Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Appl. Math. 50 (2012) 10440-10443

Elixir ISSN: 2229-712X

Rotation and radiation effects on MHD flow through porous media past an impulsively started vertical plate with variable temperature

U.S. Rajput and Surendra Kumar

Department of Mathematics and Astronomy, University of Lucknow, Lucknow-226007, India.

ARTICLE INFO

Article history:

Received: 26 July 2012;

Received in revised form: 5 September 2012;

Accepted: 12 September 2012;

Keywords

Radiation effects,

MHD,

Heat transfer,

Porous media.

ABSTRACT

Rotation and radiation effects on MHD flow through porous media past an impulsively started vertical plate with variable temperature is studied here. The fluid considered is gray, absorbing-emitting radiation but a non- scattering medium. The governing equations involved in the present analysis are solved by the Laplace transform technique. The velocity, temperature and skin friction are studied for different parameters like Thermal Grashof number, Prandtl number, magnetic field parameter, rotation parameter, radiation parameter, permeability of porous media and time.

Introduction

Study of MHD flow with heat and mass transfer plays an important role in chemical, mechanical and biological Sciences. Some important applications are cooling of nuclear reactors, liquid metals fluid, power generation system and aero dynamics. The response of laminar skin friction and heat transfer to fluctuations in the stream velocity was studied by Lighthill [6]. Free convection effects on the oscillating flow past an infinite vertical porous plate with constant suction - I, was studied by Soundalgekar [11] which was further improved by Vajravelu et al. [13]. Further researches in these areas were done by Gupta et al. [1], jaiswal et al. [4] and Soundalgekar et al. [12] by taking different models. Some effects like, radiation and mass transfer on MHD flow were studied by Muthucumaraswamy et al. [7-8] and Prasad et al. [9]. Radiation effects on mixed convection along a vertical plate with uniform surface temperature were studied by Hossain and Takhar [3]. Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux was studied by Jha, Prasad and Rai [5]. On the other hand, Radiation and free convection flow past a moving plate was considered by Raptis and Perdikis [10].

We are considering the rotation and radiation effects on MHD flow through porous media past an impulsively started vertical plate with variable temperature. The results are shown with the help of graphs (Fig-1 to Fig-5) and table-1.

Mathematical Analysis

In this paper we have consider the unsteady MHD flow of an electrically conducting fluid induced by viscous incompressible fluid past an impulsively started vertical plate with variable temperature. The fluid and the plate rotate as a rigid body with a uniform angular velocity Ω' about z'-axis in the presence of an imposed uniform magnetic field B_0 normal to the plate. Initially, the temperature of the plate near the plate is assumed to be T'_∞. At time $t'>0$, the plate starts moving with a velocity $u'=u_0$ in its own plane and the temperature from the plate is raised to T'_w. Since the plate occupying the plane $z'=0$ is of infinite extent, all the physical quantities depends only on z' and t'. It is assumed that the induced magnetic field is negligible so that $\overset{\rightharpoonup}{B_0}=(0,0,B_0)$. Then the unsteady flow which is governed by free-convective flow of an electrically conducting fluid in a rotating system under the usual Boussinesq's approximation in dimensionless form is as follows:

$$\frac{\partial u'}{\partial t'}-2\Omega' v'=g\beta\left(T'-T'_\infty\right)+\upsilon\frac{\partial^2 u'}{\partial z'^2}-\frac{\sigma B_0^2 u'}{\rho}-\upsilon\frac{u'}{K'}, \tag{1}$$

$$\frac{\partial v'}{\partial t'}+2\Omega' u'=\upsilon\frac{\partial^2 v'}{\partial z'^2}-\frac{\sigma B_0^2 u'}{\rho}-\upsilon\frac{u'}{K'}, \tag{2}$$

$$\rho C_p\frac{\partial T'}{\partial t'}=k\frac{\partial^2 T'}{\partial z'^2}-\frac{\partial q_r}{\partial z'}. \tag{3}$$

