



# Estimation of population mean using mean square error by double sampling the non-respondents

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## ARTICLE INFO

### Article history:

Received: 7 August 2012;

Received in revised form:

30 September 2012;

Accepted: 5 October 2012;

### Keywords

Bias,  
Mean square error,  
Auxiliary character,  
Non-response.

## ABSTRACT

In this paper, we have considered the problem of estimating the population mean of study character using mean square error by double sampling the non-respondents. Two generalized estimators for estimating the population mean using auxiliary character under two different cases are proposed. Further the problem has been extended to the wider classes of estimators, which include several generalized estimators as a particular member. The bias, mean square error and optimum property of the proposed classes of estimators have been obtained under different cases. The efficiency of the proposed classes of estimators has also been shown through the theoretical and empirical studies.

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## Introduction

In most of the sample surveys based on mail questionnaire, we often find incomplete information due to the occurrence of non-response. In such situation an estimate based on the incomplete information may be misleading [see Cochran [1]] when the respondents character differ significantly from the non-respondents. Hansen and Hurwitz [2] first developed the method of sub sampling on the non-respondents to collect the information on them by personal interview basis and proposed an unbiased estimator for population mean. Following the technique of Hansen and Hurwitz [2], the problem of estimation of population mean using auxiliary character in presence of non-response is considered by several authors Rao [16, 17], Khare and Srivastava [8, 9, 10, 11, 12], Okafor and Lee [14], Khare and Sinha [6, 7], Singh and Kumar [20] and Sinha and Kumar [21].

In this paper, we have considered the problem of estimating the population mean of character under study using auxiliary information in presence of non-response when the values of population mean square error of study as well as auxiliary characters are known in advance. Following Prasad and Singh [15] and Turgut and Cingi [24], two classes of estimators for estimating the population mean have been suggested and their bias and mean square error are obtained. Comparisons of the proposed classes of estimators have been carried out with the relevant estimators and the performances of the proposed classes of estimators are also shown through an empirical study.

### The Proposed Estimator

Let us consider a finite population  $U_N = (u_1, u_2, \dots, u_N)$  of size  $N$  in which  $y$  and  $x$  are the study and auxiliary characters having the non-negative  $i^{th}$  value of  $y_i$  and  $x_i$  of  $u_i$ . Let a sample  $U_n$  of size  $n$  has been drawn from  $U_N$  using simple random sampling without replacement ( $S_{wor}$ ) method and it has been observed that only  $U_{n_1}$  of  $n_1$  units respond and  $U_{n_2}$  of  $n_2 (= n - n_1)$  units do not respond. In this problem, it has been

assumed that the whole population  $U_N$  is divided into two non-overlapping strata  $U_{N_1}$  and  $U_{N_2}$  of responding and non-responding soft-core groups; however they are not known in advance. The stratum weights of responding and non-responding groups are given by  $P_1 = N_1/N$  and  $P_2 = N_2/N$  and their estimates are respectively given by  $\hat{P}_1 = p_1 = n_1/n$  and  $\hat{P}_2 = p_2 = n_2/n$ . At the second stage we draw a subsample  $U_{n_{2r}}$  of size  $r = n_2/k$ , ( $k \geq 1$ ) from  $U_{n_2}$  and obtained the information through personal interviews. Let population mean and population mean square of study ( $y$ ) and auxiliary ( $x$ ) characters are given by

$$\bar{Y} = \frac{1}{N} \sum_{i=1}^N y_i, \quad \bar{X} = \frac{1}{N} \sum_{i=1}^N x_i,$$

$$S_y^2 = \frac{1}{N-1} \sum_{i=1}^N (y_i - \bar{Y})^2,$$

$$S_x^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{X})^2.$$

Now using the information of  $(n_1 + r)$  units, Hansen and Hurwitz [2] suggested an unbiased estimator for estimating  $\bar{Y}$  as

$$\bar{y}_h = p_1 \bar{y}_1 + p_2 \bar{y}_{n_{(2r)}} \quad (1)$$

where  $\bar{y}_1$  and  $\bar{y}_{n_{(2r)}}$  are the sample means of  $y$  based on  $n_1$  and  $r$  units respectively and  $p_i = n_i/n$  ( $i = 1, 2$ ).

