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On Bayesian estimator from exponential distribution based on records with presence of outliers

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ABSTRACT

In this paper, Bayes estimator is derived for the parameter of the exponential model with presence of outliers based on records value. This estimator compared with estimator when we not use of records value.

Introduction

The exponential distribution plays an important role in life testing problems. A great deal with research has been done on estimating the parameters of exponential distribution using both classical and Bayesian techniques and a very good summary of this work can be found in Johanson, Kotz, and Balakrishnan (1994). There are also some papers on estimation and prediction for exponential distribution parameters based of record and censored samples. See for example Balasubramanian and Balakrishnan (1992), Ohandrasekar, Leo Alexander and Balakrishnan (2002), Jaheen (2004), Ahmadi, Doostparast and Parsian (2005) and references therein. In 2001, Dixit and Nasiri have estimated the parameter of exponential distribution with presence of outliers, and in 2009 Asgharzadeh has estimated the parameter of exponential model based on records value.

In section two, we estimate Bayes estimator of parameter of the exponential distribution with presence of outlier, in section three, we estimate the parameter (θ) of exponential distribution based on records value with presence of outliers, in section four, we estimate , Bayes estimation of (θ) based on records value with presence of outliers. These estimators compare in section five.

Bayes estimation of θ with presence of outliers

According to Dixit, Moor and Barnett (1996), we assume that a set of random variables $(X_1, X_2, ..., X_n)$ represent of the distribution of an infected sampled plant from a plot of plants inoculated with a virus. Some of the observations are derived from the airborne dispersal of the spores and are distributed according to the exponential distribution. The other observations out of n random variables (say k) are present because aphids which are known to be carriers of BYMDV have passed the virus into the plant when the aphids feed on the sap. Theses k (known) aphids are considered to be exponential distribution. Thus, we assume that the random variables $(X_1, X_2, ..., X_n)$ are such that k of them are distributed with probability density function (pdf) $g(x,\lambda,\theta)$ as

$$g(x|\lambda,\theta)=\frac{\lambda}{\theta}\exp\left(-\frac{\lambda}{\theta}x\right) \quad , \quad x>0 \quad , \quad \lambda>0 \quad , \quad \theta>0 \tag{1}$$

and the remaining $(n-k)$ random variables are distributed with the following pdf.

$$f(x|\theta)=\frac{1}{\theta}\exp\left(-\frac{x}{\theta}\right) \quad , \quad x>0 \quad , \quad \theta>0 \tag{2}$$

Then the joint pdf of (X_1, X_2, X_n) is

$$f(\underline{x}|\lambda,\theta)=\frac{k!(n-k)!}{n!}\prod_{i=1}^{n} f(x_i|\theta)\sum^{*}\prod_{j=1}^{k}\frac{g(x_{A_j})}{f(x_{A_j})}$$

$$=\frac{k!(n-k)!}{n!}\frac{1}{\theta^n}\exp\left(-\frac{\sum_{i=1}^{n}x_i}{\theta}\right)\sum^{*}\prod_{j=1}^{k}\frac{\frac{\lambda}{\theta}\exp\left(-\frac{\lambda_{xA_j}}{\theta}\right)}{\frac{1}{\theta}\exp\left(-\frac{x_{A_j}}{\theta}\right)} \tag{3}$$

$$=\frac{k!(n-k)!}{n!}\frac{\lambda^k\exp\left(-\frac{n\bar{x}}{\theta}\right)}{\theta n}\sum^{*}\exp\left\{-\left(\frac{\lambda}{\theta}-\frac{1}{\theta}\right)\sum_{j=1}^{k}x_{A_j}\right\} \tag{4}$$

where

$$\sum^{*}=\sum_{A_1=A_1}^{n-k+1}\;\sum_{A_2=A_1+1}^{n-k+2}\cdots\sum_{A_k=A_{k-1}+1} \tag{5}$$

From (3) the marginal distribution of X is given by

$$f(x)=\frac{k\lambda}{n\theta}\exp\left(-\frac{\lambda x}{\theta}\right)+\frac{(n-k)}{n}+\frac{1}{\theta}\exp\left(-\frac{x}{\theta}\right) \quad , \quad x>0 \tag{6}$$

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For more details, see Dixit and Nasiri (2001).