The following boundary conditions have been assumed:

$$\left.\begin{array}{l} t'\le 0: u'=0, T'=T'_\infty \text{ for all the value of } z', \\ t'>0: u'=u_0, T'=T'_\infty+\left(T'_w-T'_\infty\right)\dfrac{u_0^2 t'}{\upsilon} \text{ at } z'=0, \\ \qquad u'\to 0, T'\to T'_\infty \text{ as } z'\to\infty. \end{array}\right\} \tag{4}$$

Where the symbols are: C_p - specific heat at constant pressure, T' - temperature of the fluid near the plate, T'_∞ - temperature of the mainstream fluid, t' -time, ρ -fluid density, g - acceleration due to gravity, β - coefficient of thermal expansion, υ - kinematic viscosity, μ - coefficient of viscosity,

Tele:

E-mail addresses: rajputsurendralko@gmail.com

B_0 – external magnetic field, σ - Stefan- Boltzmann constant, k - thermal conductivity of the fluid, u' - velocity of the fluid in the x' - direction, v' - velocity of the fluid in the y' - direction, T'_w - temperature of the plate, Ω' - rotation parameter, z' - coordinate axis normal to the plate, K' - permeability of porous medium and q_r - radiative heat flux in the z' -direction.

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial z'} = -4a^*\sigma\left(T'^4_\infty - T'^4\right), \tag{5}$$

where a^* is absorption constant.

Considering the temperature difference within the flow sufficiently small, T'^4 can be expressed as the linear function of temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher-order terms

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty. \tag{6}$$

Using equations (5) and (6), equation (3) becomes

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial z'^2} + 16a^*\sigma T'^3_\infty\left(T'_\infty - T'\right). \tag{7}$$

Introducing the following non- dimensional quantities:

$$\left.\begin{aligned} & u = \frac{u'}{u_0}, v = \frac{v'}{u_0}, t = \frac{t'u_0^2}{\upsilon}, z = \frac{z'u_0}{\upsilon}, \\ & G_r = \frac{g\beta\upsilon\left(T'_w - T'_\infty\right)}{u_0^3}, K = \frac{u_0^2}{\upsilon^2}K' \\ & P_r = \frac{\mu C_p}{k}, \Omega = \frac{\Omega'\upsilon}{u_0^2}, \theta = \frac{\left(T' - T'_\infty\right)}{\left(T'_w - T'_\infty\right)}, \\ & R = \frac{16a^*\upsilon^2\sigma T'^3_\infty}{ku_0^2}, M = \frac{\sigma B_0^2\upsilon}{\rho u_0^2} \end{aligned}\right\} \tag{8}$$

where u - dimensionless velocity along x -axis, v - dimensionless velocity along y -axis, z -dimensionless coordinate axis normal to the plate, θ - dimensionless temperature, G_r - Grashof number, P_r - Prandtl number, t – dimensionless time, Ω - dimensionless rotation parameter, R - radiation parameter, K - dimensionless permeability of porous medium and M -magnetic field parameter.

Equations (1), (2) and (3) leads to

$$\frac{\partial u}{\partial t} - 2\Omega v = G_r\theta + \frac{\partial^2\theta}{\partial z^2} - \left(M + \frac{1}{K}\right)u, \tag{9}$$

$$\frac{\partial v}{\partial t} + 2\Omega u = \frac{\partial^2\theta}{\partial z^2} - \left(M + \frac{1}{K}\right)v, \tag{10}$$

$$\frac{\partial\theta}{\partial t} = \frac{1}{P_r}\left(\frac{\partial^2\theta}{\partial z^2}\right) - \frac{R}{P_r}\theta. \tag{11}$$

Adding equation (9) and (10), we have

$$\frac{\partial q}{\partial t} = G_r\theta + \frac{\partial^2 q}{\partial y^2} - mq, \tag{12}$$

where $q = u + iv$ and $m = M + 2i\Omega + \frac{1}{K}$.