The variance of the estimator  $\bar{y}_h$  upto the order  $(n^{-1})$  is given by

$$V(\bar{y}_h) = V_0 = \frac{N-n}{Nn} S_y^2 + \frac{P_2(k-1)}{n} S_{y(2)}^2 \quad (2)$$

where

$$S_{y(2)}^2 = \frac{1}{N_2-1} \sum_{U_{N_2}} (y_i - \bar{Y}_2)^2.$$

Similarly, one may define an unbiased estimator  $\bar{x}_h$  for estimating  $\bar{X}$  as

$$\bar{x}_h = p_1 \bar{x}_1 + p_2 \bar{x}_{n_{(2r)}} \quad (3)$$

and its variance upto the order ( $n^{-1}$ ) is given by

$$V(\bar{x}_h) = \frac{N-n}{Nn} S_x^2 + \frac{P_2(k-1)}{n} S_{x(2)}^2, \tag{4}$$

where  $S_{x(2)}^2 = \frac{1}{N_2-1} \sum U_{N_2} (x_i - \bar{X}_2)^2$ .

Let

$$s_{y_h}^2 = \frac{1}{n-1} \left( \sum U_{n_1} y_i^2 + k \sum U_{n_{2r}} y_i^2 - n \bar{y}_h^2 \right) \tag{5}$$

and

$$s_{x_h}^2 = \frac{1}{n-1} \left( \sum U_{n_1} x_i^2 + k \sum U_{n_{2r}} x_i^2 - n \bar{x}_h^2 \right) \tag{6}$$

which are respectively unbiased estimators of  $S_y^2$  and  $S_x^2$ . Now if  $S_y^2$  and  $S_x^2$  are known in advance, then following Isaki [3], we have considered two different cases:

**Case I-** When we have incomplete information on both study character ( $y$ ) and auxiliary character ( $x$ ), then our proposed ratio ( $T_{1r}$ ) and product ( $T_{1p}$ ) estimators for estimating  $\bar{Y}$ , which are as follows:

$$T_{1r} = \bar{y}_h \left( \frac{s_y^2}{s_{y_h}^2} \right) \left( \frac{s_x^2}{s_{x_h}^2} \right) \tag{7}$$

$$T_{1p} = \bar{y}_h \left( \frac{s_{y_h}^2}{s_y^2} \right) \left( \frac{s_{x_h}^2}{s_x^2} \right) \tag{8}$$

**Case II-** When we have incomplete information on study character ( $y$ ) but complete information on auxiliary character ( $x$ ), then we propose ratio ( $T_{2r}$ ) and product ( $T_{2p}$ ) estimators for estimating  $\bar{Y}$  as:

$$T_{2r} = \bar{y}_h \left( \frac{s_y^2}{s_{y_h}^2} \right) \left( \frac{s_x^2}{s_x^2} \right) \tag{9}$$

$$T_{2p} = \bar{y}_h \left( \frac{s_{y_h}^2}{s_y^2} \right) \left( \frac{s_x^2}{s_x^2} \right) \tag{10}$$

where  $s_x^2 = \frac{1}{n-1} \sum U_n (x_i - \bar{x})^2$ .

