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Wave propagation in ring shaped electro-magneto-elastic plate of polygonal cross-sections

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ABSTRACT

This paper describes the method for solving vibration problem of ring shaped electro–magneto -elastic plate of polygonal (Triangle, Square, Pentagon and Hexagon) cross-sections using Fourier Expansion Collocation Method. A mathematical model is developed to study the wave propagation in a electro-magneto-elastic plate of polygonal cross-sections using the theory of elasticity. The frequency equations are obtained from the arbitrary cross sectional boundary conditions, since the boundary is irregular in shape; it is difficult to satisfy the boundary conditions along the inner and outer surface of the plate directly. Hence, the Fourier Expansion Collocation Method is applied along the boundary to satisfy the boundary conditions. The roots of the frequency equations are obtained by using the secant method, applicable for complex roots. The computed non-dimensional frequencies are plotted in the form of dispersion curves and its characteristics are discussed. The problem may be extended to any kinds of cross-sections by using the proper geometrical relations.

Introduction

The wave propagation in magneto-electro-elastic materials has gained considerable importance since last decade. The electro-magneto-elastic materials exhibit a desirable coupling effect between electric and magnetic fields, which are useful in smart structure applications. These materials have the capacity to convert one form of energy namely, magnetic, electric and mechanical energy to another form of energy. The composite consisting of piezoelectric and piezomagnetic components have found increasing application in engineering structures, particularly in smart/intelligent structure system. The magneto-electro-elastic materials are used as magnetic field probes, electric packing, acoustic, hydrophones, medical, ultrasonic image processing, sensors and actuators with the responsibility of magnetic-electro-mechanical energy conversion.

A method, for solving wave propagation in arbitrary and polygonal cross-sectional plates and to find out the phase velocities in different modes of vibrations namely longitudinal, torsional and flexural, by constructing frequency equations was devised by (Nagaya, 1981a, 1981b, 1981c, 1983a, 1983b). He formulated the Fourier expansion collocation method for this purpose and the same method is used in this problem.The three-dimensional behavior of magnetoelectroelastic laminates under simple support has been studied by (Pan, 2001) and (Pan and Heyliger, 2002). An exact solution for magnetoelectroelastic laminates in cylindrical bending has also been obtained by (Pan and Heyliger, 2003). (Pan and Han, 2005) studied the exact solution for functionally graded and layered magneto-electro-elastic plates. (Feng and Pan, 2008) discussed the dynamic fracture behavior of an internal interfacial crack between two dissimilar magneto-electro-elastic plates. (Buchanan, 2003) developed the free vibration of an infinite magneto-electro-elastic cylinder. (Dai and Wang, 2005, 2006) have studied thermo-electro-elastic transient responses in piezoelectric hollow structures and hollow cylinder subjected to complex loadings. Later (Kong et al, 2009) presented the thermo-magneto-dynamic stresses and perturbation of magnetic field vector in a non-homogeneous hollow cylinder. (Annigeri et al, 2006, 2007, 2006) studied respectively, the free vibration of clamped-clamped magneto-electro-elastic cylindrical shells, free vibration behavior of multiphase and layered magneto-electro-elastic beam, free vibrations of simply supported layered and multiphase magneto-electro-elastic cylindrical shells. (Hon et al, 2008) analyzed a point heat source on the surface of a semi-infinite transversely isotropic electro-magneto-thermo-elastic materials. (Sharma and Mohinder Pal, 2004) developed the Rayleigh-Lamb waves in magneto-thermo-elastic homogeneous isotropic plate. Later (Sharma and Thakur, 2006) studied the effect of rotation on Rayleigh-Lamb waves in magneto-thermo-elastic media. (Gao and Noda, 2004) presented the thermal-induced interfacial cracking of magnetoelectroelastic materials. (Bin et al, 2008) studied the wave propagation in non-homogeneous magneto-electo-elastic plates. (Ponnusamy, 2007, 2011, 2012) have studied the wave propagation in generalized thermo-elastic cylinder of arbitrary cross section, thermoelastic and generalized thermo elastic plates of arbitrary and polygonal cross-sections respectively. (Ponnusamy and Rajagopal, 2010, 2011) have studied, the wave propagation in a generalized thermo elastic solid cylinder of arbitrary cross-section and in a homogeneous transversely isotropic thermo elastic solid cylinder of polygonal cross-sections respectively using the Fourier expansion collocation method.

Formulation of the Problem

We consider a homogeneous transversely isotropic magneto-electro-elastic ring shaped plate of polygonal cross-sections with thickness h and occupying the space $0 \le Z \le L$, L is the length of plate considered along the z- axis is shown in Figure 2. The system displacements and stresses are defined by

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the cylindrical co-ordinates r, θ and z. The governing equations of motion, electric and magnetic conduction equation in the absence of body force are

$$\sigma_{rr,r} + r^{-1}\sigma_{r\theta,\theta} + \sigma_{rz,z} + r^{-1}\left(\sigma_{rr} - \sigma_{\theta\theta}\right) = \rho u_{,tt}$$
$$\sigma_{r\theta,r} + r^{-1}\sigma_{\theta\theta,\theta} + \sigma_{\theta z,z} + 2r^{-1}\sigma_{r\theta} = \rho v_{,tt}$$
$$\sigma_{rz,r} + r^{-1}\sigma_{\theta z,\theta} + \sigma_{zz,z} + r^{-1}\sigma_{rz} = \rho w_{,tt} \quad (1)$$

The electric conduction equation is

$$D_{r,r} + r^{-1}D_{,r} + r^{-1}D_{\theta,\theta} + D_{z,z} = 0 \quad (2)$$

The Magnetic conduction equation is

$$B_{r,r} + r^{-1}B_{r} + r^{-1}B_{\theta,\theta} + B_{z,z} = 0 \quad (3)$$

Where

$$\sigma_{rr} = c_{11}e_{rr} + c_{12}e_{\theta\theta} + c_{13}e_{zz} - e_{31}E_z - q_{31}H_z$$
$$\sigma_{\theta\theta} = c_{12}e_{rr} + c_{11}e_{\theta\theta} + c_{13}e_{zz} - e_{31}E_z - q_{31}H_z$$
$$\sigma_{zz} = c_{13}e_{rr} + c_{13}e_{\theta\theta} + c_{33}e_{zz} - e_{33}E_z - q_{33}H_z$$
$$\sigma_{r\theta} = 2c_{66}e_{r\theta}$$
$$\sigma_{\theta z} = 2c_{44}e_{\theta z} - e_{15}E_\theta - q_{15}H_\theta$$
$$\sigma_{rz} = 2c_{44}e_{rz} - e_{15}E_r - q_{15}H_r \quad (4)$$

$$D_r = 2e_{15}e_{rz} + \varepsilon_{11}E_r + m_{11}H_r$$
$$D_\theta = 2e_{15}e_{\theta z} + \varepsilon_{11}E_\theta + m_{11}H_\theta$$
$$D_z = e_{31}\left(e_{rr} + e_{\theta\theta}\right) + e_{33}e_{zz} + \varepsilon_{33}E_z + m_{33}H_z \quad (5)$$

and

$$B_r = 2q_{15}e_{rz} + m_{11}E_r + \mu_{11}H_r$$
$$B_\theta = 2q_{15}e_{\theta z} + m_{11}E_\theta + \mu_{11}H_\theta$$
$$B_z = q_{31}\left(e_{rr} + e_{\theta\theta}\right) + q_{33}e_{zz} + m_{33}E_z + \mu_{33}H_z \quad (6)$$

Where $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}, \sigma_{r\theta}, \sigma_{rz}, \sigma_{\theta z}$ are the stress components, $c_{11}, c_{12}, c_{13}, c_{33}, c_{44}$, and c_{66} are elastic constants, $\varepsilon_{11}, \varepsilon_{33}$ are the dielectric constants, μ_{11}, μ_{33} are the magnetic permeability coefficients, e_{31}, e_{33}, e_{15} are the piezoelectric material coefficients, q_{31}, q_{33}, q_{15} are the piezomagnetic material coefficients, m_{11}, m_{33} are the magneto-electric material coefficients, ρ is the mass density of the material, D_r, D_θ and D_z are the electric displacements, B_r, B_θ and B_z are the magnetic displacements components.

The strain e_{ij} are related to the displacements corresponding to the cylindrical coordinates (r,θ,z) are given by

$$e_{rr} = u_{,r}, \quad e_{\theta\theta} = r^{-1}\left(v_{,\theta} + u\right), \quad e_{zz} = w_{,z}$$
$$e_{\theta z} = \frac{1}{2}\left(v_{,z} + r^{-1}w_{,\theta}\right), \quad e_{r\theta} = \frac{1}{2}\left(r^{-1}u_{,\theta} + v_{,r} - r^{-1}v\right),$$
$$e_{rz} = \frac{1}{2}\left(u_{,z} + w_{,r}\right) \quad (7)$$

where u, v and w are the mechanical displacements along the radial, circumferential and axial directions respectively.

The Electric field vector $E_i\,(i = r,\theta,z)$ is related to the electric potential E as

$$E_r = -\frac{\partial E}{\partial r}, \quad E_\theta = -\frac{1}{r}\frac{\partial E}{\partial \theta} \quad \text{and} \quad E_z = -\frac{\partial E}{\partial z} \quad (8)$$

Similarly, the magnetic field vector $H_i\,(i = r,\theta,z)$ is related to the magnetic potential H as

$$H_r = -\frac{\partial H}{\partial r}, \quad H_\theta = -\frac{1}{r}\frac{\partial H}{\partial \theta} \quad \text{and} \quad H_z = -\frac{\partial H}{\partial z} \quad (9)$$

Substituting the Eqs. (4)-(9) in the Eqs. (1)-(3), we obtain the set of displacement equations as follows;

$$c_{11}\left(u_{,rr} + r^{-1}u_{,r} - r^{-2}u\right) + c_{66}r^{-2}u_{,\theta\theta} + c_{44}u_{,zz} + \left(c_{66} + c_{12}\right)r^{-1}v_{,r\theta} - \left(c_{11} + c_{66}\right)r^{-2}v_{,\theta}$$
$$+\left(c_{44} + c_{13}\right)w_{,rz} + \left(e_{31} + e_{15}\right)E_{,rz} + \left(q_{31} + q_{15}\right)H_{,rz} = \rho u_{,tt} \quad (10a)$$

$$\left(c_{66} + c_{12}\right)r^{-1}u_{,r\theta} + \left(c_{11} + c_{66}\right)r^{-2}u_{,\theta} + c_{66}\left(v_{,rr} + r^{-1}v_{,r} - r^{-2}v\right) + c_{44}v_{,zz} + c_{11}r^{-2}v_{,\theta\theta}$$
$$+\left(c_{44} + c_{13}\right)r^{-1}w_{,\theta z} + \left(e_{31} + e_{15}\right)r^{-1}E_{,\theta z} + \left(q_{31} + q_{15}\right)r^{-1}H_{,\theta z} = \rho v_{,tt} \quad (10b)$$

$$\left(c_{44} + c_{13}\right)\left(u_{,rz} + r^{-1}u_{,z} + r^{-1}v_{,\theta z}\right) + c_{44}\left(w_{,rr} + r^{-1}w_{,r} + w_{,\theta\theta}\right) + c_{33}w_{,zz} + e_{33}E_{,zz}$$
$$+q_{33}H_{,zz} + e_{15}\left(E_{,rr} + r^{-1}E_{,r} + r^{-2}E_{,\theta\theta}\right) + q_{15}\left(H_{,rr} + r^{-1}H_{,r} + r^{-2}H_{,\theta\theta}\right) = \rho w_{,tt} \quad (10c)$$

$$e_{15}\left(w_{,rr} + r^{-1}w_{,r} + r^{-2}w_{,\theta\theta}\right) + \left(e_{31} + e_{15}\right)\left(u_{,rz} + r^{-1}u_{,z} + r^{-1}v_{,\theta z}\right) + e_{33}w_{,zz} - \varepsilon_{33}E_{,zz}$$
$$-m_{33}H_{,zz} - \varepsilon_{11}\left(E_{,rr} + r^{-1}E_{,r} + r^{-2}E_{,\theta\theta}\right) - m_{11}\left(H_{,rr} + r^{-1}H_{,r} + r^{-2}H_{,\theta\theta}\right) = 0 \quad (10d)$$

$$q_{15}\left(w_{,rr} + r^{-1}w_{,r} + r^{-2}w_{,\theta\theta}\right) + \left(q_{31} + q_{15}\right)\left(u_{,rz} + r^{-1}u_{,z} + r^{-1}v_{,\theta z}\right) + q_{33}w_{,zz} - m_{33}E_{,zz}$$
$$-\mu_{33}H_{,zz} - m_{11}\left(E_{,rr} + r^{-1}E_{,r} + r^{-2}E_{,\theta\theta}\right) - \mu_{11}\left(H_{,rr} + r^{-1}H_{,r} + r^{-2}H_{,\theta\theta}\right) = 0 \quad (10e)$$

Solution of the Problem

The Eqs. (10) is a coupled partial differential equation with three displacements and magnetic and electric conduction components. To uncouple the Eqs. (10), we follow (Sharma and Sharma, 2002) and seek the solutions in the following form

$$u = \sum \varepsilon_n\left[\left(r^{-1}\psi_{n,\theta} - \phi_{n,r}\right) + \left(r^{-1}\overline{\psi}_{n,\theta} - \overline{\phi}_{n,r}\right)\right]$$
$$v = \sum \varepsilon_n\left[\left(-r^{-1}\phi_{n,\theta} - \psi_{n,r}\right) + \left(-r^{-1}\overline{\phi}_{n,\theta} - \overline{\psi}_{n,r}\right)\right]$$
$$W = \sum \varepsilon_n\left[W_{n,z} + \overline{W}_{n,z}\right]$$
$$E = \sum \varepsilon_n\left[E_{n,z} + \overline{E}_{n,z}\right]$$
$$H = \sum \varepsilon_n\left[H_{n,z} + \overline{H}_{n,z}\right] \quad (11)$$

where $\varepsilon_n = 1/2$ for $n = 0$, $\varepsilon_n = 1$ for $n \ge 1$, $\phi_n(r,\theta), \psi_n(r,\theta), E_n(r,\theta), H_n(r,\theta)$ are the displacement potentials for the symmetric mode and $\overline{\phi}_n(r,\theta), \overline{\psi}_n(r,\theta), \overline{E}_n(r,\theta), \overline{H}_n(r,\theta)$ are the displacement potentials for the anti symmetric mode of vibrations.