Also, the boundary conditions (4) are reduced to:

$$\left.\begin{aligned} & t \le 0 : q(z,t) = 0, \theta(z,0) = 0 \text{ for all value of } z, \\ & t > 0 : q(z,t) = 1, \theta(0,t) = t \text{ at } z = 0, \\ & \qquad q(z,t) \to 0, \theta(0,t) \to 0 \text{ as } z \to \infty, \end{aligned}\right\} \tag{13}$$

Using Laplace transform technique with the help of [2], the solution of equation (11) and (12) obtained is as follows:

$$\begin{aligned} \theta(z,t) = {} & \theta_1 e^{-z\sqrt{R}} erfc\left(\eta\sqrt{P_r} - \sqrt{at}\right) \\ & + \theta_2 e^{z\sqrt{R}} erfc\left(\eta\sqrt{P_r} + \sqrt{at}\right), \end{aligned} \tag{14}$$

$$\begin{aligned} q(z,t) = {} & q_1 e^{-z\sqrt{R}} erfc\left(\eta - \sqrt{mt}\right) \\ & + q_2 e^{z\sqrt{R}} erfc\left(\eta + \sqrt{mt}\right) \\ & - q_3 e^{-z\sqrt{R}} erfc\left(\eta\sqrt{P_r} - \sqrt{at}\right) \\ & - q_4 e^{z\sqrt{R}} erfc\left(\eta\sqrt{P_r} + \sqrt{at}\right) \\ & - q_5\begin{bmatrix} e^{-z\sqrt{b+m}} erfc\left(\eta - \sqrt{(b+m)t}\right) \\ + e^{z\sqrt{b+m}} erfc\left(\eta + \sqrt{(b+m)t}\right) \end{bmatrix} \\ & + q_5\begin{bmatrix} e^{-z\sqrt{cP_r}} erfc\left(\eta\sqrt{P_r} - \sqrt{ct}\right) \\ + e^{z\sqrt{cP_r}} erfc\left(\eta\sqrt{P_r} + \sqrt{ct}\right) \end{bmatrix} \end{aligned} \tag{15}$$

For making the solution concise, the following symbols have been used above:

$$q_1 = \frac{1}{2}\left[1 + \frac{G_1}{b^2} + \frac{G_1 t}{b} - \frac{G_1 z}{2b\sqrt{m}}\right], G_1 = \frac{G_r}{1 - P_r},$$

$$q_2 = \frac{1}{2}\left[1 + \frac{G_1}{b^2} + \frac{G_1 t}{b} + \frac{G_1 z}{2b\sqrt{m}}\right], c = (a+b),$$

$$q_3 = \frac{G_1}{2b}\left[t - \frac{zP_r}{2\sqrt{R}} + \frac{1}{b}\right], b = \frac{R-m}{1-P_r}, a = \frac{R}{P_r},$$

$$q_4 = \frac{G_1}{2b}\left[t + \frac{zP_r}{2\sqrt{R}} + \frac{1}{b}\right], q_5 = \frac{G_1 e^{bt}}{2b^2}, \eta = \frac{z}{2\sqrt{t}}$$

$$\theta_1 = \left(\frac{t}{2} - \frac{zP_r}{4\sqrt{R}}\right) \text{ and } \theta_2 = \left(\frac{t}{2} + \frac{zP_r}{4\sqrt{R}}\right).$$

Now we can separate real and imaginary term by the following formula:

$$\begin{aligned} erf(a+ib) = {} & erf(a) \\ & + \frac{e^{-a^2}}{2a\pi}\left[1 - Cos(2ab) + iSin(2ab)\right] \\ & + \frac{2e^{-a^2}}{\pi}\sum_{n=1}^{\infty}\frac{e^{-n^2/4}}{n^2 + 4a^2}\left[f_n(a,b) + ig_n(a,b)\right] \\ & + \varepsilon(a,b). \end{aligned}$$

Where

$$f_n(a,b) = 2a - 2aCosh(nb)Cos(2ab) + nSinh(nb)Sin(2ab),$$

$$g_n(a,b) = 2aCosh(nb)Sin(2ab) + nSinh(nb)Cos(2ab)$$

and $\varepsilon(a,b) \approx 10^{-16}\,|erf(a+ib)|$.

