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Loss of efficiency in randomized block designs with two missing values

J.Subramani[1] and S.Balamurali[2]

[1]Department of Statistics, Pondicherry University, R V Nagar, Kalapet, Puducherry – 605 014, India.

[2]Kalasalingam University, Anandanagar, Krishnankoil- 6426 190, Srivilliputhur Via, Virudhunagar Dt. Tamilnadu, India.

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ABSTRACT

In this paper an attempt has been made to study the efficiency of randomized block designs with two missing values. Consequently, we have obtained explicit expressions for the variances of the elementary treatment contrasts, average variance and the loss of efficiency in randomized block designs due to two missing values. Further we have also tabulated the loss of efficiency for various values of $t(3 \le t \le 10)$ and $r(3 \le r \le 10)$ where t *and* r are respectively representing the number of treatments and the number of blocks in the randomized block designs.

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Introduction

The estimation of missing values is an important problem associated with the analysis of variance to retain the orthogonal structure of the design matrix and the usual orthogonal data analysis. Otherwise one has to use the non-orthogonal data analysis to analyze the experimental data from orthogonal models with missing values. Even if one uses the missing plot techniques to analyze the incomplete data due to the missing values, one cannot retain the minimum efficiency of the original experimental designs without any missing values. The various aspects of missing plot techniques and its related problems have been extensively studied in the literature, see for example Cochran and Cox (1957), Dodge (1985), Jarret (1978), Rubin (1972), Subramani and Ponnuswamy (1989), Wilkinson (1958) and Yates (1933). The explicit expressions for the variances of the elementary treatment contrasts, average variance and the loss of efficiency in randomized block designs with a single missing values are presented in several standard textbooks including Chakrabarti (1962) and Kshirsagar (1983). In the present study an attempt has been made to obtain the efficiency of randomized block designs with two missing values. Consequently, we have presented explicit expressions for the variances of the elementary treatment contrasts, average variance and the loss of efficiency in randomized block designs due to two missing values. Further we have also tabulated the loss of efficiency for various values of $t(3 \le t \le 10)$ and $r(3 \le r \le 10)$ where t *and* r are respectively representing the number of treatments and the number of blocks in the randomized block designs. The various expressions obtained and the table values are presented in Section 2.

Loss of effifiency in randomized block designs

If we consider the case of two missing values in randomized block designs with t treatments and r blocks, we can have the following three different situations. They are:

- The missing values are from a particular block.
- The missing values are of a particular treatment.
- The missing values are from different blocks and from different treatments.

Subramani and Ponnuswamy (1989) have presented a non-iterative least squares estimation of several missing values and also obtained explicit expressions of the estimates for particular patterns of missing values in randomized block designs. For the sake of convenience to the readers the procedure is briefly outlined below.

In this procedure, to estimate the m missing values one has to obtain the following system of linear equations with m equations and with the $m \times m$ symmetric matrix unknowns,

$$AX = b \qquad (2.1)$$

where A is the $m \times m$ symmetric matrix, X is the $m \times 1$ vector of unknown missing values and b is the $m \times 1$ vector of known values. By solving the system of linear equations given in (2.1) one can get the non-iterative least squares estimates of the m missing values as given below:

$$\hat{X} = A^{-1}b \qquad (2.2)$$

Let $A = (a_{ij})$ be the $m \times m$ symmetric matrix. Then the value of the element a_{ij} is obtained as given below:

$$a_{ij} = \begin{cases} (r-1)(t-1) & \text{if } i = j \\ -(r-1) & \text{if } i^{th} \text{ and } j^{th} \text{ missing values are in a particular block} \\ -(t-1) & \text{if } i^{th} \text{ and } j^{th} \text{ missing values are of a particular treatment} \\ 1 & \text{otherwise} \end{cases} \qquad (2.3)$$

The i^{th} element b_i of the vector $b = (b_1\, b_2 \ldots b_m)'$ is obtained as $b_i = rB_{(i)}' + tT_{(i)}' - G'$ (2.4)

Tele:
E-mail addresses: drjsubramani@yahoo.co.in

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where $B'_{(i)}$ and $T'_{(i)}$ are respectively the block and treatment totals corresponding to the i^{th} missing value and G' is the grand total of all known values.