Now following Srivastava [22], we proposed a generalized estimator for both the cases

$$T_{g_1}^{(i)} = \bar{y}_h (\theta_0)^{a_i} (\theta_i)^{b_i}, i = 1,2 \tag{11}$$

where  $\theta_0 = \frac{s_{y_h}^2}{s_y^2}, \theta_1 = \frac{s_{x_h}^2}{s_x^2}, \theta_2 = \frac{s_x^2}{s_x^2}$  and  $a_i, b_i; i = 1,2$  are the suitably chosen constants. It can be easily seen that the estimators ( $T_{1r}, T_{1p}$ ) and ( $T_{2r}, T_{2p}$ ) are the particular members of the proposed generalized estimators  $T_{g_1}^{(1)}$  and  $T_{g_1}^{(2)}$  respectively. However we can modify our proposed ratio and product estimators [(7), (9)] and [(8), (10)] by introducing constants ( $a_l, b_l; l = 3,4,5,6$ ) in many other ways, which are as follows-

$$T_{g_2}^{(i)} = \bar{y}_h [a_3 \theta_0 + b_3 \theta_i]^{k'}, a_3 + b_3 = 1, i = 1,2 \tag{12}$$

$$T_{g_3}^{(i)} = \bar{y}_h [a_4 \theta_0 + (1 - a_4) \theta_0] [b_4 \theta_i + (1 - b_4) \theta_i], i = 1,2 \tag{13}$$

$$T_{g_4}^{(i)} = \bar{y}_h \left[ \frac{1 + a_5 (\theta_0 - 1)}{1 + b_5 (\theta_i - 1)} \right], i = 1,2 \tag{14}$$

$$T_{g_5}^{(i)} = \bar{y}_h [1 + a_6 (\theta_0 - 1) + b_6 (\theta_i - 1)], i = 1,2. \tag{15}$$

Some more estimators may also be possible by the modification in proposed ratio and product estimators. Keeping in a view of all generalized estimator from (11) to (15), we propose a wider class of estimators for both the cases I and II as follows:

$$T_C^{(i)} = \bar{y}_h h^{(i)}(\theta_0, \theta_i), i = 1,2 \tag{16}$$

such that  $h^{(i)}(1,1)=1$  and let it satisfies the following regularity conditions:

- (i) whatever be the sample chosen,  $(\theta_0, \theta_i)$  assume values in a bounded and closed convex subset  $D_i$  of the two dimensional real space containing the point  $(1,1)$ .
- (ii) the function  $h^{(i)}(\theta_0, \theta_i)$  is continuous and bounded in  $D_i$  and
- (iii) the first and second order partial derivatives of  $h^{(i)}(\theta_0, \theta_i)$  exists and are continuous and bounded in  $D_i$ .

On occurrence of the regularity conditions imposed on  $h^{(i)}(\theta_0, \theta_i)$ , it may be easily seen that the bias and mean square error of the estimator  $T_C^{(i)}$  will always exists.

**Bias and Mean Square Error (MSE)**

Under the regularity conditions and  $h^{(i)}(1,1) = 1$ , expand the function  $h^{(i)}(\theta_0, \theta_i)$  about  $(1,1)$  using Taylor's series upto the second order partial derivative, the expressions for bias and mean square error of the proposed class of estimators  $T_C^{(i)}$  upto the order  $(n^{-1})$  for any sampling design are given by

$$\begin{aligned} Bias(T_C^{(i)}) &= \bar{Y} [E(u-1)(\theta_0-1)h_1^{(i)}(1,1) \\ &\quad + E(u-1)(\theta_i-1)h_2^{(i)}(1,1) \\ &\quad + \frac{1}{2} \{ E(\theta_0-1)^2 h_{11}^{(i)}(\theta_0^*, \theta_i^*) \\ &\quad + 2E(\theta_0-1)(\theta_i-1)h_{12}^{(i)}(\theta_0^*, \theta_i^*) \\ &\quad + E(\theta_i-1)^2 h_{22}^{(i)}(\theta_0^*, \theta_i^*) \}], \quad i = 1,2 \end{aligned} \tag{17}$$

$$\begin{aligned} MSE(T_C^{(i)}) &= \bar{Y}^2 [E(u-1)^2 + E(\theta_0-1)^2 h_1^{(i)2}(1,1) \\ &\quad + E(\theta_i-1)^2 h_2^{(i)2}(1,1) \\ &\quad + 2\{ E(u-1)(\theta_0-1)h_1^{(i)}(1,1) \\ &\quad + E(\theta_0-1)(\theta_i-1)h_1^{(i)}(1,1)h_2^{(i)}(1,1) \\ &\quad + E(u-1)(\theta_i-1)h_2^{(i)}(1,1) \}] \quad i = 1,2 \end{aligned} \tag{18}$$

where  $u = \frac{\bar{y}_h}{\bar{Y}}, \theta_0^* = 1 + \delta_0(\theta_0 - 1)$  and  $\theta_i^* = 1 + \delta_i(\theta_i - 1)$ , such that  $0 < \delta_0, \delta_i < 1 \forall i = 1,2$ .