Skin Friction

The Skin-friction components τ_x and τ_y are obtained as:

$$\tau_x + i\tau_y = -\left(\frac{\partial q}{\partial z}\right)_{z=0}, \tag{16}$$

Therefore, using equation (16), we obtain:

$$\tau_x + i\tau_y = \tau_1 erf\left(\sqrt{mt}\right) + \frac{2}{\sqrt{\pi}}\left(1 + \frac{G_1}{2b^2} + \frac{tG_1}{b}\right)e^{-mt}$$
$$-\tau_2 erf\left(\sqrt{(b+m)t}\right) - \tau_3 erf\left(\sqrt{at}\right) - \frac{tG_1\sqrt{P_r}}{b\sqrt{\pi t}}e^{-at} \quad \ldots (17)$$
$$+\frac{G_1 e^{bt}\sqrt{(a+b)P_r}}{b^2} erf\left(\sqrt{(a+b)t}\right),$$

$$\tau_1 = \frac{G_1}{b\sqrt{m}} + 2\sqrt{m}\left(1 + \frac{G_1}{b^2} + \frac{tG_1}{b}\right),$$

where $\tau_2 = \dfrac{G_1 e^{bt}\sqrt{b+m}}{b^2}$

and $\tau_3 = \dfrac{G_1 P_r}{4b\sqrt{R}} + \dfrac{G_1\sqrt{R}}{b}\left(t + \dfrac{1}{b}\right).$

Results and Discussion

The velocity profiles for different parameters $M, G_r, P_r, R, \Omega, K$ and t are shown by figures-1 to 4. Temperature profile is shown in figure-5. Primary velocity profiles are shown in figures-1 and 2. From figure-1, it is clear that the primary velocity increases when M is decreased (keeping other parameters $K = 2, G_r = 5, P_r = 0.71, R = 2, \Omega = 0.5, t = 0.2$ constant). Same pattern is observed when values of G_r are raised from 5 to 10. Primary velocity profile for different values of rotation parameter is shown in figure-2. It shows that primary velocity increases with increasing rotation parameter and it decreases when t is increased (taking $K = 2, G_r = 10, P_r = 0.71, R = 2, \Omega = 0.5,$).

Secondary velocity is shown in figures-3 and 4. In figure-3 it is observed that velocity increases when radiation parameter R is increased. Similarly the velocity increases when the value of G_r is decreased.

In case of secondary velocity, rotation parameter has similar effect as with primary velocity (figure-4).

From the temperature profile (figure-5) it is observed that temperature of the fluid increases with time and radiation parameter. Effect is reversed when value of P_r is increased. These observations demonstrate the practical feasibility of the model.

The values of skin friction are tabulated in table-1 for different parameters. When the values of M , R and P_r are increased (keeping other parameters constant) the value of τ_x is also get increased. But if values of G_r and t are increased, the value of τ_y gets decreased. Similarly, when the values of t ; G_r and P_r are increased (keeping other parameters constant) the value of τ_y is gets increased. But if values of M and R are increased the value of τ_y gets decreased.

