical example.

For the sake of convenience and easy reference to the readers, the step-by-step ANOME procedure to analyze the data from any of the experimental designs is given below:

1. Write the model of the experimental design
2. Estimate the treatment effects

3. Obtain the estimate of the experimental error ($\hat{\sigma}$). Normally the estimate $\hat{\sigma}$ is obtained as the square root of the error mean square from the ANOVA table
4. Determine the degrees of freedom ($q$) for the treatment effects
5. Determine the degrees of freedom ($f$) for error sum of squares. It is the degrees of freedom for error sum of squares obtained from the ANOVA table.
6. Determine the decision lines (LDL and UDL) for the desired $\alpha$ risk as

$$0 \pm \hat{\sigma} h_\alpha \sqrt{q/n}$$

Where $n$: total number of observations in the experimental design, $q$: degrees of freedom for the treatment effects to be plotted, $h_\alpha : h_\alpha$ (k, f), critical factor obtained from the table of Schilling (1973), $k$ is the number of points to be plotted.

7. Plot the treatment effects against the decision lines and draw the statistical inference. That is, if any of the treatment effects plotted on the ANOME chart falls outside of either UDL or LDL, conclude that the treatment effects are not homogeneous. Otherwise conclude that the treatment effects are homogeneous at the given level of significance

## Orthogonal Array Experiments

Orthogonal Array Experiment (OAE) is an alternate to the fractional factorial design. The main and important advantage of the OAE is its flexibility and simplicity in assigning the main factors and interaction factors into the experiment.

The associated linear graphs is very much useful to assign the treatments to an OAE. In the linear graphs the vertices are used to denote the main factors and the edges are used to represent the interaction of the main factors represented by the two vertices connected by the edge.

It is to be noted that the interaction factor can be replaced to accommodate a main factor but not the reverse. An example of OAE $L_8(2^7)$ and its associated linear graphs are given below:

**Table 1:** $L_8(2^7)$ **Orthogonal Array**

|    | C1 | C2 | C3 | C4 | C5 | C6 | C7 |
|----|----|----|----|----|----|----|----|
| R1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  |
| R2 | 1  | 1  | 1  | 2  | 2  | 2  | 2  |
| R3 | 1  | 2  | 2  | 1  | 1  | 2  | 2  |
| R4 | 1  | 2  | 2  | 2  | 2  | 1  | 1  |
| R5 | 2  | 1  | 2  | 1  | 2  | 1  | 2  |
| R6 | 2  | 1  | 2  | 2  | 1  | 2  | 1  |
| R7 | 2  | 2  | 1  | 1  | 2  | 2  | 1  |
| R8 | 2  | 2  | 1  | 2  | 1  | 1  | 2  |
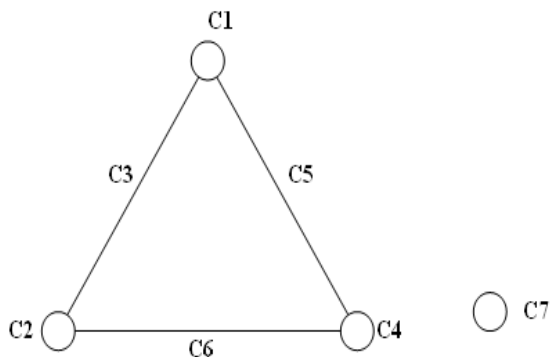


**Fig.1: Linear Graph of** $L_8(2^7)$ **Orthogonal Array**

From the linear graph given in Fig.1, we can use the $L_8(2^7)$ OA to conduct a fractional factorial design to analyze 4 main factors each at 2 levels and 3 interaction factors. In case, if we need to analyze some more main factors we can use the columns corresponding to the interaction factors. That is, one can use at most 7 main factors without interaction factors. The ad vantage of the OAEs is the easy computation of the treatment effects. The steps involved in computing the main effects and interaction effects are given below:

● Compute the total of all observations from each of the rows
● For computing the main effect corresponding to the jth column:

○ Add the row totals corresponding to the value 1 in jth column and denote it by $[M_1]$.

○ Add the row totals corresponding to the value 2 in jth column and denote it by $[M_2]$.

○ The main effect is obtained as $[M] = 2([M_2] - [M_1])/n$, where $n$ is the total number of observations.

● For computing the interaction effect corresponding to the kth column:

○ Add the row totals corresponding to the value 1 in kth column and denote it by $[I_1]$.

○ Add the row totals corresponding to the value 2 in kth column and denote it by $[I_2]$.

○ The interaction effect is obtained as $[I] = 2([I_2] - [I_1])/n$, where $n$ is the total number of observations.