Here,  $h_1^{(i)}(\theta_0, \theta_i)$  and  $h_2^{(i)}(\theta_0, \theta_i)$  denote the first partial derivative of  $h^{(i)}(\theta_0, \theta_i)$  with respect to  $\theta_0$  and  $\theta_i$  respectively. The second order partial derivatives of  $h^{(i)}(\theta_0, \theta_i)$  with respect to  $\theta_0$  and  $\theta_i$  are denoted by  $h_{11}^{(i)}(\theta_0, \theta_i)$  and  $h_{22}^{(i)}(\theta_0, \theta_i)$  while first partial derivative of  $h_2(\theta_0, \theta_i)$  with respect to  $\theta_0$  is denoted by  $h_{12}^{(i)}(\theta_0, \theta_i)$ .

The mean square error of  $T_C^{(i)}$  will attain its minimum value for  $h_1^{(i)}(1,1) = -\frac{E(u-1)(\theta_0-1)}{E(\theta_0-1)^2} - \frac{E(\theta_0-1)(\theta_i-1)}{E(\theta_0-1)^2} h_2^{(i)}(1,1)$  (19)

and  $h_2^{(i)}(1,1) = \frac{A}{B}$  (20)

where

$$A = \{ E(u-1)(\theta_0-1) \{ E(\theta_0-1)(\theta_i-1) \} - \{ E(u-1)(\theta_i-1) \} \{ E(\theta_0-1)^2 \} \}$$

and

$$B = E(\theta_0-1)^2 E(\theta_i-1)^2 - \{ E(\theta_0-1)(\theta_i-1) \}^2$$

Using the value of  $h_1^{(i)}(1,1)$  and  $h_2^{(i)}(1,1)$  from (19) and (20), the minimum value of mean square error of  $T_C^{(i)}$  is given by

$$MSE(T_C^{(i)})_{min.} = V(\bar{y}_h) - \frac{\bar{y}^2}{E(\theta_0-1)^2} \left[ \{E(u-1)(\theta_0-1)\}^2 + \frac{A^2}{B} \right] \quad (21)$$

To derive the expressions for bias and mean square error of the proposed estimator  $T_C^{(i)}$  under simple random sampling without replacement ( $S_{wor}$ ) method of sampling upto the order ( $n^{-1}$ ), we assume that

$$\bar{y}_h = \bar{Y}(1+\epsilon), \quad s_{y_h}^2 = S_y^2(1+\epsilon_0), \quad s_{x_h}^2 = S_x^2(1+\epsilon_1), \\ s_x^2 = S_x^2(1+\epsilon_2)$$

such that  $E(\epsilon) = E(\epsilon_0) = E(\epsilon_1) = E(\epsilon_2) = 0$ .

Following the results discussed in Kendall and Stuart [5] and Kadilar and Cingi [4], we derived the expressions of bias and mean square of  $T_C^{(1)}$  and  $T_C^{(2)}$  upto the terms of order ( $n^{-1}$ ) under ( $S_{wor}$ ), which are as follows:

$$Bias(T_C^{(1)}) = \frac{\lambda_{30}}{S_y^2} h_1^{(1)}(1,1) + \frac{\lambda_{12}}{S_x^2} h_2^{(1)}(1,1) \\ + \frac{\bar{y}}{2} \left[ \frac{V_1}{(S_y^2)^2} h_{11}^{(1)}(\theta_0^*, \theta_1^*) + 2 \frac{\lambda_{22}}{S_x^2 S_y^2} h_{12}^{(1)}(\theta_0^*, \theta_1^*) + \frac{V_2}{(S_x^2)^2} h_{22}^{(1)}(\theta_0^*, \theta_1^*) \right] \quad (22)$$