Conclusions

In this paper a theoretical analysis has been done to study the rotation and radiation effects on MHD flow past an impulsively started vertical plate with variable temperature. Solutions for the model have been derived by using Laplace - transform technique. Some conclusions of the study are as below:

1. Primary velocity increases with the increase in G_r and M , and decreases with increase in Ω and t .
2. Secondary velocity increases with the increase in R , and decreases with increase in G_r and Ω .
3. Temperature of the fluid increases when R and t are increased.
4. Skin friction :

- τ_x increases when radiation parameter, Prandtl number, rotation parameter and magnetic field parameter are increased but decreases when thermal Grashof number and time are increased.
- τ_y increases when thermal Grashof number, Prandtl number and t are increased but decreases when radiation parameter, rotation parameter and magnetic field are increased.

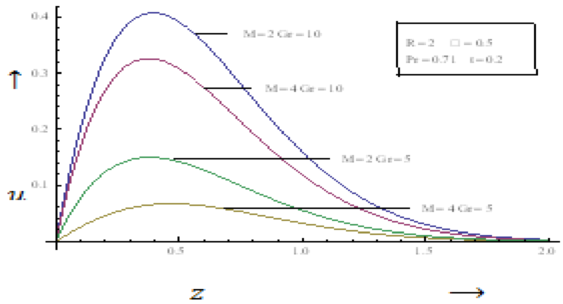

Figure 1: Primary velocity Profiles for different parameters at $K = 0.5$

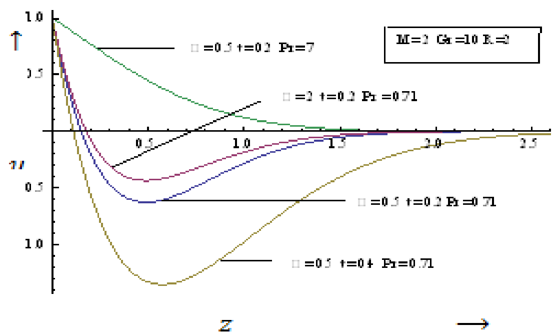

Figure 2: Primary velocity Profiles for different parameters at $K = 0.5$

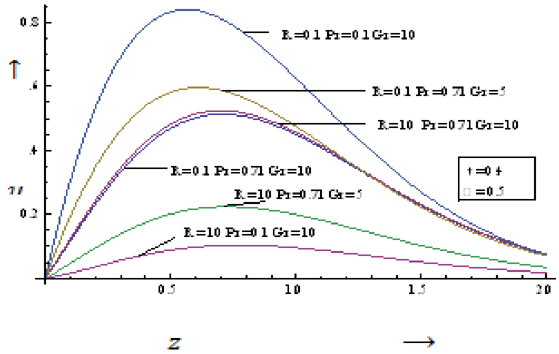

Figure 3: Secondary velocity Profiles at $M = 2$ **and** $K = 0.5$

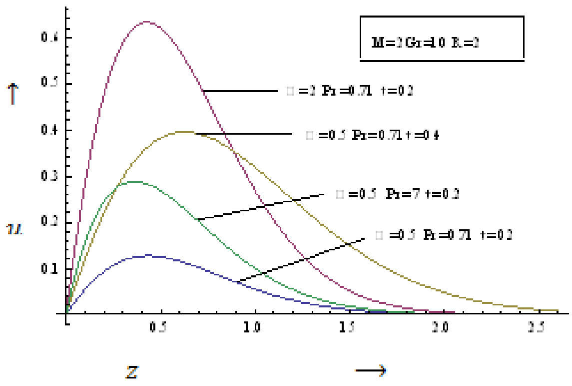

Figure 4: Secondary velocity Profiles for different parameters at $K = 0.5$

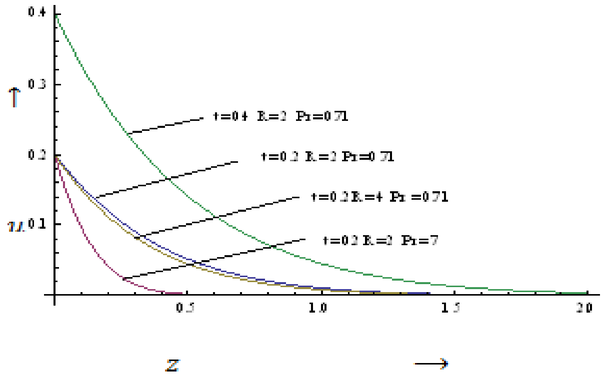

Figure 5: Secondary velocity Profiles for different parameters at $K = 0.5$

Table 1: Skin friction for different parameters at $K = 0.5$

G_r	M	Ω	R	P_r	t	τ_x	τ_y
5	2	0.5	2.0	0.71	0.2	0.63	3.42
5	2	0.5	2.0	0.71	0.4	0.01	11.5
5	2	0.5	2.0	7.0	0.2	55.4	5.17
5	2	0.5	4.0	0.71	0.2	4.83	1.59
5	2	0.2	2.0	0.71	0.2	3.19	2.51
5	4	0.5	2.0	0.71	0.2	3.11	0.59
10	2	0.5	2.0	0.71	0.2	-2.2	6.39
10	2	0.5	4.0	0.71	0.2	6.19	2.75

Acknowledgement

We acknowledge the U.G.C. (University Grant Commission) and thank for providing financial support for the research work.

References:

[1] A.S. Gupta, I. Pop and V.M. Soundalgekar, Free convection effects on the flow past an accelerated vertical plate in an incompressible dissipative fluid, Rev. Roum. Sci. Techn. -Mec. Apl, 24 (1979), 561 - 568.