● The sum of squares due to the main effects are obtained as $SSME = ([M_2] - [M_1])^2 / n$

● The sum of squares due to the interaction effects are obtained as $SSIE = ([I_1] - [I_2])^2 / n$

● Total sum of squares is obtained as $TSS = \sum_{i=1}^{k} \sum_{j=1}^{r} Y_{ij}^2 - CF$, where $CF = \dfrac{1}{n} \left( \sum_{i=1}^{k} \sum_{j=1}^{r} Y_{ij} \right)^2$

● Error sum of squares is obtained by subtracting all the sum of squares due to the main effects and the interaction effects from the total sum of squares.
● By dividing the sum of squares by its degrees of freedom one may get the mean squares due to the main effects, interaction effects and the error factors.
● By dividing each of the mean square due to main effects (interaction effects) by the mean square due to error one may get the calculated F values.
● By comparing the calculated F value with that of the Table Value of F, we may draw the necessary inference.

**ANOME to Orthogonal Array Experiments**

In this section, the step by step ANOME procedure of analyzing data from Orthogonal Array Experiments is presented and also illustrated with the help of a numerical example. Further the ANOME results are compared with that of the ANOVA table obtained for the OAE. The proposed ANOME procedure is given below:

**Step 1:** Write down the OAE together with the observations
**Step 2:** Decide the number of main effects and the interaction effects to be tested. Assign these factors to the columns of the OA with the help of linear graphs.
**Step 3:** Obtain the total of all observations corresponding to each of the rows.
**Step 4:** Obtain the treatment effects due to main factors and the interaction factors as per the procedure given in Section 2.
**Step 5:** Obtain the sum of squares due to main effects, Interactions effects, Total sum of squares and Sum of squares due to error factor.
**Step 6:** Determine the degrees of freedom for error sum of squares as $f = n - 1 - m$, where $m =$ number of main factors and interactions considered
**Step 7:** The estimate of the experimental error $\hat{\sigma}$ is obtained as

$$\hat{\sigma} = \sqrt{\left\{ \left( \sum_{i=1}^{k} \sum_{j=1}^{r} y_{ij}^2 - CF - \sum_{j=1}^{m} (SSME / SSIE) \right) / f \right\}}$$

**Step 8**: Compute the decision lines (LDL and UDL) for the desired risk $\alpha$ as follows:

$$0 \pm \hat{\sigma} \sqrt{q * F_{\alpha, q, f} / n},$$

**Step 9:** Plot the main effects $[M]$ and the interaction effects $[I]$ against the decision lines and draw the conclusion that the main (or interaction) effect is significant if the plotted point falls outside the decision lines. Otherwise conclude that the effects are not significant.

**Numerical Example**

Consider the data given in Bagchi (1993, page. 88), in which an investigator used the $L_8$ OA to conduct replicated experiments involving four parameters namely, Adhesive Type (AT), Conductor Material (CM), Curing Time (CT) and Integrated Circuit Post Coating (Co) to maximize bonding of mounted ICs on a metalized glass substrate. The parameter settings and the observations were as given in Table 2 given below.

**Table 2: IC Bonding Test Results**

| Set | AT | CM | CT | CO | Replicated Observations | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | D | Cu | 90 | Sn | 73.000 | 73.200 | 72.800 | 72.200 | 76.200 |
| 2 | D | Cu | 120 | Ag | 87.700 | 86.400 | 86.900 | 87.900 | 86.400 |
| 3 | D | Ni | 90 | Ag | 80.500 | 81.400 | 82.600 | 81.300 | 82.100 |
| 4 | D | Ni | 120 | Sn | 79.800 | 77.800 | 81.300 | 79.800 | 78.200 |
| 5 | H | Cu | 90 | Ag | 85.200 | 85.000 | 80.400 | 85.200 | 83.600 |
| 6 | H | Cu | 120 | Ag | 78.000 | 75.500 | 83.100 | 81.200 | 79.900 |
| 7 | H | Ni | 90 | Sn | 78.400 | 72.800 | 80.500 | 78.400 | 67.900 |
| 8 | H | Ni | 120 | Ag | 90.200 | 87.400 | 92.900 | 90.000 | 91.100 |

By applying AT, CM, CT and CO respectively to the columns 1,2,4 and 7, we get the following main and interaction effects to be tested corresponding to the 7 columns of the $L_8$ OA:

AT, CM, ATxCM, CT, ATxCT, CMxCT and CO.

By applying the ANOVA technique to analyze the above data we get the following:

**Table 3: Anova Table**

```
------------------------------------------------------------
SOURCE       DF        SS          MSS       F_CAL    F_table
------------------------------------------------------------
TREATMENT    7      1108.3743   158.3392    25.8183   2.41*

AT           1        38.4159    38.4159     6.2640   4.24*
CM           1         5.3291     5.3291     0.8689   4.24
ATxCM        1         2.7039     2.7039     0.4409   4.24
CT           1       295.9356   295.9356    48.2543   4.24*
ATxCT        1         0.6250     0.6250     0.1019   4.24
CMxCT        1         6.7241     6.7241     1.0964   4.24
CO           1       758.6406   758.6406   123.7014   4.24*

ERROR       32       196.2507     6.1328

------------------------------------------------------------
TOTAL       39      1304.6250
------------------------------------------------------------
```

From the ANOVA table we conclude that the main effects AT, CT and CO are significant at 5% level of significance.