$$MSE(T_C^{(1)}) = V_0 + \bar{Y}^2 \left[ \frac{V_1}{(S_y^2)^2} h_1^{(1)2}(1,1) + \frac{V_2}{(S_x^2)^2} h_2^{(1)2}(1,1) \right. \\ \left. + 2 \left\{ \frac{\lambda_{30}}{\bar{y} S_y^2} h_1^{(1)}(1,1) + \frac{\lambda_{22}}{S_x^2 S_y^2} h_1^{(1)}(1,1) h_2^{(1)}(1,1) + \frac{\lambda_{12}}{\bar{y} S_x^2} h_2^{(1)}(1,1) \right\} \right] \quad (23)$$

$$Bias(T_C^{(2)}) = \frac{\lambda_{30}}{S_y^2} h_1^{(2)}(1,1) + \frac{\lambda'_{12}}{S_x^2} h_2^{(2)}(1,1) \\ + \frac{\bar{y}}{2} \left[ \frac{V_1}{(S_y^2)^2} h_{11}^{(2)}(\theta_0^*, \theta_2^*) + 2 \frac{\lambda'_{22}}{S_x^2 S_y^2} h_{12}^{(2)}(\theta_0^*, \theta_2^*) + \frac{V'_2}{(S_x^2)^2} h_{22}^{(2)}(\theta_0^*, \theta_2^*) \right] \quad (24)$$

$$MSE(T_C^{(2)}) = V_0 + \bar{Y}^2 \left[ \frac{V_1}{(S_y^2)^2} h_1^{(2)2}(1,1) + \frac{V'_2}{(S_x^2)^2} h_2^{(2)2}(1,1) \right. \\ \left. + 2 \left\{ \frac{\lambda_{30}}{\bar{y} S_y^2} h_1^{(2)}(1,1) + \frac{\lambda'_{22}}{S_x^2 S_y^2} h_1^{(2)}(1,1) h_2^{(2)}(1,1) + \frac{\lambda'_{12}}{S_x^2} h_2^{(2)}(1,1) \right\} \right] \quad (25)$$

The mean square error of  $T_C^{(1)}$  will be minimum when

$$h_1^{(1)}(1,1) = -\frac{S_y^2 \lambda_{30}}{\bar{y} V_1} - \frac{S_y^2 \lambda_{22}}{\bar{y} V_1} \left[ \frac{\Delta_1}{\Delta_2} \right] \quad (26)$$

$$\text{and } h_2^{(1)}(1,1) = \frac{S_x^2}{\bar{y}} \left[ \frac{\Delta_1}{\Delta_2} \right] \quad (27)$$

and the minimum value of mean square error of  $T_C^{(1)}$  is given by

$$MSE(T_C^{(1)})_{min.} = V_0 - \frac{\lambda_{30}^2 V_2 + \lambda_{12}^2 V_1 - 2 \lambda_{30} \lambda_{12} \lambda_{22}}{V_1 V_2 - \lambda_{22}^2} \quad (28)$$

Similarly, the mean square error of  $T_C^{(2)}$  will attain the minimum value when

$$h_1^{(2)}(1,1) = -\frac{S_y^2 \lambda_{30}}{\bar{y} V_1} - \frac{S_y^2 \lambda'_{22}}{\bar{y} V_1} \left[ \frac{\Delta'_1}{\Delta'_2} \right] \quad (29)$$

$$\text{and } h_2^{(2)}(1,1) = \frac{S_x^2}{\bar{y}} \left[ \frac{\Delta'_1}{\Delta'_2} \right] \quad (30)$$