Substituting the Eq.(11) in (10), we get

$$\left(c_{11}\nabla_1^2 + c_{44}\frac{\partial^2}{\partial z^2} - \rho\frac{\partial^2}{\partial t^2}\right)\phi_n - \left(c_{13} + c_{44}\right)\frac{\partial W_n}{\partial z} - \left(e_{31} + e_{15}\right)\frac{\partial E_n}{\partial z} - \left(q_{31} + q_{15}\right)\frac{\partial H_n}{\partial z} = 0 \quad (12a)$$

$$\left(c_{11}\nabla_1^2 + c_{44}\frac{\partial^2}{\partial z^2} - \rho\frac{\partial^2}{\partial t^2}\right)\phi_n - \left(c_{13} + c_{44}\right)\frac{\partial W_n}{\partial z} - \left(e_{31} + e_{15}\right)\frac{\partial E_n}{\partial z} - \left(q_{31} + q_{15}\right)\frac{\partial H_n}{\partial z} = 0 \quad (12b)$$

$$\left(c_{44}\nabla_1^2+c_{33}\frac{\partial^2}{\partial z^2}-\rho\frac{\partial^2}{\partial t^2}\right)W_n-\left(c_{13}+c_{44}\right)\frac{\partial}{\partial z}\nabla_1^2\phi_n+\left(e_{15}\nabla_1^2+e_{33}\frac{\partial^2}{\partial z^2}\right)E_n+\left(q_{15}\nabla_1^2+q_{33}\frac{\partial^2}{\partial z^2}\right)H_n=0 \quad (12c)$$

$$\left(e_{15}\nabla_1^2+e_{33}\frac{\partial^2}{\partial z^2}\right)W_n-\left(e_{31}+e_{15}\right)\frac{\partial}{\partial z}\nabla_1^2\phi_n-\left(\varepsilon_{11}\nabla_1^2+\varepsilon_{33}\frac{\partial^2}{\partial z^2}\right)E_n-\left(m_{11}\nabla_1^2+m_{33}\frac{\partial^2}{\partial z^2}\right)H_n=0 \quad (12d)$$

$$\left(q_{15}\nabla_1^2+q_{33}\frac{\partial^2}{\partial z^2}\right)W_n-\left(q_{31}+q_{15}\right)\nabla_1^2\phi_n-\left(m_{11}\nabla_1^2+m_{33}\frac{\partial^2}{\partial z^2}\right)E_n-\left(\mu_{11}\nabla_1^2+\mu_{33}\frac{\partial^2}{\partial z^2}\right)H_n=0 \quad (12e)$$

and

$$\left(c_{66}\nabla_1^2+c_{44}\frac{\partial^2}{\partial z^2}-\rho\frac{\partial^2}{\partial t^2}\right)\psi_n=0 \quad (13)$$

where

$$\nabla_1^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$$

The Eq. (13) gives purely transverse wave, which is not affected by the electric and magnetic field. This wave is polarized in the planes perpendicular to the z-axis and it may be referred as the simple harmonic wave. We assume that the disturbance is time harmonic through the factor $e^{i\omega t}$, ω is angular velocity and hence, the system of Eqs. (12a)-(12e) becomes

$$\left(c_{11}\nabla_1^2+c_{44}\frac{\partial^2}{\partial z^2}+\rho\omega^2\right)\phi_n-\left(c_{13}+c_{44}\right)\frac{\partial W_n}{\partial z}-\left(e_{31}+e_{15}\right)\frac{\partial E_n}{\partial z}-\left(q_{31}+q_{15}\right)\frac{\partial H_n}{\partial z}=0 \quad (14a)$$

$$\left(c_{11}\nabla_1^2+c_{44}\frac{\partial^2}{\partial z^2}+\rho\omega^2\right)\phi_n-\left(c_{13}+c_{44}\right)\frac{\partial W_n}{\partial z}-\left(e_{31}+e_{15}\right)\frac{\partial E_n}{\partial z}-\left(q_{31}+q_{15}\right)\frac{\partial H_n}{\partial z}=0 \quad (14b)$$

$$\left(c_{44}\nabla_1^2+c_{33}\frac{\partial^2}{\partial z^2}+\rho\omega^2\right)W_n-\left(c_{13}+c_{44}\right)\frac{\partial}{\partial z}\nabla_1^2\phi_n+\left(e_{15}\nabla_1^2+e_{33}\frac{\partial^2}{\partial z^2}\right)E_n+\left(q_{15}\nabla_1^2+q_{33}\frac{\partial^2}{\partial z^2}\right)H_n=0 \quad (14c)$$

$$\left(e_{15}\nabla_1^2+e_{33}\frac{\partial^2}{\partial z^2}\right)W_n-\left(e_{31}+e_{15}\right)\frac{\partial}{\partial z}\nabla_1^2\phi_n-\left(\varepsilon_{11}\nabla_1^2+\varepsilon_{33}\frac{\partial^2}{\partial z^2}\right)E_n-\left(m_{11}\nabla_1^2+m_{33}\frac{\partial^2}{\partial z^2}\right)H_n=0 \quad (14d)$$

$$\left(q_{15}\nabla_1^2+q_{33}\frac{\partial^2}{\partial z^2}\right)W_n-\left(q_{31}+q_{15}\right)\frac{\partial}{\partial z}\nabla_1^2\phi_n-\left(m_{11}\nabla_1^2+m_{33}\frac{\partial^2}{\partial z^2}\right)E_n-\left(\mu_{11}\nabla_1^2+\mu_{33}\frac{\partial^2}{\partial z^2}\right)H_n=0 \quad (14e)$$

We consider the free vibration of polygonal cross-sectional plate, so we assume the

$$\phi_n(r,\theta,z,t)=\phi_n(r)\cos(m\pi z)\cos n\theta$$
$$W_n(r,\theta,z,t)=W_n(r)\sin(m\pi z)\cos n\theta$$
$$E_n(r,\theta,z,t)=\left(\frac{c_{44}}{q_{33}}\right)E_n(r)\sin(m\pi z)\cos n\theta$$
$$H_n(r,\theta,z,t)=\left(\frac{c_{44}}{q_{33}}\right)H_n(r)\sin(m\pi z)\cos n\theta \quad (15)$$

and

$$\psi_n(r,\theta,z,t)=\psi_n(r)\cos(m\pi z)\sin n\theta \quad (16)$$

where $z=\frac{a}{L}$

Introducing the dimensionless quantities such as

$x=\frac{r}{a}$, $\varsigma=a/L$, $t_L=\frac{m\pi a}{L}$, $\bar{c}_{ij}=\frac{c_{ij}}{c_{44}}$, $\bar{e}_{ij}=\frac{e_{ij}}{e_{33}}$, $\bar{q}_{ij}=\frac{q_{ij}}{q_{33}}$, $\Omega^2=\frac{\rho\omega^2a^2}{c_{44}}$, $\bar{m}_{ij}=\frac{m_{ij}c_{44}}{q_{33}e_{33}}$, $\bar{\mu}_{ij}=\frac{\mu_{ij}c_{44}}{q_{33}^2}$, $\bar{\varepsilon}_{ij}=\frac{\varepsilon_{ij}c_{44}}{e_{33}^2}$ and using the Eqs. (15) and (16) in the Eqs. (14) and (13), we get

$$\left(\bar{c}_{11}\nabla_2^2-t_L^2+\Omega^2\right)\phi_n-\left(1+\bar{c}_{13}\right)t_LW_n-\left(\bar{e}_{31}+\bar{e}_{15}\right)t_LE_n-\left(\bar{q}_{31}+\bar{q}_{15}\right)t_LH_n=0$$
$$\left(\nabla_2^2+\Omega^2-\bar{c}_{33}t_L^2\right)W_n+\left(1+\bar{c}_{13}\right)t_L\nabla_2^2\phi_n+\left(\bar{e}_{15}\nabla_2^2-t_L^2\right)E_n+\left(\bar{q}_{15}\nabla_2^2-t_L^2\right)H_n=0$$
$$\left(\bar{e}_{15}\nabla_2^2-t_L^2\right)W_n+\left(\bar{e}_{31}+\bar{e}_{15}\right)t_L\nabla_2^2\phi_n+\left(\bar{\varepsilon}_{33}t_L^2-\bar{\varepsilon}_{11}\nabla_2^2\right)E_n+\left(\bar{m}_{33}t_L^2-\bar{m}_{11}\nabla_2^2\right)H_n=0$$
$$\left(\bar{q}_{15}\nabla_2^2-t_L^2\right)W_n+\left(\bar{q}_{31}+\bar{q}_{15}\right)t_L\nabla_2^2\phi_n+\left(\bar{m}_{33}t_L^2-\bar{m}_{11}\nabla_2^2\right)E_n+\left(\bar{\mu}_{33}t_L^2-\bar{\mu}_{11}\nabla_2^2\right)H_n=0 \quad (17)$$

and

$$\left(\bar{c}_{66}\nabla_2^2-t_L^2+\Omega^2\right)\psi_n=0 \quad (18)$$

Where $\nabla_2^2=\frac{\partial^2}{\partial x^2}+\frac{1}{x}\frac{\partial}{\partial x}-\frac{n^2}{x^2}$.

The Eq. (17) is a homogeneous linear equation which has a trivial solution to obtain the non-trivial solution, the determinant of the coefficient matrix is equal to zero. Thus we get

$$\begin{vmatrix}\left(\bar{c}_{11}\nabla_2^2+g_1\right) & -g_2t_L & -g_3t_L & -g_4t_L\\ g_2t_L\nabla_2^2 & \left(\nabla_2^2+g_5\right) & \left(\bar{e}_{15}\nabla_2^2-t_L^2\right) & \left(\bar{q}_{15}\nabla_2^2-t_L^2\right)\\ g_3t_L\nabla_2^2 & \left(\bar{e}_{15}\nabla_2^2-t_L^2\right) & \left(g_6-\bar{\varepsilon}_{11}\nabla_2^2\right) & \left(g_7-\bar{m}_{11}\nabla_2^2\right)\\ g_4t_L\nabla_2^2 & \left(\bar{q}_{15}\nabla_2^2-t_L^2\right) & \left(g_7-\bar{m}_{11}\nabla_2^2\right) & \left(g_8-\bar{\mu}_{11}\nabla_2^2\right)\end{vmatrix}\left(\phi_n,W_n,E_n,H_n\right)=0 \quad (19)$$

where

$g_1=\Omega^2-t_L^2$, $g_2=1+\bar{c}_{13}$, $g_3=\left(\bar{e}_{31}+\bar{e}_{15}\right)$, $g_4=\left(\bar{q}_{31}+\bar{q}_{15}\right)$, $g_5=\left(\Omega^2-\bar{c}_{33}t_L^2\right)$, $g_6=\bar{\varepsilon}_{33}t_L^2$, $g_7=\bar{m}_{33}t_L^2$, $g_8=\bar{\mu}_{33}t_L^2$

Evaluating the determinant given in Eq. (19), we obtain the partial differential equation of the form

$$\left(A\nabla_2^8+B\nabla_2^6+C\nabla_2^4+D\nabla_2^2+E\right)\phi_n=0 \quad (20)$$

where

$$A=\bar{c}_{11}\left\{\bar{\varepsilon}_{11}\left[\bar{\mu}_{11}+\bar{q}_{15}^2\right]+\bar{\mu}_{11}\bar{e}_{15}^2-\bar{m}_{11}\left[\bar{m}_{11}+2\bar{e}_{15}\bar{q}_{15}\right]\right\}$$