For the definition and derivation of the variances of elementary treatment contrasts, average variance and the loss of efficiency in randomized block designs one may refer to Chakrabarti (1962). Throughout this paper we use the notations $\bar{y}_{im}$ and $\bar{y}_{ie}$ to denote the mean of the i^{th} treatment with and without missing values. After a little algebra we have obtained the explicit expressions for the estimates of the missing values, variances of the various treatment contrasts, average variances and loss of efficiency to each of the three different situations mentioned earlier. They are given below:

Case (i): When the two missing values are from a particular block

The estimates of the missing values are obtained as given below:

$$\hat{X}_i = \frac{(t-1)b_i + \sum_{j=1}^{2} b_j}{t(r-1)(t-2)} \quad i = 1,2 \tag{2.5}$$

where b_i is as defined in (2.4)

The variances of different elementary contrasts are obtained as given below:

$$V(\bar{y}_{im} - \bar{y}_{jm}) = \frac{2\sigma^2}{(r-1)}$$

$$V(\bar{y}_{im} - \bar{y}_{je}) = \frac{2\sigma^2}{r}\left[1 + \frac{(t-1)}{2(r-1)(t-2)}\right] \tag{2.6}$$

$$V(\bar{y}_{ie} - \bar{y}_{je}) = \frac{2\sigma^2}{r}$$

The average variance is obtained as given below:

Average Variance

$$= \frac{2\sigma^2}{r}\left[1 + \frac{2}{(r-1)(t-1)}\right] \tag{2.7}$$

The loss of efficiency is obtained as given below:

Loss of Efficiency

$$= \frac{2}{(r-1)(t-1)+2} \tag{2.8}$$

Further we have also computed the loss of efficiency for various values of $t\,(3 \le t \le 10)$ and $r\,(3 \le r \le 10)$, where t and r are respectively representing the number of treatments and the number of blocks in the randomized block designs and are presented in the Table 2.1.

Case (ii): When the two missing values are of a particular treatment

The estimates of the missing values are obtained as given below:

$$\hat{X}_i = \frac{(r-2)b_i + \sum_{j=1}^{2} b_j}{r(r-2(t-1)} \quad i = 1,2 \tag{2.9}$$

where b_i is as defined in (2.4)

The variances of different elementary contrasts are obtained as given below:

$$V(\bar{y}_{im} - \bar{y}_{je}) = \frac{2\sigma^2}{r}\left[1 + \frac{t}{(r-2)(t-1)}\right]$$

$$V(\bar{y}_{ie} - \bar{y}_{je}) = \frac{2\sigma^2}{r} \tag{2.10}$$

The average variance is obtained as given below:

Average Variance

$$= \frac{2\sigma^2}{r}\left[1 + \frac{2}{(r-2)(t-1)}\right] \tag{2.11}$$

The loss of efficiency is obtained as given below:

Loss of Efficiency

$$= \frac{2}{(r-2)(t-1)+2} \tag{2.12}$$

Further we have also computed the loss of efficiency for various values of $t\,(3 \le t \le 10)$ and $r\,(3 \le r \le 10)$, where t and r are respectively representing the number of treatments and the number of blocks in the randomized block designs and are presented in the Table 2.2.