and the minimum value of the mean square error of  $T_C^{(2)}$  is given by

$$MSE(T_C^{(2)})_{min.} = V_0 - \frac{\lambda_{30}^2 V'_2 + \lambda'^2_{12} V_1 - 2 \lambda_{30} \lambda'_{12} \lambda'_{22}}{V_1 V'_2 - \lambda'^2_{22}} \quad (31)$$

where

$$V_0 = \left( \frac{N-n}{Nn} \right) S_y^2 + \frac{P_2(k-1)}{n} S_{y(2)}^2, \\ V'_2 = \left( \frac{N-n}{Nn} \right) (S_x^2)^2 (\beta_2(x) - 1), \\ V_1 = \left( \frac{N-n}{Nn} \right) (S_y^2)^2 (\beta_2(y) - 1) + \frac{P_2(k-1)}{n} (S_{y(2)}^2)^2 (\beta_{2(2)}(y) - 1), \\ V_2 = \left( \frac{N-n}{Nn} \right) (S_x^2)^2 (\beta_2(x) - 1) + \frac{P_2(k-1)}{n} (S_{x(2)}^2)^2 (\beta_{2(2)}(x) - 1), \\ \lambda_{30} = \left( \frac{N-n}{Nn} \right) \mu_{30} + \frac{P_2(k-1)}{n} \mu_{30(2)}, \\ \lambda_{12} = \left( \frac{N-n}{Nn} \right) \mu_{12} + \frac{P_2(k-1)}{n} \mu_{12(2)}, \\ \lambda_{22} = \left( \frac{N-n}{Nn} \right) S_y^2 S_x^2 \left( \frac{\mu_{22}}{\mu_{20} \mu_{02}} - 1 \right) + \frac{P_2(k-1)}{n} S_{y(2)}^2 S_{x(2)}^2 \left( \frac{\mu_{22(2)}}{\mu_{20(2)} \mu_{02(2)}} - 1 \right), \\ \lambda'_{12} = \left( \frac{N-n}{Nn} \right) \mu_{12}, \lambda'_{22} = \left( \frac{N-n}{Nn} \right) S_y^2 S_x^2 \left( \frac{\mu_{22}}{\mu_{20} \mu_{02}} - 1 \right), \\ \frac{\Delta_1}{\Delta_2} = \frac{\lambda_{30} \lambda_{22} - \lambda_{12} V_1}{V_1 V_2 - \lambda_{22}^2}, \\ \frac{\Delta'_1}{\Delta'_2} = \frac{\lambda_{30} \lambda'_{22} - \lambda'_{12} V_1}{V_1 V'_2 - \lambda'^2_{22}}, \\ \mu_{rs} = \frac{1}{N} \sum_{i=1}^N (y_i - \bar{Y})^r (x_i - \bar{X})^s, \\ \mu_{rs(2)} = \frac{1}{N} \sum_{i=1}^N (y_i - \bar{Y}_2)^r (x_i - \bar{X}_2)^s, \\ \beta_2(y) = \frac{\mu_{40}}{(\mu_{20})^2}, \\ \beta_2(x) = \frac{\mu_{04}}{(\mu_{02})^2}, \\ \beta_{2(2)}(y) = \frac{\mu_{40(2)}}{(\mu_{20(2)})^2}, \\ \beta_{2(2)}(x) = \frac{\mu_{04(2)}}{(\mu_{02(2)})^2}, \\ \bar{X}_2 = \frac{\sum U_{N_2} x_i}{N_2} \quad \text{and} \quad \bar{Y}_2 = \frac{\sum U_{N_2} y_i}{N_2}.$$

**Comparison of the Proposed Estimators with the Relevant Estimators**

Since all the generalized estimators  $T_{g_j}^{(1)}$  and  $T_{g_j}^{(2)}$ , ( $j = 1, 2, \dots, 5$ ) are the particular members of the proposed classes of estimators  $T_C^{(1)}$  and  $T_C^{(2)}$ , so their bias and mean square error can be obtained from [(22), (23)] and [(24), (25)] respectively. If the constants involved in  $T_{g_j}^{(1)}$  and  $T_{g_j}^{(2)}$  are respectively calculated by [(26), (27)] and [(29), (30)] then  $T_{g_j}^{(1)}$  and  $T_{g_j}^{(2)}$ ;  $j = 1, 2, \dots, 5$  will attain the minimum mean square error equal to the expressions given in (28) and (31) respectively.