$$B=\bar{c}_{11}\{-g_6\bar{\mu}_{11}-g_8\bar{\varepsilon}_{11}+2g_7\bar{m}_{11}+g_5\left[\bar{\varepsilon}_{11}\bar{\mu}_{11}-\bar{m}_{11}^2\right]-\bar{e}_{15}\left[g_8\bar{e}_{15}+2t_L^2\left(\bar{\mu}_{11}-\bar{m}_{11}\right)\right]$$
$$+\bar{q}_{15}\left[-g_6\bar{q}_{15}+2t_L^2\left(\bar{m}_{11}-\bar{\varepsilon}_{11}\right)\right]+2g_7\bar{e}_{15}\bar{q}_{15}\}+g_1\left\{\bar{\varepsilon}_{11}\left[\bar{\mu}_{11}+\bar{q}_{15}^2\right]+\bar{\mu}_{11}\bar{e}_{15}^2-\bar{m}_{11}\left[\bar{m}_{11}+2\bar{e}_{15}\bar{q}_{15}\right]\right\}$$
$$+g_2t_L^2\left\{g_2\left[\bar{\varepsilon}_{11}\bar{\mu}_{11}-\bar{m}_{11}^2\right]-\bar{e}_{15}\left[-g_3\bar{\mu}_{11}+g_4\bar{m}_{11}\right]+\bar{q}_{15}\left[-g_3\bar{m}_{11}+g_4\bar{\varepsilon}_{11}\right]\right\}$$
$$-g_3t_L^2\left\{g_2\left[-\bar{\mu}_{11}\bar{e}_{15}+\bar{m}_{11}\bar{q}_{15}\right]+g_3\bar{\mu}_{11}+g_4\bar{m}_{11}+\bar{q}_{15}\left[g_3\bar{q}_{15}-g_4\bar{e}_{15}\right]\right\}$$
$$+g_4t_L^2\left\{g_2\left[-\bar{m}_{11}\bar{e}_{15}+\bar{\varepsilon}_{11}\bar{q}_{15}\right]+g_3\bar{m}_{11}-g_4\bar{\varepsilon}_{11}+\bar{e}_{15}\left[g_3\bar{q}_{15}-g_4\bar{e}_{15}\right]\right\}$$
$$C=\bar{c}_{11}\{g_6g_8-g_7^2+g_5\left[-g_6\bar{\mu}_{11}-g_8\bar{\varepsilon}_{11}+2g_7\bar{m}_{11}\right]+2t_L^2\left[-\bar{e}_{15}\left(-g_8+g_7\right)+\bar{q}_{15}\left(-g_7+g_8\right)\right]$$
$$+t_L^4\left[\bar{\mu}_{11}-2\bar{m}_{11}+\bar{\varepsilon}_{11}\right]\}+g_1\{-g_6\bar{\mu}_{11}-g_8\bar{\varepsilon}_{11}+2g_7\bar{m}_{11}+g_5\left[\bar{\varepsilon}_{11}\bar{\mu}_{11}-\bar{m}_{11}^2\right]$$
$$-\bar{e}_{15}\left[g_8\bar{e}_{15}+2t_L^2\left(\bar{\mu}_{11}-\bar{m}_{11}\right)\right]+\bar{q}_{15}\left[-g_6\bar{q}_{15}+2t_L^2\left(\bar{m}_{11}-\bar{\varepsilon}_{11}\right)\right]+2g_7\bar{e}_{15}\bar{q}_{15}\}$$
$$+g_2t_L^2\{g_2\left[-g_6\bar{\mu}_{11}-g_8\bar{\varepsilon}_{11}+2g_7\bar{m}_{11}\right]-\bar{e}_{15}\left[g_3g_8-g_4g_7\right]+\bar{q}_{15}\left[g_3g_7-g_4g_6\right]$$
$$+t_L^2\left[-g_3\bar{\mu}_{11}+g_4\bar{m}_{11}+g_3\bar{m}_{11}-g_4\bar{\varepsilon}_{11}\right]\}-g_3t_L^2\{g_2\left[g_8\bar{e}_{15}-g_7\bar{q}_{15}+t_L^2\left(\bar{\mu}_{11}-\bar{m}_{11}\right)\right]-g_3g_8$$
$$+g_4g_7-g_5\left[-g_3\bar{\mu}_{11}+g_4\bar{m}_{11}\right]+\bar{q}_{15}t_L^2\left[-g_3+g_4\right]-t_L^2\left[g_3\bar{q}_{15}-g_4\bar{e}_{15}\right]\}$$
$$+g_4t_L^2\{g_2\left[g_7\bar{e}_{15}-g_6\bar{q}_{15}+t_L^2\left(\bar{m}_{11}-\bar{\varepsilon}_{11}\right)\right]-g_3g_7+g_4g_6-g_5\left[-g_3\bar{m}_{11}+g_4\bar{\varepsilon}_{11}\right]$$
$$+\bar{e}_{15}t_L^2\left[-g_3+g_4\right]-t_L^2\left[g_3\bar{q}_{15}-g_4\bar{e}_{15}\right]\}$$
$$D=\bar{c}_{11}\left\{g_5\left[g_6g_8-g_7^2\right]+t_L^4\left[-g_8+2g_7-g_6\right]\right\}+g_4t_L^2\{g_2t_L^2\left[-g_7+g_6\right]$$
$$-g_5\left[g_3g_7-g_4g_6\right]-t_L^4\left[-g_3+g_4\right]\}+g_1\{g_6g_8-g_7^2+g_5\left[-g_6\bar{\mu}_{11}-g_8\bar{\varepsilon}_{11}+2g_7\bar{m}_{11}\right]$$
$$+2t_L^2\left[-\bar{e}_{15}\left(-g_8+g_7\right)+\bar{q}_{15}\left(-g_7+g_8\right)\right]+t_L^4\left[\bar{\mu}_{11}-2\bar{m}_{11}+\bar{\varepsilon}_{11}\right]\}$$
$$+g_2t_L^2\left\{g_2\left[g_6g_8-g_7^2\right]+t_L^2\left[g_3g_8-g_4g_7-g_3g_7+g_4g_6\right]\right\}$$
$$-g_3t_L^2\left\{g_2t_L^2\left[-g_8+g_7\right]-g_5\left[g_3g_8-g_4g_7\right]-t_L^4\left[-g_3+g_4\right]\right\}$$
$$E=g_1\left\{g_5\left[g_6g_8-g_7^2\right]+t_L^4\left[-g_8+2g_7-g_6\right]\right\}$$

Solving the Eq. (20), the solution for the symmetric mode obtained as

$$\phi_n^*=\sum_{i=1}^{4}\left(A_{in}J_n\left(\alpha_ir\right)+B_{in}Y_n\left(\alpha_ir\right)\right)\cos n\theta$$
$$W_n^*=\sum_{i=1}^{4}a_i\left(A_{in}J_n\left(\alpha_ir\right)+B_{in}Y_n\left(\alpha_ir\right)\right)\cos n\theta$$
$$E_n^*=\sum_{i=1}^{4}b_i\left(A_{in}J_n\left(\alpha_ir\right)+B_{in}Y_n\left(\alpha_ir\right)\right)\cos n\theta$$
$$H_n^*=\sum_{i=1}^{4}c_i\left(A_{in}J_n\left(\alpha_ir\right)+B_{in}Y_n\left(\alpha_ir\right)\right)\cos n\theta \qquad (21)$$

The solutions to the anti symmetric modes of vibrations $\bar{\phi}_n^*,\bar{W}_n^*,\bar{E}_n^*,\bar{H}_n^*$, are obtained by changing $\cos n\theta$ by $\sin n\theta$ in the Eq. (21), we get

$$\bar{\phi}_n^*=\sum_{i=1}^{4}\left(\bar{A}_{in}J_n\left(\alpha_ir\right)+\bar{B}_{in}Y_n\left(\alpha_ir\right)\right)\sin n\theta$$
$$\bar{W}_n^*=\sum_{i=1}^{4}a_i\left(\bar{A}_{in}J_n\left(\alpha_ir\right)+\bar{B}_{in}Y_n\left(\alpha_ir\right)\right)\sin n\theta$$
$$\bar{E}_n^*=\sum_{i=1}^{4}b_i\left(\bar{A}_{in}J_n\left(\alpha_ir\right)+\bar{B}_{in}Y_n\left(\alpha_ir\right)\right)\sin n\theta$$
$$\bar{H}_n^*=\sum_{i=1}^{4}c_i\left(\bar{A}_{in}J_n\left(\alpha_ir\right)+\bar{B}_{in}Y_n\left(\alpha_ir\right)\right)\sin n\theta \qquad (22)$$

where J_n is the Bessel function of first kind of order n and Y_n is the Bessel function of second kind of order n. The constants a_i,b_i and c_i defined in the Eqs. (21) and (22) are calculated using the following equations

$$-g_2t_La_i-g_3t_Lb_i-g_4t_Lc_i=\bar{c}_{11}\alpha_i^2-g_1$$
$$\left(-\alpha_i^2+g_5\right)a_i-\left(\bar{e}_{15}\alpha_i^2+t_L^2\right)b_i-\left(\bar{q}_{15}\alpha_i^2+t_L^2\right)c_i=g_2t_L\alpha_i^2$$
$$-\left(\bar{e}_{15}\alpha_i^2+t_L^2\right)a_i+\left(g_6+\bar{\varepsilon}_{11}\alpha_i^2\right)b_i+\left(g_7+\bar{m}_{11}\alpha_i^2\right)c_i=g_3t_L\alpha_i^2$$
$$-\left(\bar{q}_{15}\alpha_i^2+t_L^2\right)a_i+\left(g_7+\bar{m}_{11}\alpha_i^2\right)b_i+\left(g_8+\bar{\mu}_{11}\alpha_i^2\right)c_i=g_4t_L\alpha_i^2 \qquad (23)$$

Solving the Eq. (23), we obtain

$$a_i=\frac{-g_3\left(g_7+\bar{m}_{11}\alpha_i^2\right)+g_4\left(g_6+\bar{\varepsilon}_{11}\alpha_i^2\right)}{-g_2\left(g_6+\bar{\varepsilon}_{11}\alpha_i^2\right)-g_3\left(\bar{e}_{15}\alpha_i^2+t_L^2\right)}$$
$$b_i=\frac{g_3g_4t_L^2\alpha_i^2+\left(\bar{c}_{11}\alpha_i^2-g_1\right)\left(g_7+\bar{m}_{11}\alpha_i^2\right)}{t_L\left[g_2\left(g_6+\bar{\varepsilon}_{11}\alpha_i^2\right)+g_3\left(\bar{e}_{15}\alpha_i^2+t_L^2\right)\right]}$$
$$c_i=\frac{\left(\bar{c}_{11}\alpha_i^2-g_1\right)\left(\bar{e}_{15}\alpha_i^2+t_L^2\right)-g_2g_3t_L^2\alpha_i^2}{t_L\left[g_2\left(g_6+\bar{\varepsilon}_{11}\alpha_i^2\right)+g_3\left(\bar{e}_{15}\alpha_i^2+t_L^2\right)\right]}$$

Solving the Eq. (13), we obtain the solution for symmetric mode as

$$\psi_n^*=\left(A_5J_n\left(\alpha_5r\right)+B_5J_n\left(\alpha_5r\right)\right)\sin n\theta \qquad (24)$$

and the solution for the anti symmetric mode is obtained by changing $\cos n\theta$ by $\sin n\theta$ in the Eq. (24), we obtain

$$\bar{\psi}_n^*=\left(\bar{A}_5J_n\left(\alpha_5r\right)+\bar{B}_5J_n\left(\alpha_5r\right)\right)\cos n\theta \qquad (25)$$

where J_n is the Bessel function of first kind of order n and Y_n is the Bessel function of second kind of order n, and $\alpha_5^2=\left(t_L^2-\Omega^2\right)/c_{66}$ If $\left(\alpha_ia\right)^2<0\left(i=1,2,3,4,5\right)$ then the Bessel functions J_n and Y_n are replaced by the modified Bessel function I_n and K_n respectively.

Boundary conditions and frequency equations

In this problem, the vibration of polygonal cross-sectional plate is considered. Since the boundary is irregular in shape, it is difficult to satisfy the boundary conditions along inner and outer the surface of the plate directly. Hence, the Fourier expansion collocation method is applied to satisfy the boundary conditions. For the plate, the normal stress $\sigma_{xx}^{'}$ and shearing stresses $\sigma_{xy}^{'},\sigma_{xz}^{'}$, the electric field $D_x^{'}$ and the magnetic field $B_x^{'}$ is equal to zero for stress free inner boundary. Similarly, normal stress σ_{xx} and shearing stresses σ_{xy},σ_{xz}, the electric field D_x and the magnetic field B_x is equal to zero for stress free outer boundary. Thus the following types of boundary conditions for the inner of the plate is obtained as

$$\left(\sigma_{xx}^{'}\right)_i=\left(\sigma_{xy}^{'}\right)_i=\left(\sigma_{xz}^{'}\right)_i=\left(D_x^{'}\right)_i=\left(B_x^{'}\right)_i=0 \qquad (26a)$$

and for the outer boundary, the boundary condition is obtained as

$$\left(\sigma_{xx}\right)_i=\left(\sigma_{xy}\right)_i=\left(\sigma_{xz}\right)_i=\left(D_x\right)_i=\left(B_x\right)_i=0 \qquad (26b)$$

where $(\ )_i$ is the value at the boundary Γ_i is shown in Figure 1 (Geometry of segments). Since the vibration displacements are expressed in terms of the coordinates r and θ, it is convenient to treat the boundary conditions when the derivatives in the equations of the stresses are transformed in terms of the coordinates r and θ instead of the coordinates x_i and y_i. The relations between the displacements are as follows for $i-th$ segment of straight-line boundaries are

$$u = u\cos(\theta-\gamma_i) - v\sin(\theta-\gamma_i)$$
$$v = v\cos(\theta-\gamma_i) + u\sin(\theta-\gamma_i) \qquad (27)$$

Since the angle γ_i between the reference axis and normal of the $i-th$ boundary has a constant value in a segment Γ_i, we obtain

$$\frac{\partial r}{\partial x_i} = \cos(\theta-\gamma_i), \quad \frac{\partial \theta}{\partial x_i} = -\left(\frac{1}{r}\right)\sin(\theta-\gamma_i)$$
$$\frac{\partial r}{\partial y_i} = \sin(\theta-\gamma_i), \quad \frac{\partial \theta}{\partial y_i} = \left(\frac{1}{r}\right)\cos(\theta-\gamma_i) \qquad (28)$$

Using the Eqs. (27) and (28), the normal and shearing stresses are transformed as

$$\sigma_{xx} = \left(c_{11}\cos^2(\theta-\gamma_i) + c_{12}\sin^2(\theta-\gamma_i)\right)u_{,r} + r^{-1}\left(c_{11}\sin^2(\theta-\gamma_i) + c_{12}\cos^2(\theta-\gamma_i)\right)(u+v_{,\theta})$$
$$+ c_{66}\left(r^{-1}(v-u_{,\theta}) - v_{,r}\right)\sin 2(\theta-\gamma_i) + c_{13}W_{,z} + e_{31}E_{,z} + q_{31}H_{,z} = 0$$
$$\sigma_{xy} = c_{66}\left(\left(u_{,r} - r^{-1}(v_{,\theta}+u)\right)\sin 2(\theta-\gamma_i) + \left(r^{-1}(u_{,\theta}-v) + v_{,r}\right)\cos 2(\theta-\gamma_i)\right) = 0$$
$$\sigma_{xz} = c_{44}\left((u_{,z}+W_{,r})\cos(\theta-\gamma_i) - (v_{,z}+r^{-1}W_{,\theta})\sin(\theta-\gamma_i)\right) + e_{15}E_{,r} + q_{15}H_{,r} = 0$$
$$D_x = e_{15}(u_{,z}+W_{,r}) - \varepsilon_{11}E_{,r} - m_{11}H_{,r} = 0$$
$$B_x = q_{15}(u_{,z}+W_{,r}) - m_{11}E_{,r} - \mu_{11}H_{,r} = 0 \qquad (29)$$