[2] R.B. Hetnarski, An algorithm for generating some inverse Laplace transforms of exponential form, ZAMP, 26 (1975), 249 - 253.

[3] M.A. Hossain and H. S. Takhar, Radiation effect on mixed convection along a vertical plate with uniform surface temperature, Heat and Mass Transfer,31 (1996), 243 - 248.

[4] B.S. Jaiswal and V.M. Soundalgekar, Oscillating plate temperature effects on a flow past an infinite porous plate with constant suction and embedded in a porous medium, Heat and Mass Transfer, 37 (2001), 125 - 131.

[5] B.K Jha, R. Prasad and S. Rai, Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux, Astrophysics and Space Science, 181(1991), 125 - 134.

[6] M.J. Lighthill, The response of laminar skin friction and heat transfer to fluctuations in the stream velocity, Proc. R. Soc., A, 224 (1954), 1 - 23.

[7] R. Muthucumaraswamy and Janakiraman, MHD and radiation effects on moving isothermal vertical plate with variable mass diffusion, Theoret. Appl. Mech., 33, no. 1 (2006), 17 - 29.

[8] R. Muthucumaraswamy, K.E. Sathappan and R. Natarajan, Mass transfer effects on exponentially accelerated isothermal vertical plate, Int. J. of Appl. Math. and Mech., 4, no. 6 (2008), 19 - 25.

[9] V.R. Prasad, N.B. Reddy and R. Muthucumaraswamy, Radiation and mass transfer effects on two- dimensional flow past an impulsively started infinite vertical plate, Int. J. Thermal Sci., 46, no. 12 (2007), 1251 - 1258.

[10] A. Raptis and C. Perdikis, Radiation and free convection flow past a moving plate, Int. J. of App. Mech. and Engineering., 4 (1999), 817 - 821.

[11] V. M. Soundalgekar, Free convection effects on the oscillatory flow an infinite, vertical porous, plate with constant suction { I, Proc. R. Soc., A, 333 (1973), 25 - 36.

[12] V.M. Soundalgekar and H.S. Takhar, Radiation effects on free convection flow past a semi-infinite vertical plate, Modeling, Measurement and Control, B 51 (1993), 31 - 40.

[13] K. Vajravelu and K.S. Sastri, Correction to `free convection effects on the oscillatory flow an infinite, vertical porous, plate with constant suction I', Proc. R. Soc., A, 51 (1977), 31 – 40.