Sometimes the optimum value of  $h_1^{(i)}(1,1)$  and  $h_2^{(i)}(1,1)$ ;  $i = 1, 2$  are obtained in the form of constants along with some parameters and sometimes in the form of some conditions between parameters. The later one is difficult to realize in practice and rarely used. In former case, the value of constants in the form of parameters can be computed on the basis of past data or from pilot sample surveys. Reddy [18] has shown that such values are stable over time and region. For more details, the reader are referred to Murthy [13] and Sahai and Sahai [19]. It has been shown by Srivastava and Jhaji [23] that the minimum value of the mean square error of the estimator is unchanged upto the order ( $n^{-1}$ ), if we estimate the optimum value of the constants by using the sample values.

On comparing the mean square error of  $T_C^{(1)}$  with  $V(\bar{y}_h)$  from equations (28) and (2), we find that

$$V(\bar{y}_h) - MSE(T_C^{(1)})_{min.} = \frac{\lambda_{30}^2}{V_1} + \frac{1}{V_1} \left[ \frac{\Delta_1^2}{\Delta_2} \right] \geq 0$$

and the comparison of mean square error of  $T_C^{(2)}$  with  $V(\bar{y}_h)$  from equations (31) and (2) shows that

$$V(\bar{y}_h) - MSE(T_C^{(2)})_{min.} = \frac{\lambda_{30}^2}{V_1} + \frac{1}{V_1} \left[ \frac{\Delta_1^2}{\Delta_2} \right] \geq 0.$$

Moreover it should be noted here that mean square error of the proposed class of estimators  $T_C^{(i)}, i = 1,2$  will be less compare to the variance of  $\bar{y}_h$  for the suitably chosen constants iff  $-\frac{V_0}{\bar{y}^2} \leq Q \leq 0$ ,

where

$$Q = \left[ \frac{V_1}{(S_y^2)^2} h_1^{(i)2}(1,1) + \frac{V_2}{(S_x^2)^2} h_2^{(i)2}(1,1) + 2 \left\{ \frac{\lambda_{30}}{\bar{y} S_y^2} h_1^{(i)}(1,1) + \frac{\lambda_{22}}{S_x^2 S_y^2} h_1^{(i)}(1,1) h_2^{(i)}(1,1) + \frac{\lambda_{12}}{\bar{y} S_x^2} h_2^{(i)}(1,1) \right\} \right]$$

**Empirical Study**

One hundred nine village population of urban area under Police station-Baria, Tahsil-Champua, Orissa, India has been taken under consideration from Distinct Census Handbook, 1981, Orissa, published by Government of India.

**Data Set I** –In this data set the first 25% villages (i.e 27 villages) have been considered as non-responding group of the population. Here we have taken total population of the village as study character (y) and area as auxiliary character (x). The values of the parameters of the population under study are as follows:

$\bar{Y} = 485.9174$	$\bar{X} = 255.9652$	$S_y^2 = 101593.3100$	$S_x^2 = 23881.5700$
$S_{y(2)}^2 = 127050.3484$	$S_{x(2)}^2 = 31172.0396$	$\mu_{30} = 51990402.19$	$\mu_{12} = 9867584.919$
$\mu_{22} = 12722032499$	$\mu_{40} = 67105355398.07$	$\mu_{04} = 3335576151.47$	$\mu_{30(2)} = 74395310.88$
$\mu_{12(2)} = 17717815.03$	$\mu_{22(2)} = 2329483525$	$\mu_{40(2)} = 1.02022 \times 10^{11}$	$\mu_{04(2)} = 6176566081$