Substituting the equations (21) - (25) in the equation (26), the boundary conditions are transformed for stress free polygonal cross-sectional inner surface of plate as follows:

$$\left[(S^1_{xx})_i + (\overline{S}^1_{xx})_i\right]e^{i\Omega T_a} = 0,$$
$$\left[(S^1_{xy})_i + (\overline{S}^1_{xy})_i\right]e^{i\Omega T_a} = 0$$
$$\left[(S^1_{xz})_i + (\overline{S}^1_{xz})_i\right]e^{i\Omega T_a} = 0$$
$$\left[(E^1_x)_i + (\overline{E}^1_x)_i\right]e^{i\Omega T_a} = 0$$
$$\left[(H^1_x)_i + (\overline{H}^1_x)_i\right]e^{i\Omega T_a} = 0 \qquad (30a)$$

and for the outer surface

$$\left[(S_{xx})_i + (\overline{S}_{xx})_i\right]e^{i\Omega T_a} = 0,$$
$$\left[(S_{xy})_i + (\overline{S}_{xy})_i\right]e^{i\Omega T_a} = 0$$
$$\left[(S_{xz})_i + (\overline{S}_{xz})_i\right]e^{i\Omega T_a} = 0,$$
$$\left[(E_x)_i + (\overline{E}_x)_i\right]e^{i\Omega T_a} = 0$$
$$\left[(H_x)_i + (\overline{H}_x)_i\right]e^{i\Omega T_a} = 0 \qquad (30b)$$

where

$$S^1_{xx} = 0.5\left(e_0^1A_{10} + e_0^2B_{10} + e_0^3A_{20} + e_0^4B_{20} + e_0^5A_{30} + e_0^6B_{30} + e_0^7A_{40} + e_0^8B_{40}\right)$$
$$+\sum_{n=1}^{\infty}\left(e_n^1A_{1n} + e_n^2B_{1n} + e_n^3A_{2n} + e_n^4B_{2n} + e_n^5A_{3n} + e_n^6B_{3n} + e_n^7A_{4n} + e_n^8B_{4n} + e_n^9A_{5n} + e_n^{10}B_{5n}\right)$$
$$S^1_{xy} = 0.5\left(f_0^1A_{10} + f_0^2B_{10} + f_0^3A_{20} + f_0^4B_{20} + f_0^5A_{30} + f_0^6B_{30} + f_0^7A_{40} + f_0^8B_{40}\right)$$
$$+\sum_{n=1}^{\infty}\left(f_n^1A_{1n} + f_n^2B_{1n} + f_n^3A_{2n} + f_n^4B_{2n} + f_n^5A_{3n} + f_n^6B_{3n} + f_n^7A_{4n} + f_n^8B_{4n} + f_n^9A_{5n} + f_n^{10}B_{5n}\right)$$
$$S^1_{xz} = 0.5\left(ge_0^1A_{10} + g_0^2B_{10} + g_0^3A_{20} + g_0^4B_{20} + g_0^5A_{30} + g_0^6B_{30} + g_0^7A_{40} + g_0^8B_{40}\right)$$
$$+\sum_{n=1}^{\infty}\left(g_n^1A_{1n} + g_n^2B_{1n} + g_n^3A_{2n} + g_n^4B_{2n} + g_n^5A_{3n} + g_n^6B_{3n} + g_n^7A_{4n} + g_n^8B_{4n} + g_n^9A_{5n} + g_n^{10}B_{5n}\right)$$
$$E^1_x = 0.5\left(h_0^1A_{10} + h_0^2B_{10} + h_0^3A_{20} + h_0^4B_{20} + h_0^5A_{30} + h_0^6B_{30} + h_0^7A_{40} + h_0^8B_{40}\right)$$
$$+\sum_{n=1}^{\infty}\left(h_n^1A_{1n} + h_n^2B_{1n} + h_n^3A_{2n} + h_n^4B_{2n} + h_n^5A_{3n} + h_n^6B_{3n} + h_n^7A_{4n} + h_n^8B_{4n} + h_n^9A_{5n} + h_n^{10}B_{5n}\right)$$
$$H^1_x = 0.5\left(e_0^1A_{10} + e_0^2B_{10} + e_0^3A_{20} + e_0^4B_{20} + e_0^5A_{30} + e_0^6B_{30} + e_0^7A_{40} + e_0^8B_{40}\right)$$
$$+\sum_{n=1}^{\infty}\left(e_n^1A_{1n} + e_n^2B_{1n} + e_n^3A_{2n} + e_n^4B_{2n} + e_n^5A_{3n} + e_n^6B_{3n} + e_n^7A_{4n} + e_n^8B_{4n} + e_n^9A_{5n} + e_n^{10}B_{5n}\right) \qquad (31a)$$

$$S_{xx} = 0.5\left(e_0^1A_{10} + e_0^2B_{10} + e_0^3A_{20} + e_0^4B_{20} + e_0^5A_{30} + e_0^6B_{30} + e_0^7A_{40} + e_0^8B_{40}\right)$$
$$+\sum_{n=1}^{\infty}\left(e_n^1A_{1n} + e_n^2B_{1n} + e_n^3A_{2n} + e_n^4B_{2n} + e_n^5A_{3n} + e_n^6B_{3n} + e_n^7A_{4n} + e_n^8B_{4n} + e_n^9A_{5n} + e_n^{10}B_{5n}\right)$$
$$S_{xy} = 0.5\left(f_0^1A_{10} + f_0^2B_{10} + f_0^3A_{20} + f_0^4B_{20} + f_0^5A_{30} + f_0^6B_{30} + f_0^7A_{40} + f_0^8B_{40}\right)$$
$$+\sum_{n=1}^{\infty}\left(f_n^1A_{1n} + f_n^2B_{1n} + f_n^3A_{2n} + f_n^4B_{2n} + f_n^5A_{3n} + f_n^6B_{3n} + f_n^7A_{4n} + f_n^8B_{4n} + f_n^9A_{5n} + f_n^{10}B_{5n}\right)$$
$$S_{xz} = 0.5\left(ge_0^1A_{10} + g_0^2B_{10} + g_0^3A_{20} + g_0^4B_{20} + g_0^5A_{30} + g_0^6B_{30} + g_0^7A_{40} + g_0^8B_{40}\right)$$
$$+\sum_{n=1}^{\infty}\left(g_n^1A_{1n} + g_n^2B_{1n} + g_n^3A_{2n} + g_n^4B_{2n} + g_n^5A_{3n} + g_n^6B_{3n} + g_n^7A_{4n} + g_n^8B_{4n} + g_n^9A_{5n} + g_n^{10}B_{5n}\right)$$
$$E_x = 0.5\left(h_0^1A_{10} + h_0^2B_{10} + h_0^3A_{20} + h_0^4B_{20} + h_0^5A_{30} + h_0^6B_{30} + h_0^7A_{40} + h_0^8B_{40}\right)$$
$$+\sum_{n=1}^{\infty}\left(h_n^1A_{1n} + h_n^2B_{1n} + h_n^3A_{2n} + h_n^4B_{2n} + h_n^5A_{3n} + h_n^6B_{3n} + h_n^7A_{4n} + h_n^8B_{4n} + h_n^9A_{5n} + h_n^{10}B_{5n}\right)$$
$$H_x = 0.5\left(e_0^1A_{10} + e_0^2B_{10} + e_0^3A_{20} + e_0^4B_{20} + e_0^5A_{30} + e_0^6B_{30} + e_0^7A_{40} + e_0^8B_{40}\right)$$
$$+\sum_{n=1}^{\infty}\left(e_n^1A_{1n} + e_n^2B_{1n} + e_n^3A_{2n} + e_n^4B_{2n} + e_n^5A_{3n} + e_n^6B_{3n} + e_n^7A_{4n} + e_n^8B_{4n} + e_n^9A_{5n} + e_n^{10}B_{5n}\right) \qquad (31b)$$

For the anti symmetric mode as

$$\overline{S}^1_{xx} = 0.5\left(\bar{e}_0^9\overline{A}_{10} + \bar{e}_0^{10}\overline{B}_{10}\right)$$
$$+\sum_{n=1}^{\infty}\left(\bar{e}_n^1\overline{A}_{1n} + \bar{e}_n^2\overline{B}_{1n} + \bar{e}_n^3\overline{A}_{2n} + \bar{e}_n^4\overline{B}_{2n} + \bar{e}_n^5\overline{A}_{3n} + \bar{e}_n^6\overline{B}_{3n} + \bar{e}_n^7\overline{A}_{4n} + \bar{e}_n^8\overline{B}_{4n} + \bar{e}_n^9\overline{A}_{5n} + \bar{e}_n^{10}\overline{B}_{5n}\right)$$
$$\overline{S}^1_{xy} = 0.5\left(\bar{f}_0^9\overline{A}_{10} + \bar{f}_0^{10}\overline{B}_{10}\right)$$
$$+\sum_{n=1}^{\infty}\left(\bar{f}_n^1\overline{A}_{1n} + \bar{f}_n^2\overline{B}_{1n} + \bar{f}_n^3\overline{A}_{2n} + \bar{f}_n^4\overline{B}_{2n} + \bar{f}_n^5\overline{A}_{3n} + \bar{f}_n^6\overline{B}_{3n} + \bar{f}_n^7\overline{A}_{4n} + \bar{f}_n^8\overline{B}_{4n} + \bar{f}_n^9\overline{A}_{5n} + \bar{f}_n^{10}\overline{B}_{5n}\right)$$
$$\overline{S}^1_{xz} = 0.5\left(\bar{g}_0^9\overline{A}_{10} + \bar{g}_0^{10}\overline{B}_{10}\right)$$
$$+\sum_{n=1}^{\infty}\left(\bar{g}_n^1\overline{A}_{1n} + \bar{g}_n^2\overline{B}_{1n} + \bar{g}_n^3\overline{A}_{2n} + \bar{g}_n^4\overline{B}_{2n} + \bar{g}_n^5\overline{A}_{3n} + \bar{g}_n^6\overline{B}_{3n} + \bar{g}_n^7\overline{A}_{4n} + \bar{g}_n^8\overline{B}_{4n} + \bar{g}_n^9\overline{A}_{5n} + \bar{g}_n^{10}\overline{B}_{5n}\right)$$
$$\overline{E}^1_x = 0.5\left(\bar{h}_0^9\overline{A}_{10} + \bar{h}_0^{10}\overline{B}_{10}\right)$$
$$+\sum_{n=1}^{\infty}\left(\bar{h}_n^1\overline{A}_{1n} + \bar{h}_n^2\overline{B}_{1n} + \bar{h}_n^3\overline{A}_{2n} + \bar{h}_n^4\overline{B}_{2n} + \bar{h}_n^5\overline{A}_{3n} + \bar{h}_n^6\overline{B}_{3n} + \bar{h}_n^7\overline{A}_{4n} + \bar{h}_n^8\overline{B}_{4n} + \bar{h}_n^9\overline{A}_{5n} + \bar{h}_n^{10}\overline{B}_{5n}\right)$$
$$\overline{H}^1_x = 0.5\left(\bar{i}_0^9\overline{A}_{10} + \bar{i}_0^{10}\overline{B}_{10}\right)$$
$$+\sum_{n=1}^{\infty}\left(\bar{i}_n^1\overline{A}_{1n} + \bar{i}_n^2\overline{B}_{1n} + \bar{i}_n^3\overline{A}_{2n} + \bar{i}_n^4\overline{B}_{2n} + \bar{i}_n^5\overline{A}_{3n} + \bar{i}_n^6\overline{B}_{3n} + \bar{i}_n^7\overline{A}_{4n} + \bar{i}_n^8\overline{B}_{4n} + \bar{i}_n^9\overline{A}_{5n} + \bar{i}_n^{10}\overline{B}_{5n}\right) \qquad (32a)$$

$$\bar{S}_{xx}=0.5\left(\bar{e}_0^{9}\bar{A}_{10}+\bar{e}_0^{10}\bar{B}_{10}\right)$$
$$+\sum_{n=1}^{\infty}\left(\bar{e}_n^{1}\bar{A}_{1n}+\bar{e}_n^{2}\bar{B}_{1n}+\bar{e}_n^{3}\bar{A}_{2n}+\bar{e}_n^{4}\bar{B}_{2n}+\bar{e}_n^{5}\bar{A}_{3n}+\bar{e}_n^{6}\bar{B}_{3n}+\bar{e}_n^{7}\bar{A}_{4n}+\bar{e}_n^{8}\bar{B}_{4n}+\bar{e}_n^{9}\bar{A}_{5n}+\bar{e}_n^{10}\bar{B}_{5n}\right)$$
$$\bar{S}_{xy}=0.5\left(\bar{f}_0^{9}\bar{A}_{10}+\bar{f}_0^{10}\bar{B}_{10}\right)$$
$$+\sum_{n=1}^{\infty}\left(\bar{f}_n^{1}\bar{A}_{1n}+\bar{f}_n^{2}\bar{B}_{1n}+\bar{f}_n^{3}\bar{A}_{2n}+\bar{f}_n^{4}\bar{B}_{2n}+\bar{f}_n^{5}\bar{A}_{3n}+\bar{f}_n^{6}\bar{B}_{3n}+\bar{f}_n^{7}\bar{A}_{4n}+\bar{f}_n^{8}\bar{B}_{4n}+\bar{f}_n^{9}\bar{A}_{5n}+\bar{f}_n^{10}\bar{B}_{5n}\right)$$
$$\bar{S}_{xz}=0.5\left(\bar{g}_0^{9}\bar{A}_{10}+\bar{g}_0^{10}\bar{B}_{10}\right)$$
$$+\sum_{n=1}^{\infty}\left(\bar{g}_n^{1}\bar{A}_{1n}+\bar{g}_n^{2}\bar{B}_{1n}+\bar{g}_n^{3}\bar{A}_{2n}+\bar{g}_n^{4}\bar{B}_{2n}+\bar{g}_n^{5}\bar{A}_{3n}+\bar{g}_n^{6}\bar{B}_{3n}+\bar{g}_n^{7}\bar{A}_{4n}+\bar{g}_n^{8}\bar{B}_{4n}+\bar{g}_n^{9}\bar{A}_{5n}+\bar{g}_n^{10}\bar{B}_{5n}\right)$$
$$\bar{E}_{x}=0.5\left(\bar{h}_0^{9}\bar{A}_{10}+\bar{h}_0^{10}\bar{B}_{10}\right)$$
$$+\sum_{n=1}^{\infty}\left(\bar{h}_n^{1}\bar{A}_{1n}+\bar{h}_n^{2}\bar{B}_{1n}+\bar{h}_n^{3}\bar{A}_{2n}+\bar{h}_n^{4}\bar{B}_{2n}+\bar{h}_n^{5}\bar{A}_{3n}+\bar{h}_n^{6}\bar{B}_{3n}+\bar{h}_n^{7}\bar{A}_{4n}+\bar{h}_n^{8}\bar{B}_{4n}+\bar{h}_n^{9}\bar{A}_{5n}+\bar{h}_n^{10}\bar{B}_{5n}\right)$$
$$\bar{H}_{x}=0.5\left(\bar{i}_0^{9}\bar{A}_{10}+\bar{i}_0^{10}\bar{B}_{10}\right)$$
$$+\sum_{n=1}^{\infty}\left(\bar{i}_n^{1}\bar{A}_{1n}+\bar{i}_n^{2}\bar{B}_{1n}+\bar{i}_n^{3}\bar{A}_{2n}+\bar{i}_n^{4}\bar{B}_{2n}+\bar{i}_n^{5}\bar{A}_{3n}+\bar{i}_n^{6}\bar{B}_{3n}+\bar{i}_n^{7}\bar{A}_{4n}+\bar{i}_n^{8}\bar{B}_{4n}+\bar{i}_n^{9}\bar{A}_{5n}+\bar{i}_n^{10}\bar{B}_{5n}\right)$$

(32b)

The coefficients e_n^i : $\bar{i}_n^i$ are given in the Appendix A.