By Applying the ANOME procedure for the OAE data given above, we get the following:

**THE ROW TOTALS ARE:**
367.400    435.300    407.900    396.900    419.400    397.700 378.000   451.600

THE TREATMENT TOTALS AT LOWER LEVEL ARE:
1607.500   1619.800   1632.300   1572.700   1624.600   1635.300 1540.000

THE TREATMENT TOTALS AT HIGHER LEVEL ARE:
1646.700   1634.400   1621.900   1681.500   1629.600   1618.900 1714.200

**THE TREATMENT EFFECTS ARE**

| | |
|---|---|
| AT | 1.96000 |
| CM | 0.73000 |
| ATxCM | 0.52000 |
| CT | 5.44000 |
| ATxCT | -0.25000 |
| CMxCT | 0.82001 |
| CO | 8.71000 |

The estimate of the experimental error $\hat{\sigma}$ is obtained as $\hat{\sigma} = 2.476449$

The degrees of freedom for error component $\sigma^2$ is

$f = (40-1) - 7 = 32$

The critical factor or table value is $F_{\alpha,q,f} = F_{0.05,1,32} = 4.24$.

Further the value of $q = 1$ and $n = 40$

The decision lines at $\alpha = 0.05$ are obtained as

$LDL = 0 - 2.476449\sqrt{4.24/40} = -0.806$

$UDL = 0 + 2.476449\sqrt{4.24/40} = +0.806$

Now plot the treatment effects AT, CM, ATxCM, CT, ATxCT, CMxCT and CO as in Figure 2. From the Figure 2 it is observed that the plotted points corresponding to the main effects AT, CT and CO fall outside the UDL and hence we conclude that these main effects are significant at 5% level of significance. Further it is to be noted that these results are agreeing with the results obtained through the ANOVA Technique.
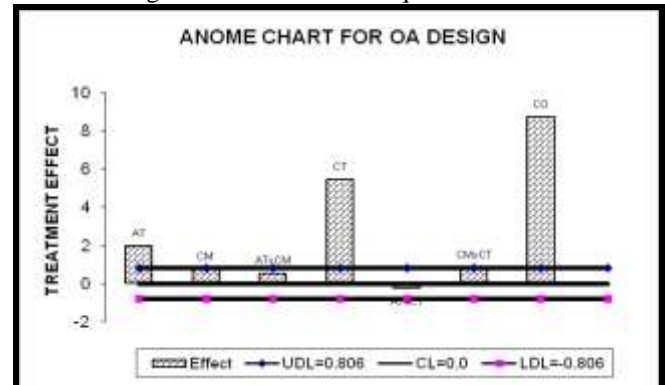


**Figure 2: ANOME CHART FOR OAE**

**Summary**

It is well known that the ANOME procedure is useful to assess the Engineering significance as well as statistical significance from the experimental data with factors at fixed levels. In this paper, we have presented a step-by-step method for the use of ANOME procedure to analyze the data from Orthogonal Array Experiments. The proposed ANOME procedure is also illustrated with the help of numerical examples. Further it is observed that the results obtained through ANOVA and ANOME are the same.

**References**

1. Bagchi, T.P. (1993): Taguchi Methods Explained: Practical Steps to Robust Design, Prentice-Hall of India Private Ltd, New Delhi.
2. OTT. E.R.(1967): Analysis of Means – A Graphical Procedure, Industrial Quality Control, 24, 101-109.
3. SCHILLING, E.G. (1973): A Systematic Approach to Analysis of Means- PART I, II and III, Journal Of Qaulity Technology, 5, 92-108,147-159
4. SUBRAMANI, J. (1992): Analysis of Means for Experimental Designs with Missing Observations, Communi. in Statistics – Theory and Methods, 21, 2045-2057.
5. SUBRAMANI, J. (2008a): Analysis of Means for Analysing Missing Data from Experimental Designs- Part I – Journal of Manufacturing Engineering, 3, 136-145.
6. SUBRAMANI J (2008b): of Means for Analysing Missing Data from Experimental Designs- Part II – Journal of Manufacturing Engineering, 3, 211-222.
7. SUBRAMANI, J.and PONNUSWAMY, K.N. (1989): A Non-iterative Least Squares Estimation of Missing Values in Experimental Designs, Journal Of Applied Statistics, 16, 77-86.
8. WILKINSON, G.N. (1958): Estimation of Missing Values for the Analysis of Incomplete Data, Biometrics, 14,257-286.
9. YATES, F. (1933): The Analysis of Replicated Experiments when the Field Results are Incomplete, Empire Journal of Experimental Agriculture, 129-142.