**Data Set II** –In this data set, we have taken same non-responding group as Data Set-I. Here we have considered total population of the village and number of literates persons in the village as study character (y) and auxiliary character (x) respectively. The values of the parameters of the population under study are as follows:

$\bar{Y} = 485.9174$	$\bar{X} = 145.3000$	$S_y^2 = 101593.3100$	$S_x^2 = 12292.4700$
$S_{y(2)}^2 = 127050.3484$	$S_{x(2)}^2 = 14146.3978$	$\mu_{30} = 51990402.19$	$\mu_{12} = 6797325.07$
$\mu_{22} = 7748514264$	$\mu_{40} = 67105355398.07$	$\mu_{04} = 1185322665.42$	$\mu_{30(2)} = 74395310.88$
$\mu_{12(2)} = 8520158.845$	$\mu_{22(2)} = 10776393123$	$\mu_{40(2)} = 1.02022 \times 10^{11}$	$\mu_{04(2)} = 1251886520$

To study the performance of the proposed class of estimators  $T_C^{(i)}, i = 1,2$  with respect to  $\bar{y}_h$ , we have considered  $T_{g_1}^{(1)}$  and  $T_{g_1}^{(2)}$  which are the members of the proposed classes of estimators  $T_C^{(1)}$  and  $T_C^{(2)}$  respectively. The mean square error and relative efficiency (in %) of different estimators for population mean  $\bar{Y}$  with respect to usual unbiased estimator  $\bar{y}_h$  for different value of sub sampling fraction are given in Table 1. We have computed the relative efficiency in percentage by the formula  $R.E. = \frac{var(\bar{y}_h)}{var(T_{g_1}^{(i)})} \times 100, i = 1,2$ . For the analysis of both Data Sets, we assume that 10%, 20% and 33% data have been sub-sampled from non-responding group and information has been collected by extra efforts.

**Conclusion**

Table 1 exhibits for both Data Sets, that the mean square error of both the estimators  $T_{g_1}^{(1)}$  and  $T_{g_1}^{(2)}$  are much efficient as compared to the usual estimator  $\bar{y}_h$  at all the different levels of sub-samplings fraction (1/k). It is also shows that the mean square error of  $T_{g_1}^{(1)}$  and  $T_{g_1}^{(2)}$  are decreasing by increasing the value of sub-sampling fraction (1/k). The percentage relative efficiency of the estimators  $T_{g_1}^{(1)}$  and  $T_{g_1}^{(2)}$  decreases as the value of (1/k) increases, this is because the variance of  $\bar{y}_h$  decreases at a faster rate comparative to the mean square error of  $T_{g_1}^{(1)}$  and  $T_{g_1}^{(2)}$ . As  $T_{g_1}^{(1)}$  and  $T_{g_1}^{(2)}$  are the particular members of  $T_C^{(1)}$  and  $T_C^{(2)}$  respectively, so the classes of estimators  $T_C^{(1)}$  and  $T_C^{(2)}$  are recommended for the use in practice under their respective circumstances as discussed in the text.

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**Table 1. Mean Square Error(.) and R. E.(in %) of the Estimators for  $n = 35$**

	Estimators	1/k		
		1/10	1/5	1/3
Data Set I	$\bar{y}_h$	100.00 (10064.0174)*	100.00 (5566.4351)	100.00 (3775.0251)
	$T_{g_1}^{(1)}$	202.34 (4973.9190)	199.26 (2793.5029)	196.27 (1923.3354)
	$T_{g_1}^{(2)}$	200.02 (5031.4274)	197.72 (2815.3529)	195.33 (1932.6419)
Data Set II	$\bar{y}_h$	100.00 (10064.0174)	100.00 (5566.4351)	100.00 (3775.0251)
	$T_{g_1}^{(1)}$	213.27 (4718.9912)	209.22 (2660.5271)	205.61 (1836.0702)
	$T_{g_1}^{(2)}$	200.84 (5010.9000)	199.27 (2793.3801)	197.80 (1908.4849)

(\*figures in parenthesis show the mean square error of the estimators)