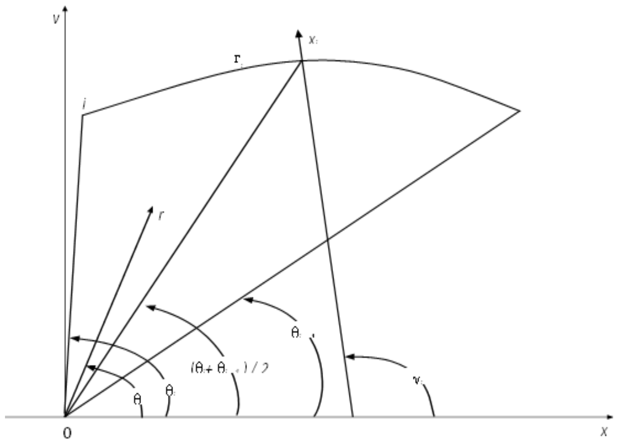

Fig. 1. Geometry of a straight line segment

The boundary conditions along the whole range of boundary cannot be satisfied directly. To satisfy the boundary conditions, the Fourier expansion is performed on the equations of the boundary conditions along the boundary line. For the present case, one straight line is considered to be one segment, while curved line must be divided into many segments according to the convergence of the solution. The Fourier coefficients are therefore obtained by the addition of these coefficients is therefore obtained by the addition of these for the separately considered boundaries. When the plate is symmetric about an axis, the analysis can be separated into symmetric and anti symmetric cases. Hence, when the co-ordinate θ is taken from the axis of symmetry, the boundary conditions along the inner boundary are expanded into the following Fourier series.

$$\sum_{m=0}^{\infty}\varepsilon_m[\tilde{E}_{m0}^{1}A_{10}+\tilde{E}_{m0}^{2}B_{10}+\tilde{E}_{m0}^{3}A_{20}+\tilde{E}_{m0}^{4}B_{20}+\tilde{E}_{m0}^{5}A_{30}+\tilde{E}_{m0}^{6}B_{30}+\tilde{E}_{m0}^{7}A_{40}+\tilde{E}_{m0}^{8}B_{40}$$
$$+\sum_{n=1}^{\infty}\begin{pmatrix}\tilde{E}_{mn}^{1}A_{1n}+\tilde{E}_{mn}^{2}B_{1n}+\tilde{E}_{mn}^{3}A_{2n}+\tilde{E}_{mn}^{4}B_{2n}+\tilde{E}_{mn}^{5}A_{3n}+\tilde{E}_{mn}^{6}B_{3n}+\tilde{E}_{mn}^{7}A_{4n}+\tilde{E}_{mn}^{8}B_{4n}\\+\tilde{E}_{mn}^{9}A_{5n}+\tilde{E}_{mn}^{10}B_{5n}\end{pmatrix}]\cos m\theta=0$$
$$\sum_{m=0}^{\infty}\varepsilon_m[\tilde{F}_{m0}^{1}A_{10}+\tilde{F}_{m0}^{2}B_{10}+\tilde{F}_{m0}^{3}A_{20}+\tilde{F}_{m0}^{4}B_{20}+\tilde{F}_{m0}^{5}A_{30}+\tilde{F}_{m0}^{6}B_{30}+\tilde{F}_{m0}^{7}A_{40}+\tilde{F}_{m0}^{8}B_{40}$$
$$+\sum_{n=1}^{\infty}\begin{pmatrix}\tilde{F}_{mn}^{1}A_{1n}+\tilde{F}_{mn}^{2}B_{1n}+\tilde{F}_{mn}^{3}A_{2n}+\tilde{F}_{mn}^{4}B_{2n}+\tilde{F}_{mn}^{5}A_{3n}+\tilde{F}_{mn}^{6}B_{3n}+\tilde{F}_{mn}^{7}A_{4n}+\tilde{F}_{mn}^{8}B_{4n}\\+\tilde{F}_{mn}^{9}A_{5n}+\tilde{F}_{mn}^{10}B_{5n}\end{pmatrix}]\sin m\theta=0$$
$$\sum_{m=0}^{\infty}\varepsilon_m[\tilde{G}_{m0}^{1}A_{10}+\tilde{G}_{m0}^{2}B_{10}+\tilde{G}_{m0}^{3}A_{20}+\tilde{G}_{m0}^{4}B_{20}+\tilde{G}_{m0}^{5}A_{30}+\tilde{G}_{m0}^{6}B_{30}+\tilde{G}_{m0}^{7}A_{40}+\tilde{G}_{m0}^{8}B_{40}$$
$$+\sum_{n=1}^{\infty}\begin{pmatrix}\tilde{G}_{mn}^{1}A_{1n}+\tilde{G}_{mn}^{2}B_{1n}+\tilde{G}_{mn}^{3}A_{2n}+\tilde{G}_{mn}^{4}B_{2n}+\tilde{G}_{mn}^{5}A_{3n}+\tilde{G}_{mn}^{6}B_{3n}+\tilde{G}_{mn}^{7}A_{4n}+\tilde{G}_{mn}^{8}B_{4n}\\+\tilde{G}_{mn}^{9}A_{5n}+\tilde{G}_{mn}^{10}B_{5n}\end{pmatrix}]\cos m\theta=0$$
$$\sum_{m=0}^{\infty}\varepsilon_m[\tilde{H}_{m0}^{1}A_{10}+\tilde{H}_{m0}^{2}B_{10}+\tilde{H}_{m0}^{3}A_{20}+\tilde{H}_{m0}^{4}B_{20}+\tilde{H}_{m0}^{5}A_{30}+\tilde{H}_{m0}^{6}B_{30}+\tilde{H}_{m0}^{7}A_{40}+\tilde{H}_{m0}^{8}B_{40}$$
$$+\sum_{n=1}^{\infty}\begin{pmatrix}\tilde{H}_{mn}^{1}A_{1n}+\tilde{H}_{mn}^{2}B_{1n}+\tilde{H}_{mn}^{3}A_{2n}+\tilde{H}_{mn}^{4}B_{2n}+\tilde{H}_{mn}^{5}A_{3n}+\tilde{H}_{mn}^{6}B_{3n}+\tilde{H}_{mn}^{7}A_{4n}+\tilde{H}_{mn}^{8}B_{4n}\\+\tilde{H}_{mn}^{9}A_{5n}+\tilde{H}_{mn}^{10}B_{5n}\end{pmatrix}]\cos m\theta=0$$
$$\sum_{m=0}^{\infty}\varepsilon_m[\tilde{I}_{m0}^{1}A_{10}+\tilde{I}_{m0}^{2}B_{10}+\tilde{I}_{m0}^{3}A_{20}+\tilde{I}_{m0}^{4}B_{20}+\tilde{I}_{m0}^{5}A_{30}+\tilde{I}_{m0}^{6}B_{30}+\tilde{I}_{m0}^{7}A_{40}+\tilde{I}_{m0}^{8}B_{40}$$
$$+\sum_{n=1}^{\infty}\begin{pmatrix}\tilde{I}_{mn}^{1}A_{1n}+\tilde{I}_{mn}^{2}B_{1n}+\tilde{I}_{mn}^{3}A_{2n}+\tilde{I}_{mn}^{4}B_{2n}+\tilde{I}_{mn}^{5}A_{3n}+\tilde{I}_{mn}^{6}B_{3n}+\tilde{I}_{mn}^{7}A_{4n}+\tilde{I}_{mn}^{8}B_{4n}\\+\tilde{I}_{mn}^{9}A_{5n}+\tilde{I}_{mn}^{10}B_{5n}\end{pmatrix}]\cos m\theta=0$$

(33)

where

$$\tilde{E}_{mn}^{j}=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}e_n^j\left(\tilde{R}_i,\theta\right)\cos m\theta d\theta,$$
$$\tilde{F}_{mn}^{j}=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}f_n^j\left(\tilde{R}_i,\theta\right)\sin m\theta d\theta$$
$$\tilde{G}_{mn}^{j}=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}g_n^j\left(\tilde{R}_i,\theta\right)\cos m\theta d\theta,$$
$$\tilde{H}_{mn}^{j}=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}h_n^j\left(\tilde{R}_i,\theta\right)\cos m\theta d\theta,$$
$$\tilde{I}_{mn}^{j}=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}i_n^j\left(\tilde{R}_i,\theta\right)\cos m\theta d\theta$$

(34)

and for the outer surface

$$\sum_{m=0}^{\infty}\varepsilon_m[E_{m0}^{1}A_{10}+E_{m0}^{2}B_{10}+E_{m0}^{3}A_{20}+E_{m0}^{4}B_{20}+E_{m0}^{5}A_{30}+E_{m0}^{6}B_{30}+E_{m0}^{7}A_{40}+E_{m0}^{8}B_{40}$$
$$+\sum_{n=1}^{\infty}\begin{pmatrix}E_{mn}^{1}A_{1n}+E_{mn}^{2}B_{1n}+E_{mn}^{3}A_{2n}+E_{mn}^{4}B_{2n}+E_{mn}^{5}A_{3n}+E_{mn}^{6}B_{3n}+E_{mn}^{7}A_{4n}+E_{mn}^{8}B_{4n}\\+E_{mn}^{9}A_{5n}+E_{mn}^{10}B_{5n}\end{pmatrix}]\cos m\theta=0$$

$$\sum_{m=0}^{\infty}\varepsilon_m[F_{m0}^{1}A_{10}+F_{m0}^{2}B_{10}+F_{m0}^{3}A_{20}+F_{m0}^{4}B_{20}+F_{m0}^{5}A_{30}+F_{m0}^{6}B_{30}+F_{m0}^{7}A_{40}+F_{m0}^{8}B_{40}$$
$$+\sum_{n=1}^{\infty}\begin{pmatrix}F_{mn}^{1}A_{1n}+F_{mn}^{2}B_{1n}+F_{mn}^{3}A_{2n}+F_{mn}^{4}B_{2n}+F_{mn}^{5}A_{3n}+F_{mn}^{6}B_{3n}+F_{mn}^{7}A_{4n}+F_{mn}^{8}B_{4n}\\+F_{mn}^{9}A_{5n}+F_{mn}^{10}B_{5n}\end{pmatrix}]\sin m\theta=0$$

$$\sum_{m=0}^{\infty}\varepsilon_m[G_{m0}^{1}A_{10}+G_{m0}^{2}B_{10}+G_{m0}^{3}A_{20}+G_{m0}^{4}B_{20}+G_{m0}^{5}A_{30}+G_{m0}^{6}B_{30}+G_{m0}^{7}A_{40}+G_{m0}^{8}B_{40}$$
$$+\sum_{n=1}^{\infty}\begin{pmatrix}G_{mn}^{1}A_{1n}+G_{mn}^{2}B_{1n}+G_{mn}^{3}A_{2n}+G_{mn}^{4}B_{2n}+G_{mn}^{5}A_{3n}+G_{mn}^{6}B_{3n}+G_{mn}^{7}A_{4n}+G_{mn}^{8}B_{4n}\\+G_{mn}^{9}A_{5n}+G_{mn}^{10}B_{5n}\end{pmatrix}]\cos m\theta=0$$

$$\sum_{m=0}^{\infty}\varepsilon_m[H_{m0}^1A_{10}+H_{m0}^2B_{10}+H_{m0}^3A_{20}+H_{m0}^4B_{20}+H_{m0}^5A_{30}+H_{m0}^6B_{30}+H_{m0}^7A_{40}+H_{m0}^8B_{40}$$
$$+\sum_{n=1}^{\infty}\begin{pmatrix}H_{mn}^1A_{1n}+H_{mn}^2B_{1n}+H_{mn}^3A_{2n}+H_{mn}^4B_{2n}+H_{mn}^5A_{3n}+H_{mn}^6B_{3n}+H_{mn}^7A_{4n}+H_{mn}^8B_{4n}\\+H_{mn}^9A_{5n}+H_{mn}^{10}B_{5n}\end{pmatrix}]\cos m\theta=0$$

$$\sum_{m=0}^{\infty}\varepsilon_m[I_{m0}^1A_{10}+I_{m0}^2B_{10}+I_{m0}^3A_{20}+I_{m0}^4B_{20}+I_{m0}^5A_{30}+I_{m0}^6B_{30}+I_{m0}^7A_{40}+I_{m0}^8B_{40}$$
$$+\sum_{n=1}^{\infty}\begin{pmatrix}I_{mn}^1A_{1n}+I_{mn}^2B_{1n}+I_{mn}^3A_{2n}+I_{mn}^4B_{2n}+I_{mn}^5A_{3n}+I_{mn}^6B_{3n}+I_{mn}^7A_{4n}+I_{mn}^8B_{4n}\\+I_{mn}^9A_{5n}+I_{mn}^{10}B_{5n}\end{pmatrix}]\cos m\theta=0 \quad (35)$$

where

$$E_{mn}^j=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}e_n^j\left(R_i,\theta\right)\cos m\theta d\theta,$$
$$F_{mn}^j=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}f_n^j\left(R_i,\theta\right)\sin m\theta d\theta$$
$$G_{mn}^j=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}g_n^j\left(R_i,\theta\right)\cos m\theta d\theta,$$
$$H_{mn}^j=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}h_n^j\left(R_i,\theta\right)\cos m\theta d\theta,$$
$$I_{mn}^j=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}i_n^j\left(R_i,\theta\right)\cos m\theta d\theta \quad (36)$$