Case (iii): When the two missing values are from different blocks and from different treatments

The estimates of the missing values are obtained as given below:

$$\hat{X}_i = \frac{[(r-1)(t-1)+1]b_i - \sum_{j=1}^{2} b_j}{[(r-1)(t-1)+1][(r-1)(t-1)-1]} \quad i = 1,2 \tag{2.13}$$

where b_i is as defined in (2.4)

The variances of different elementary contrasts are obtained as given below:

$$V(\bar{y}_{im} - \bar{y}_{jm}) = \frac{2\sigma^2}{(r-1)}\left[1 + \frac{t}{(r-1)(t-1)-1}\right]$$

$$V(\bar{y}_{im} - \bar{y}_{je}) = \frac{2\sigma^2}{r}\left[1 + \frac{t(r-1)(t-1)}{2[(r-2)^2(t-1)^2-1]}\right] \tag{2.14}$$

$$V(\bar{y}_{ie} - \bar{y}_{je}) = \frac{2\sigma^2}{r}$$

The average variance is obtained as given below:

Average Variance

$$= \frac{2\sigma^2}{r}\left[1 + \frac{2(r-1)(t-1)^2 + 2}{(t-1)[(r-1)^2(t-1)^2 - 1]}\right] \tag{2.15}$$

The loss of efficiency is obtained as given below:

Loss of Efficiency

$$= \frac{2\{(t-1)[(r-1)(t-1)+1] - (t-2)\}}{\{(t-1)[(r-1)(t-1)+1]^2 - (t-2)\}} \tag{2.16}$$

Further we have also computed the loss of efficiency for various values of $t\,(3 \le t \le 10)$ and $r\,(3 \le r \le 10)$, where t and r are respectively representing the number of treatments and the number of blocks in the randomized block designs and are presented in the Table 2.3.

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Table 2.1: Loss of efficiency in RBD with two missing values and are from a particular block

r \ t	3	4	5	6	7	8	9	10
3	0.333	0.250	0.200	0.167	0.143	0.125	0.111	0.100
4	0.250	0.182	0.143	0.112	0.100	0.087	0.077	0.069
5	0.200	0.143	0.111	0.091	0.077	0.067	0.059	0.053
6	0.167	0.118	0.091	0.074	0.063	0.054	0.048	0.043
7	0.143	0.100	0.077	0.063	0.053	0.046	0.040	0.036
8	0.125	0.087	0.067	0.054	0.046	0.039	0.035	0.031
9	0.111	0.077	0.059	0.048	0.040	0.035	0.030	0.027
10	0.100	0.069	0.053	0.043	0.036	0.031	0.027	0.024

Table 2.2: Loss of efficiency in RBD with two missing values and are of a particular treatment

r \ t	3	4	5	6	7	8	9	10
3	0.500	0.400	0.333	0.286	0.250	0.222	0.200	0.182
4	0.333	0.250	0.200	0.167	0.143	0.125	0.111	0.100
5	0.250	0.182	0.143	0.112	0.100	0.087	0.077	0.069
6	0.200	0.143	0.111	0.091	0.077	0.067	0.059	0.053
7	0.167	0.118	0.091	0.074	0.063	0.054	0.048	0.043
8	0.143	0.100	0.077	0.063	0.053	0.046	0.040	0.036
9	0.125	0.087	0.067	0.054	0.046	0.039	0.035	0.031
10	0.111	0.077	0.059	0.048	0.040	0.035	0.030	0.027

Table 2.3: Loss of efficiency in RBD with two missing values and are from different blocks and from different treatments

r \ t	3	4	5	6	7	8	9	10
3	0.367	0.262	0.206	0.170	0.145	0.126	0.112	0.101
4	0.268	0.188	0.146	0.119	0.101	0.088	0.077	0.069
5	0.211	0.147	0.113	0.092	0.077	0.067	0.059	0.053
6	0.174	0.120	0.092	0.075	0.063	0.054	0.048	0.043
7	0.148	0.102	0.078	0.063	0.053	0.046	0.040	0.036
8	0.129	0.086	0.067	0.054	0.046	0.037	0.035	0.031
9	0.114	0.078	0.059	0.048	0.040	0.035	0.030	0.027
10	0.103	0.070	0.053	0.043	0.036	0.031	0.027	0.024