Similarly, for the anti symmetric mode, the boundary conditions for the inner surface are

$$\sum_{m=1}^{\infty}[\overline{\tilde{E}}_{m0}^9\bar{A}_{50}+\overline{\tilde{E}}_{m0}^{10}\bar{B}_{50}$$
$$+\sum_{n=1}^{\infty}\begin{pmatrix}\overline{\tilde{E}}_{mn}^1\bar{A}_{1n}+\overline{\tilde{E}}_{mn}^2\bar{B}_{1n}+\overline{\tilde{E}}_{mn}^3\bar{A}_{2n}+\overline{\tilde{E}}_{mn}^4\bar{B}_{2n}+\overline{\tilde{E}}_{mn}^5\bar{A}_{3n}+\overline{\tilde{E}}_{mn}^6\bar{B}_{3n}\\+\overline{\tilde{E}}_{mn}^7\bar{A}_{4n}+\overline{\tilde{E}}_{mn}^8\bar{B}_{4n}+\overline{\tilde{E}}_{mn}^9\bar{A}_{5n}+\overline{\tilde{E}}_{mn}^{10}\bar{B}_{5n}\end{pmatrix}]\sin m\theta=0$$

$$\sum_{m=1}^{\infty}[\overline{\tilde{F}}_{m0}^9\bar{A}_{50}+\overline{\tilde{F}}_{m0}^{10}\bar{B}_{50}$$
$$+\sum_{n=1}^{\infty}\begin{pmatrix}\overline{\tilde{F}}_{mn}^1\bar{A}_{1n}+\overline{\tilde{F}}_{mn}^2\bar{B}_{1n}+\overline{\tilde{F}}_{mn}^3\bar{A}_{2n}+\overline{\tilde{F}}_{mn}^4\bar{B}_{2n}+\overline{\tilde{F}}_{mn}^5\bar{A}_{3n}+\overline{\tilde{F}}_{mn}^6\bar{B}_{3n}\\+\overline{\tilde{F}}_{mn}^7\bar{A}_{4n}+\overline{\tilde{F}}_{mn}^8\bar{B}_{4n}+\overline{\tilde{F}}_{mn}^9\bar{A}_{5n}+\overline{\tilde{F}}_{mn}^{10}\bar{B}_{5n}\end{pmatrix}]\cos m\theta=0$$

$$\sum_{m=1}^{\infty}[\overline{\tilde{G}}_{m0}^9\bar{A}_{50}+\overline{\tilde{G}}_{m0}^{10}\bar{B}_{50}$$
$$+\sum_{n=1}^{\infty}\begin{pmatrix}\overline{\tilde{G}}_{mn}^1\bar{A}_{1n}+\overline{\tilde{G}}_{mn}^2\bar{B}_{1n}+\overline{\tilde{G}}_{mn}^3\bar{A}_{2n}+\overline{\tilde{G}}_{mn}^4\bar{B}_{2n}+\overline{\tilde{G}}_{mn}^5\bar{A}_{3n}+\overline{\tilde{G}}_{mn}^6\bar{B}_{3n}\\+\overline{\tilde{G}}_{mn}^7\bar{A}_{4n}+\overline{\tilde{G}}_{mn}^8\bar{B}_{4n}+\overline{\tilde{G}}_{mn}^9\bar{A}_{5n}+\overline{\tilde{G}}_{mn}^{10}\bar{B}_{5n}\end{pmatrix}]\sin m\theta=0$$

$$\sum_{m=1}^{\infty}[\overline{\tilde{H}}_{m0}^9\bar{A}_{50}+\overline{\tilde{H}}_{m0}^{10}\bar{B}_{50}$$
$$+\sum_{n=1}^{\infty}\begin{pmatrix}\overline{\tilde{H}}_{mn}^1\bar{A}_{1n}+\overline{\tilde{H}}_{mn}^2\bar{B}_{1n}+\overline{\tilde{H}}_{mn}^3\bar{A}_{2n}+\overline{\tilde{H}}_{mn}^4\bar{B}_{2n}+\overline{\tilde{H}}_{mn}^5\bar{A}_{3n}+\overline{\tilde{H}}_{mn}^6\bar{B}_{3n}\\+\overline{\tilde{H}}_{mn}^7\bar{A}_{4n}+\overline{\tilde{H}}_{mn}^8\bar{B}_{4n}+\overline{\tilde{H}}_{mn}^9\bar{A}_{5n}+\overline{\tilde{H}}_{mn}^{10}\bar{B}_{5n}\end{pmatrix}]\sin m\theta=0$$

$$\sum_{m=1}^{\infty}[\overline{\tilde{I}}_{m0}^9\bar{A}_{50}+\overline{\tilde{I}}_{m0}^{10}\bar{B}_{50}$$
$$+\sum_{n=1}^{\infty}\begin{pmatrix}\overline{\tilde{I}}_{mn}^1\bar{A}_{1n}+\overline{\tilde{I}}_{mn}^2\bar{B}_{1n}+\overline{\tilde{I}}_{mn}^3\bar{A}_{2n}+\overline{\tilde{I}}_{mn}^4\bar{B}_{2n}+\overline{\tilde{I}}_{mn}^5\bar{A}_{3n}+\overline{\tilde{I}}_{mn}^6\bar{B}_{3n}\\+\overline{\tilde{I}}_{mn}^7\bar{A}_{4n}+\overline{\tilde{I}}_{mn}^8\bar{B}_{4n}+\overline{\tilde{I}}_{mn}^9\bar{A}_{5n}+\overline{\tilde{I}}_{mn}^{10}\bar{B}_{5n}\end{pmatrix}]\sin m\theta=0 \quad (37)$$

where

$$\overline{\tilde{E}}_{mn}^j=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}e_n^j\left(\tilde{R}_i,\theta\right)\sin m\theta d\theta,$$
$$\overline{\tilde{F}}_{mn}^j=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}f_n^j\left(\tilde{R}_i,\theta\right)\cos m\theta d\theta$$
$$\overline{\tilde{G}}_{mn}^j=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}g_n^j\left(\tilde{R}_i,\theta\right)\sin m\theta d\theta,$$
$$\overline{\tilde{H}}_{mn}^j=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}h_n^j\left(\tilde{R}_i,\theta\right)\sin m\theta d\theta,$$
$$\overline{\tilde{I}}_{mn}^j=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}i_n^j\left(\tilde{R}_i,\theta\right)\sin m\theta d\theta \quad (38)$$

and for the outer surface are

$$\sum_{m=1}^{\infty}[\bar{E}_{m0}^5\bar{A}_{30}+\bar{E}_{m0}^6\bar{B}_{30}+\sum_{n=1}^{\infty}\begin{pmatrix}\bar{E}_{mn}^1\bar{A}_{1n}+\bar{E}_{mn}^2\bar{B}_{1n}+\bar{E}_{mn}^3\bar{A}_{2n}+\bar{E}_{mn}^4\bar{B}_{2n}+\bar{E}_{mn}^5\bar{A}_{3n}\\+\bar{E}_{mn}^6\bar{B}_{3n}+\bar{E}_{mn}^7\bar{A}_{4n}+\bar{E}_{mn}^8\bar{B}_{4n}+\bar{E}_{mn}^9\bar{A}_{5n}+\bar{E}_{mn}^{10}\bar{B}_{5n}\end{pmatrix}]\sin m\theta=0$$

$$\sum_{m=1}^{\infty}[\bar{F}_{m0}^5\bar{A}_{30}+\bar{F}_{m0}^6\bar{B}_{30}+\sum_{n=1}^{\infty}\begin{pmatrix}\bar{F}_{mn}^1\bar{A}_{1n}+\bar{F}_{mn}^2\bar{B}_{1n}+\bar{F}_{mn}^3\bar{A}_{2n}+\bar{F}_{mn}^4\bar{B}_{2n}+\bar{F}_{mn}^5\bar{A}_{3n}\\+\bar{F}_{mn}^6\bar{B}_{3n}+\bar{F}_{mn}^7\bar{A}_{4n}+\bar{F}_{mn}^8\bar{B}_{4n}+\bar{F}_{mn}^9\bar{A}_{5n}+\bar{F}_{mn}^{10}\bar{B}_{5n}\end{pmatrix}]\cos m\theta=0$$

$$\sum_{m=1}^{\infty}[\bar{G}_{m0}^5\bar{A}_{30}+\bar{G}_{m0}^6\bar{B}_{30}+\sum_{n=1}^{\infty}\begin{pmatrix}\bar{G}_{mn}^1\bar{A}_{1n}+\bar{G}_{mn}^2\bar{B}_{1n}+\bar{G}_{mn}^3\bar{A}_{2n}+\bar{G}_{mn}^4\bar{B}_{2n}+\bar{G}_{mn}^5\bar{A}_{3n}\\+\bar{G}_{mn}^6\bar{B}_{3n}+\bar{G}_{mn}^7\bar{A}_{4n}+\bar{G}_{mn}^8\bar{B}_{4n}+\bar{G}_{mn}^9\bar{A}_{5n}+\bar{G}_{mn}^{10}\bar{B}_{5n}\end{pmatrix}]\sin m\theta=0$$

$$\sum_{m=1}^{\infty}[\bar{H}_{m0}^5\bar{A}_{30}+\bar{H}_{m0}^6\bar{B}_{30}+\sum_{n=1}^{\infty}\begin{pmatrix}\bar{H}_{mn}^1\bar{A}_{1n}+\bar{H}_{mn}^2\bar{B}_{1n}+\bar{H}_{mn}^3\bar{A}_{2n}+\bar{H}_{mn}^4\bar{B}_{2n}+\bar{H}_{mn}^5\bar{A}_{3n}\\+\bar{H}_{mn}^6\bar{B}_{3n}+\bar{H}_{mn}^7\bar{A}_{4n}+\bar{H}_{mn}^8\bar{B}_{4n}+\bar{H}_{mn}^9\bar{A}_{5n}+\bar{H}_{mn}^{10}\bar{B}_{5n}\end{pmatrix}]\sin m\theta=0 \quad (39)$$

where

$$\bar{E}_{mn}^j=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}e_n^j\left(R_i,\theta\right)\sin m\theta d\theta,$$
$$\bar{F}_{mn}^j=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}f_n^j\left(R_i,\theta\right)\cos m\theta d\theta$$
$$\bar{G}_{mn}^j=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}g_n^j\left(R_i,\theta\right)\sin m\theta d\theta,$$
$$\bar{H}_{mn}^j=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}h_n^j\left(R_i,\theta\right)\sin m\theta d\theta,$$
$$\bar{I}_{mn}^j=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}i_n^j\left(R_i,\theta\right)\sin m\theta d\theta \quad (40)$$

where $j=1,2,3,4,5,6,7,8,9$ and 10, $\varepsilon_m=1/2$ for $m=0$ and $\varepsilon_m=1$ for $m\geq 0$, I is the number of segments, $\tilde{R}_i$ is the coordinate r at the inner boundary, and R_i is the coordinate r at the outer boundary.

The frequency equations are obtained from the inner and outer boundary conditions of the Eqs. (33) and (35), for the symmetric mode, and for the anti symmetric mode, the frequency equations are obtained from the Eqs. (37) and (39) by truncating the series to N+1 terms, and equating the determinant of the coefficients of the amplitudes $A_{in}, \bar{A}_{in}, B_{in}$ and $\bar{B}_{in}$ $(i = 1,2,3,4,5)$ to zero. Thus the frequency equation for the symmetric mode is obtained in the form of matrix as follows;

$$\begin{bmatrix} \hat{E}^{1}_{00} & \hat{E}^{2}_{00} & \cdots & \cdots & \hat{E}^{8}_{00} & \hat{E}^{1}_{01} & \cdots & \hat{E}^{1}_{0N} & \hat{E}^{2}_{01} & \cdots & \hat{E}^{2}_{0N} & \cdots & \cdots & \hat{E}^{10}_{01} & \cdots & \hat{E}^{10}_{0N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \cdots & \cdots & \vdots & & \vdots \\ \hat{E}^{1}_{N0} & \hat{E}^{2}_{N0} & \cdots & \cdots & \hat{E}^{8}_{N0} & \hat{E}^{1}_{N1} & \cdots & \hat{E}^{1}_{NN} & \hat{E}^{2}_{N1} & \cdots & \hat{E}^{2}_{NN} & \cdots & \cdots & \hat{E}^{10}_{N1} & \cdots & \hat{E}^{10}_{NN} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \vdots \\ \hat{F}^{1}_{00} & \hat{F}^{2}_{00} & \cdots & \cdots & \hat{F}^{8}_{00} & \hat{F}^{1}_{01} & \cdots & \hat{F}^{1}_{0N} & \hat{F}^{2}_{01} & \cdots & \hat{F}^{2}_{0N} & \cdots & \cdots & \hat{F}^{10}_{01} & \cdots & \hat{F}^{10}_{0N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & & \vdots \\ \hat{F}^{1}_{N0} & \hat{F}^{2}_{N0} & \cdots & \cdots & \hat{F}^{8}_{N0} & \hat{F}^{1}_{N1} & \cdots & \hat{F}^{1}_{NN} & \hat{F}^{2}_{N1} & \cdots & \hat{F}^{2}_{NN} & \cdots & \cdots & \hat{F}^{10}_{N1} & \cdots & \hat{F}^{10}_{NN} \\ E^{1}_{00} & E^{2}_{00} & \cdots & \cdots & E^{8}_{00} & E^{1}_{01} & \cdots & E^{1}_{0N} & E^{2}_{01} & \cdots & E^{2}_{0N} & \cdots & \cdots & E^{2}_{01} & \cdots & E^{2}_{0N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & & & & & \vdots \\ E^{1}_{N0} & E^{2}_{N0} & \cdots & \cdots & E^{8}_{N0} & E^{1}_{N1} & \cdots & E^{1}_{NN} & E^{2}_{N1} & \cdots & E^{2}_{NN} & \cdots & \cdots & E^{2}_{N1} & \cdots & E^{2}_{NN} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & & & \vdots & & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ I^{1}_{00} & I^{2}_{00} & \cdots & \cdots & I^{8}_{00} & I^{1}_{00} & \cdots & I^{1}_{0N} & I^{2}_{00} & \cdots & I^{2}_{0N} & \cdots & \cdots & I^{10}_{00} & \cdots & I^{10}_{0N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & & & \vdots & & \vdots \\ I^{1}_{N0} & I^{2}_{N0} & \cdots & \cdots & I^{8}_{N0} & I^{1}_{N1} & \cdots & I^{1}_{NN} & I^{2}_{N1} & \cdots & I^{2}_{NN} & \cdots & \cdots & I^{10}_{N1} & \cdots & I^{10}_{NN} \end{bmatrix} \begin{bmatrix} A_{10} \\ B_{10} \\ \vdots \\ A_{80} \\ B_{80} \\ A_{11} \\ \vdots \\ A_{1N} \\ B_{11} \\ \vdots \\ B_{1N} \\ \vdots \\ \vdots \\ B_{101} \\ \vdots \\ \vdots \\ B_{10N} \end{bmatrix} = 0 \tag{41}$$

The frequency equation for the anti symmetric mode is obtained in the form of matrix as follows;

$$\begin{bmatrix} \bar{\hat{E}}^{9}_{00} & \bar{\hat{E}}^{10}_{00} & \cdots & \cdots & \cdots & \bar{\hat{E}}^{1}_{01} & \cdots & \bar{\hat{E}}^{1}_{0N} & \bar{\hat{E}}^{2}_{01} & \cdots & \bar{\hat{E}}^{2}_{0N} & \cdots & \cdots & \bar{\hat{E}}^{10}_{01} & \cdots & \bar{\hat{E}}^{10}_{0N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \cdots & \cdots & \vdots & & \vdots \\ \bar{\hat{E}}^{9}_{N0} & \bar{\hat{E}}^{10}_{N0} & \cdots & \cdots & \cdots & \bar{\hat{E}}^{1}_{N1} & \cdots & \bar{\hat{E}}^{1}_{NN} & \bar{\hat{E}}^{2}_{N1} & \cdots & \bar{\hat{E}}^{2}_{NN} & \cdots & \cdots & \bar{\hat{E}}^{10}_{N1} & \cdots & \bar{\hat{E}}^{10}_{NN} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \vdots \\ \bar{\hat{F}}^{1}_{00} & \bar{\hat{F}}^{2}_{00} & \cdots & \cdots & \cdots & \bar{\hat{F}}^{1}_{01} & \cdots & \bar{\hat{F}}^{1}_{0N} & \bar{\hat{F}}^{2}_{01} & \cdots & \bar{\hat{F}}^{2}_{0N} & \cdots & \cdots & \bar{\hat{F}}^{10}_{01} & \cdots & \bar{\hat{F}}^{10}_{0N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & & \vdots \\ \bar{\hat{F}}^{1}_{N0} & \bar{\hat{F}}^{2}_{N0} & \cdots & \cdots & \vdots & \bar{\hat{F}}^{1}_{N1} & \cdots & \bar{\hat{F}}^{1}_{NN} & \bar{\hat{F}}^{2}_{N1} & \cdots & \bar{\hat{F}}^{2}_{NN} & \cdots & \cdots & \bar{\hat{F}}^{10}_{N1} & \cdots & \bar{\hat{F}}^{10}_{NN} \\ \bar{E}^{1}_{00} & \bar{E}^{2}_{00} & \cdots & \cdots & \vdots & \bar{E}^{1}_{01} & \cdots & \bar{E}^{1}_{0N} & \bar{E}^{2}_{01} & \cdots & \bar{E}^{2}_{0N} & \cdots & \cdots & \bar{E}^{2}_{01} & \cdots & \bar{E}^{2}_{0N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & & & & & \vdots \\ \bar{E}^{1}_{N0} & \bar{E}^{2}_{N0} & \cdots & \cdots & \cdots & \bar{E}^{1}_{N1} & \cdots & \bar{E}^{1}_{NN} & \bar{E}^{2}_{N1} & \cdots & \bar{E}^{2}_{NN} & \cdots & \cdots & \bar{E}^{2}_{N1} & \cdots & \bar{E}^{2}_{NN} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & & & \vdots & & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ \bar{I}^{1}_{00} & \bar{I}^{2}_{00} & \cdots & \cdots & \cdots & \bar{I}^{1}_{00} & \cdots & \bar{I}^{1}_{0N} & \bar{I}^{2}_{00} & \cdots & \bar{I}^{2}_{0N} & \cdots & \cdots & \bar{I}^{10}_{00} & \cdots & \bar{I}^{10}_{0N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & & & \vdots & & \vdots \\ \bar{I}^{1}_{N0} & \bar{I}^{2}_{N0} & \cdots & \cdots & \cdots & \bar{I}^{1}_{N1} & \cdots & \bar{I}^{1}_{NN} & \bar{I}^{2}_{N1} & \cdots & \bar{I}^{2}_{NN} & \cdots & \cdots & \bar{I}^{10}_{N1} & \cdots & \bar{I}^{10}_{NN} \end{bmatrix} \begin{bmatrix} \bar{A}_{50} \\ \bar{B}_{50} \\ \vdots \\ \vdots \\ \vdots \\ \bar{A}_{11} \\ \vdots \\ \bar{A}_{1N} \\ \bar{B}_{11} \\ \vdots \\ \bar{B}_{1N} \\ \vdots \\ \vdots \\ \vdots \\ \bar{B}_{51} \\ \vdots \\ \vdots \\ \bar{B}_{5N} \end{bmatrix} = 0 \tag{42}$$

Numerical results and Discussions

The numerical analysis of the frequency equation is carried out for electro-magneto-elastic plate of polygonal (square, triangle, pentagon and hexagon) cross-sections, and the dimensions of each plate used in the numerical calculation are shown in Figure 2. The axis of symmetry is denoted by the lines in the Figure 2. The electro-magnetic material constants based on graphical results of Aboudi (2001) used for the numerical calculations. The material constants are given in the Table 1.

In the numerical calculation, the angle θ is taken as an independent variable and the coordinate R_i at the $i-th$ segment of the boundary is expressed in terms of θ. Substituting R_i and the angle γ_i, between the reference axis and the normal to the $i-th$ boundary line, the integrations of the Fourier coefficients $e^i_n, f^i_n, g^i_n, h^i_n, i^i_n$ and $\bar{e}^i_n, \bar{f}^i_n, \bar{g}^i_n, \bar{h}^i_n, \bar{i}^i_n$ can be expressed in terms of the angle θ. Using these coefficients in to the equations (34) and (36), the frequencies are obtained for electro-magneto-elastic polygonal plate.

Polygonal cross-sectional plate

The geometry of the ring shaped polygonal (triangle, square, pentagon and hexagon) cross-sectional plates are shown in the Figure 2. The numerical analysis of the frequency equation is carried out for magneto-electro-elastic polygonal (square, triangle, pentagon and hexagon) cross-sectional plates, and the dimensions of each plate used in the numerical calculation are shown in Figure 2. The geometrical relations for the polygonal cross-sectional ring shaped plates are given by Nagaya (1981a) as follows.

$$R_i/a = \left[\cos(\theta - \gamma_i)\right]^{-1}$$

$$R_i/b = \left[\cos(\theta - \gamma_i)\right]^{-1}$$

$$\gamma_i = \gamma_i \tag{43}$$

where $a = (b+h)$ and b is the apothems as shown in the Figure 2, and h is the thickness of the plate and $0 \le z \le L$ is the length of the plate along the z-axis. Here the apothem b is taken as the reference length which is used to obtain the dimensionless expressions, and γ_i is the angle between the reference axis and the normal to the segment as shown in the Figure 1. In the present problem, there are three kinds of basic independent modes of wave propagation have been considered, namely, the longitudinal and two flexural (symmetric and anti symmetric) modes of vibrations.

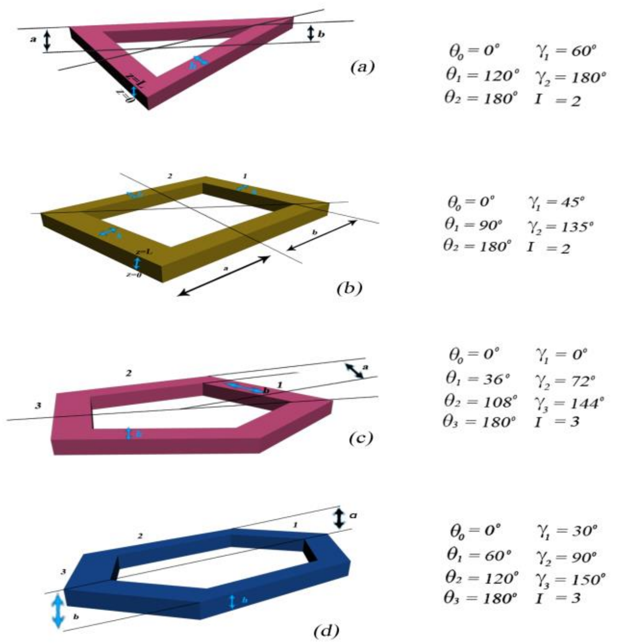

Fig.2 (a) Triangle (b) Square (c) Pentagon (d) Hexagonal cross sections

Longitudinal mode

In longitudinal mode of square and hexagonal cross-section, the cross-section vibrates along the axis of the plate, so that the vibration displacements in the cross-sections are symmetrical about both the major and the minor axes. Hence the frequency equations are obtained by choosing both the terms of m and n as $0,2,4,6....$ in the Eq. (41) for the numerical calculations. In the case of triangle and pentagonal cross-sectional plate, the vibration and displacements are symmetrical about the major axis alone, hence the frequency equations are obtained from the

Eq. (41) by choosing m and n as $0,1,2,3,....$. Since the boundary of the cross-sections namely, triangle, square, pentagon and hexagon are irregular in shape, it is difficult to satisfy the boundary conditions along the curved surface, and hence Fourier expansion collocation method is applied. That is the curved surface, in the range $\theta = 0$ and $\theta = \pi$ is divided into 20 segments, such that the distance between any two segments is negligible and the integrations is performed for each segment numerically by using the Gauss five point formula .The non-dimensional frequencies are computed for $0 \le \Omega \le 1.0$, using the secant method (applicable for the complex roots, (Anita, 2002).

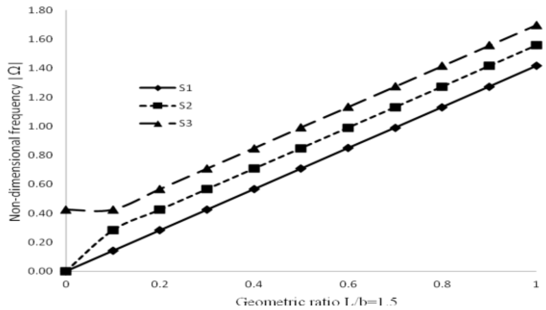

Fig. 3 Geometric ratio L/b versus non-dimensional frequency |Ω| of longitudinal modes of vibrations for the triangular cross-sectional plate

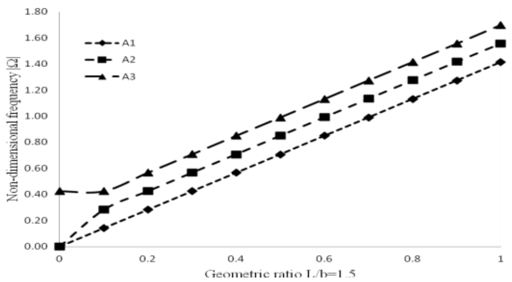

Fig. 4 Geometric ratio L/b versus non-dimensional frequency |Ω| of flexural anti symmetric modes of vibrations for the triangular cross-sectional plate

Flexural Mode

In the case of flexural mode of square and hexagonal cross-section, the vibration and displacements are anti symmetrical about the major axis and symmetrical about the minor axis. Hence the frequency equation may be obtained from Eq. (42) by choosing $n, m = 1, 3, 5, 7, ...$ In the case of triangle and pentagonal cross-sections, the vibration and displacements are anti symmetrical about the minor axis, hence the frequency equations may be obtained from Eq. (42) by choosing $n, m = 1, 2, 3....$

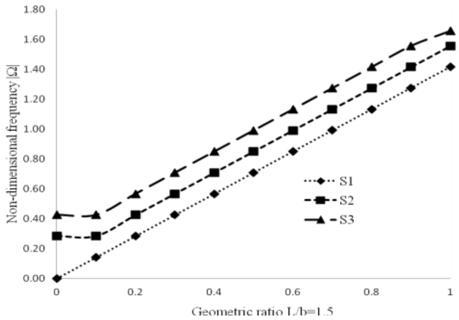

Fig. 5 Geometric ratio L/b versus non-dimensional frequency |Ω| of longitudinal modes of vibrations for the pentagonal cross-sectional plate

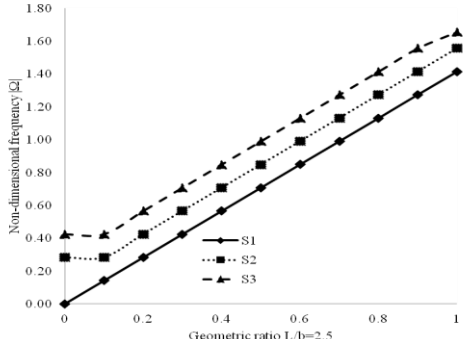

Fig. 6 Geometric ratio L/b versus non-dimensional frequency |Ω| for the longitudinal modes of square cross-sectional plate

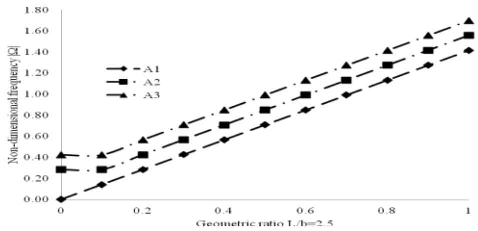

Fig. 7 Geometric ratio L/b versus non-dimensional frequency |Ω| for the flexural anti symmetric modes of square cross-sectional plate

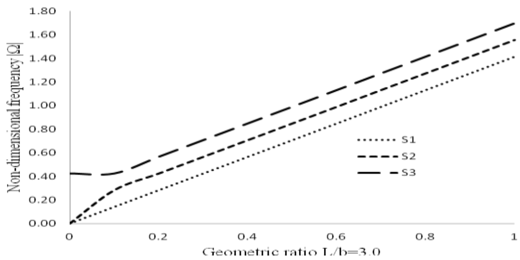

Fig. 8 Geometric ratio L/b versus non-dimensional frequency |Ω| of longitudinal modes of vibrations for the hexagonal cross-sectional plate

The geometric relation for the polygonal cross-section is given in Eq. (43), which is used for the numerical calculation. The non-dimensional frequencies of longitudinal and flexural anti symmetric modes are plotted in the form of dispersion curves as shown in the Figures 3 – 7. The notations namely, S_1, S_2 , S_3 and A_1, A_2, A_3 used in the graphs respectively represents the symmetric and anti symmetric modes vibration, and the subscripts 1, 2, 3 etc.. represents the first, second, third modes vibrations.

A graph is drawn between the geometric ratio L/a =1.5 versus non-dimensional frequency $|\Omega|$ for longitudinal modes of triangular cross-sectional ring shaped plate is shown in Fig.3. From the Fig.3, it is observed that the non-dimensional frequency is linearly increases with respect to its mode of vibrations S1, S2 and S3.The similar behavior is observed for flexural antisymmetric modes of triangular cross-sectional plate. A dispersion curve is drawn between the Geometric ratio L/a=1.5 versus non-dimensional frequency $|\Omega|$ for longitudinal modes of pentagonal cross-sectional plate is shown in the Fig. 5. From the Fig. 5, it is observed that, the similar behavior as discussed in the Fig. 3. This the proper physical behavior of a electro-magneto-elastic plates.

The Figs. 6 and 7 respectively represents the dispersion curve drawn for Geometric ratio L/a versus non-dimensional frequency c for longitudinal and flexural anti symmetric modes of square cross-sectional plate. From the Fig. 6 and 7, it is observed that the non-dimensional frequency is increase by increasing the modes of vibrations. A graph is drawn between the geometric ratio L/a=3.0 versus dimensionless frequency for the longitudinal modes of hexagonal cross-sectional electro-magneto-elastic plates is shown in the Fig. 8. From the Fig.8, it is observed the non- dimensional frequency is increases with respect to is modes of vibrations.

Conclusions

In this paper, the wave propagation in a electro-magneto-elastic ring shaped plate of polygonal (triangle, square, pentagon and hexagon) cross section are analyzed by satisfying the boundary conditions on the irregular boundary using the Fourier expansion collocation method and the frequency equations for the longitudinal and flexural (symmetric and anti symmetric) modes of vibrations are obtained. Numerically the frequency equations are analyzed for the plate of different cross-sections such as triangular, square, pentagon and hexagon. The computed dimensionless frequencies are plotted in graphs for longitudinal and flexural (symmetric and anti symmetric) modes of vibrations. The problem can be analyzed for any other cross-section by using the proper geometric relation.

Acknowledgement

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Appendix

$$e_n^i = [-2\bar{c}_{66}\cos 2(\theta-\gamma_i)\{n(n-1)J_n(\alpha_i ax)+(\alpha_i ax)J_{n+1}(\alpha_i ax)\} + x^2[(\alpha_i a)^2(\bar{c}_{11}\cos^2(\theta-\gamma_i)+\bar{c}_{12}\sin^2(\theta-\gamma_i)) + t_L(\bar{c}_{13}a_i+\bar{e}_{31}b_i+\bar{q}_{31}c_i)J_n(\alpha_i ax)]\cos(m\pi\varsigma)\cos n\theta - 2n\bar{c}_{66}\{n(n-1)J_n(\alpha_i ax)-(\alpha_i ax)J_{n+1}(\alpha_i ax)\}\sin 2(\theta-\gamma_i)\cos(m\pi\varsigma)\sin n\theta, i=1,2,3,4 \quad \text{(A1)}$$

$$e_n^5 = [2\bar{c}_{66}\cos 2(\theta-\gamma_i)n\{(n-1)J_n(a_5ax)-(a_5ax)J_{n+1}(a_5ax)\}\cos(m\pi\varsigma)\cos n\theta + \bar{c}_{66}[2\{n(n-1)J_n(a_5ax)+(a_5ax)J_{n+1}(a_5ax)\}-(a_5ax)^2J_n(a_5ax)]\cos(m\pi\varsigma)\sin n\theta\sin 2(\theta-\gamma_i) \quad \text{(A2)}$$

$$e_n^i = [-2\bar{c}_{66}\cos 2(\theta-\gamma_i)\{n(n-1)Y_n(\alpha_i ax)+(\alpha_i ax)Y_{n+1}(\alpha_i ax)\} + x^2[(\alpha_i a)^2(\bar{c}_{11}\cos^2(\theta-\gamma_i)+\bar{c}_{12}\sin^2(\theta-\gamma_i)) + t_L(\bar{c}_{13}a_i+\bar{e}_{31}b_i+\bar{q}_{31}c_i)Y_n(\alpha_i ax)]\cos(m\pi\varsigma)\cos n\theta - 2n\bar{c}_{66}\{n(n-1)Y_n(\alpha_i ax)-(\alpha_i ax)Y_{n+1}(\alpha_i ax)\}\sin 2(\theta-\gamma_i)\cos(m\pi\varsigma)\sin n\theta, i=6,7,8,9 \quad \text{(A3)}$$

$$e_n^{10} = [2\bar{c}_{66}\cos 2(\theta-\gamma_i)n\{(n-1)Y_n(a_{10}ax)-(a_{10}ax)Y_{n+1}(a_{10}ax)\}\cos(m\pi\varsigma)\cos n\theta + \bar{c}_{66}[2\{n(n-1)Y_n(a_{10}ax)+(a_{10}ax)Y_{n+1}(a_{10}ax)\}-(a_{10}ax)^2Y_n(a_{10}ax)]\cos(m\pi\varsigma)\sin n\theta\sin 2(\theta-\gamma_i) \quad \text{(A4)}$$

$$f_n^i = \left[-2\{n(n-1)J_n(\alpha_i ax)+(\alpha_i ax)J_{n+1}(\alpha_i ax)\}+(\alpha_i ax)^2J_n(\alpha_i ax)\right]\cos(m\pi\varsigma)\cos n\theta\sin 2(\theta-\gamma_i) + 2n\{(n-1)J_n(\alpha_i ax)-(\alpha_i ax)J_{n+1}(\alpha_i ax)\}\cos(m\pi\varsigma)\sin n\theta\cos 2(\theta-\gamma_i), i=1,2,3,4. \quad \text{(A5)}$$

$$f_n^5 = 2n\{(n-1)J_n(a_5ax)-(a_5ax)J_{n+1}(a_5ax)\}\cos(m\pi\varsigma)\cos n\theta\sin 2(\theta-\gamma_i) + \left[-2\{n(n-1)J_n(a_5ax)+(a_5ax)J_{n+1}(a_5ax)\}+(a_5ax)^2J_n(a_5ax)\right]\cos(m\pi\varsigma)\sin n\theta\cos 2(\theta-\gamma_i) \quad \text{(A6)}$$

$$f_n^i = \left[-2\{n(n-1)Y_n(\alpha_i ax)+(\alpha_i ax)Y_{n+1}(\alpha_i ax)\}+(\alpha_i ax)^2Y_n(\alpha_i ax)\right]\cos(m\pi\varsigma)\cos n\theta\sin 2(\theta-\gamma_i) + 2n\{(n-1)Y_n(\alpha_i ax)-(\alpha_i ax)Y_{n+1}(\alpha_i ax)\}\cos(m\pi\varsigma)\sin n\theta\cos 2(\theta-\gamma_i), i=6,7,8,9. \quad \text{(A7)}$$

$$f_n^{10} = 2n\{(n-1)Y_n(a_{10}ax)-(a_{10}ax)Y_{n+1}(a_{10}ax)\}\cos(m\pi\varsigma)\cos n\theta\sin 2(\theta-\gamma_i) + \left[-2\{n(n-1)Y_n(a_{10}ax)+(a_{10}ax)Y_{n+1}(a_{10}ax)\}+(a_{10}ax)^2Y_n(a_{10}ax)\right]\cos(m\pi\varsigma)\sin n\theta\cos 2(\theta-\gamma_i) \quad \text{(A8)}$$

$$g_n^i = \left[(t_L+a_i)\cos(\theta-\gamma_i)+\bar{e}_{15}b_i+\bar{q}_{15}c_i\right]\{nJ_n(\alpha_i ax)-(\alpha_i ax)J_{n+1}(\alpha_i ax)\}\sin(m\pi\varsigma)\cos n\theta + (t_L+a_i)nJ_n(\alpha_i ax)\sin(m\pi\varsigma)\sin n\theta\sin(\theta-\gamma_i), i=1,2,3,4. \quad \text{(A9)}$$

$$g_n^5 = -nt_LJ_n(a_{10}ax)\sin(m\pi\varsigma)\cos n\theta\cos(\theta-\gamma_i) - \{nJ_n(a_{10}ax)-(a_{10}ax)J_{n+1}(a_{10}ax)\}t_L\sin(m\pi\varsigma)\sin n\theta\sin(\theta-\gamma_i) \quad \text{(A10)}$$

$$g_n^i = \left[(t_L+a_i)\cos(\theta-\gamma_i)+\bar{e}_{15}b_i+\bar{q}_{15}c_i\right]\{nY_n(\alpha_i ax)-(\alpha_i ax)Y_{n+1}(\alpha_i ax)\}\sin(m\pi\varsigma)\cos n\theta + (t_L+a_i)nY_n(\alpha_i ax)\sin(m\pi\varsigma)\sin n\theta\sin(\theta-\gamma_i), i=6,7,8,9. \quad \text{(A11)}$$

$$g_n^{10} = -nt_LY_n(a_{10}ax)\sin(m\pi\varsigma)\cos n\theta\cos(\theta-\gamma_i) - \{nY_n(a_{10}ax)-(a_{10}ax)Y_{n+1}(a_{10}ax)\}t_L\sin(m\pi\varsigma)\sin n\theta\sin(\theta-\gamma_i) \quad \text{(A12)}$$

$$h_n^i = \left[\bar{e}_{15}(t_L+a_i)-\bar{\varepsilon}_{11}b_i-\bar{m}_{11}c_i\right]\{nJ_n(\alpha_i ax)-(\alpha_i ax)J_{n+1}(\alpha_i ax)\}\sin(m\pi\varsigma)\cos n\theta, i=1,2,3,4 \quad \text{(A13)}$$

$$h_n^5 = -\bar{e}_{15}nt_LJ_n(a_5ax) \quad \text{(A14)}$$

$$h_n^i = \left[\bar{e}_{15}(t_L+a_i)-\bar{\varepsilon}_{11}b_i-\bar{m}_{11}c_i\right]\{nY_n(\alpha_i ax)-(\alpha_i ax)Y_{n+1}(\alpha_i ax)\}\sin(m\pi\varsigma)\cos n\theta, i=6,7,8,9 \quad \text{(A15)}$$

$$h_n^{10} = -\bar{e}_{15}nt_LY_n(a_{10}ax) \quad \text{(A16)}$$

$$i_n^i = \left[\bar{q}_{15}(t_L+a_i)-\bar{m}_{11}b_i-\bar{\mu}_{11}c_i\right]\{nJ_n(\alpha_i ax)-(\alpha_i ax)J_{n+1}(\alpha_i ax)\}, i=1,2,3,4. \quad \text{(A17)}$$

$$i_n^5 = -\bar{q}_{15} n t_L J_n\left(a_5 a x\right) \quad \text{(A18)}$$

$$i_n^i = \left[\bar{q}_{15}\left(t_L + a_i\right) - \bar{m}_{11} b_i - \bar{\mu}_{11} c_i\right]\left\{n Y_n\left(\alpha_i a x\right) - \left(\alpha_i a x\right) Y_{n+1}\left(\alpha_i a x\right)\right\}, i = 6,7,8,9. \quad \text{(A19)}$$

$$i_n^{10} = -\bar{q}_{15} n t_L Y_n\left(a_{10} a x\right) \quad \text{(A20)}$$

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Table 1: The material properties of the electro-magnetic material based on graphical results of Aboudi (2001) composites

c_{11}	c_{12}	c_{13}	c_{33}	c_{44}	c_{66}
218	120	120	215	50	49
e_{15}	e_{31}	e_{33}	q_{15}	q_{31}	q_{33}
0	-2.5	7.5	200	265	345
ε_{11}	ε_{33}	μ_{11}	μ_{33}	m_{11}	m_{33}
0.4	5.8	-200	95	0.0074	2.82

Units: $c_{ij}\left(10^9 N/m^2\right), \varepsilon_{ij}\left(10^{-9} C/Vm\right), e_{ij}\left(C/m^2\right)$
$q_{ij}\left(N/Am\right), \mu_{ij}\left(10^{-6} Ns^2/C^2\right), m_{ij}\left(10^{-9} Ns/VC\right)$