Let $\prod(\theta|\alpha,\beta)$ is the prior pdf of θ as

$$\prod(\theta|\alpha,\beta)=\frac{\beta^{\alpha}}{\Gamma(\alpha)}\left(\frac{1}{\theta}\right)^{\alpha-1}\exp\left(-\frac{\beta}{\theta}\right)\quad,\quad\theta>0$$

Using (3) and (5), $f(x,\theta|\alpha,\beta,\lambda)$ is given by

$$f(\theta|\alpha,\beta,\lambda)=\frac{k!(n-k)!}{n!}\frac{\beta^{\alpha}}{\Gamma(\alpha)}\frac{\lambda^{k}\exp\left(\frac{-n\bar{x}}{\theta}-\frac{\beta}{\theta}\right)}{\theta^{n+\alpha-1}}\sum^{*}\exp\left\{-\left(\frac{\lambda}{\theta}-\frac{1}{\theta}\right)\sum_{j=1}^{k}x_{Ai_j}\right\}\tag{7}$$

Then the posterior distribution of θ is given by

$$\prod(\theta|\underline{x})=\frac{\dfrac{\exp\left(-\dfrac{n\bar{x}+\beta}{\theta}\right)}{\theta^{n+\alpha-1}}\sum^{*}\exp\left(-\dfrac{(\lambda-1)\sum_{j=1}^{k}x_{A_j}}{\theta}\right)}{\int_{0}^{\infty}\dfrac{\exp\left(-\dfrac{n\bar{x}+\beta}{\theta}\right)}{\theta^{n+\alpha-1}}\sum^{*}\left(-\dfrac{(\lambda-1)\sum_{j=1}^{k}x_{A_j}}{\theta}\right)d\theta}$$

Therefore, by using the notation of I as the integral

$$E(\theta^{r}|\underline{x})=\int_{0}^{\infty}\theta^{r}\prod(\theta|\underline{x})d\theta$$

$$=\frac{1}{I_{0}}\int_{0}^{\infty}\frac{\theta^{r}\exp\left(-\frac{n\bar{x}+\beta}{\theta}\right)}{\theta^{n+\alpha-1}}\sum^{*}\exp\left(-\frac{(\lambda-1)\sum_{j=1}^{k}x_{A_j}}{\theta}\right)d\theta\tag{8}$$

$$=\frac{I_{r}}{I_{0}}\tag{9}$$

Where

$$I_{r}=\int_{0}^{\infty}\frac{\theta^{r}\exp\left(-\frac{n\bar{x}+\beta}{\theta}\right)}{\theta^{n+\alpha-1}}\sum^{*}\exp\left(-\frac{(\lambda-1)\sum_{j=1}^{k}x_{A_j}}{\theta}\right)d\theta\tag{10}$$

For $r=1$, the bayes estimator of θ is given by

$$\theta^{*}=E(\theta|\underline{x})=\frac{I_{1}}{I_{0}}$$

We can use the Lindely approximation to estimate θ

Estimation of θ based on records value with presence of outliers

In this section, we shall be concerned with estimation of the parameter of the exponential distribution based on upper record values with the presence of outliers. Suppose that we observe n upper record values $X_{u(1)}=x_{1}, X_{u(2)}=x_{2}, \ldots, X_{u(n)}=x_{n}$ from the exponential model with presence of outliers given by (4). Then the joint pdf of

$$X_{u(1)}=x_{1}, X_{u(2)}=x_{2}, \ldots, X_{u(n)}=x_{n}\tag{11}$$

is given as

$$f(\underline{x}|\theta)=f(x_{n},\theta)\prod_{i=1}^{n-1}h(x_{i},\theta)\quad,\quad x_{1}<x_{2}<\ldots<x_{n}$$

where

$$h(x_{i},\theta)=\frac{f(x_{i},\theta)}{1-F(x_{i},\theta)}$$

For more details, refer to Asgharzadeh (2009). So

$$f(\underline{x}|\theta)=\left[\frac{k\lambda}{n\theta}\exp\left(-\frac{\lambda x_{n}}{\theta}\right)+\frac{n-k}{n\theta}\exp\left(-\frac{x_{n}}{\theta}\right)\right]\prod_{i=1}^{n-1}\left[\frac{\frac{k\lambda}{n\theta}\exp\left(-\frac{\lambda x_{i}}{\theta}\right)+\frac{n-k}{n\theta}\exp\left(-\frac{x_{i}}{\theta}\right)}{\frac{k}{n}\exp\left(-\frac{\lambda x_{i}}{\theta}\right)+\frac{n-k}{n}\exp\left(-\frac{x_{i}}{\theta}\right)}\right]\tag{12}$$

$$=\frac{\prod_{i=1}^{n}\left[\frac{k\lambda}{n\theta}\exp\left(-\frac{\lambda x_{i}}{\theta}\right)+\frac{n-k}{n\theta}\exp\left(-\frac{x_{i}}{\theta}\right)\right]}{\prod_{i=1}^{n-1}\left[\frac{k}{n}\exp\left(-\frac{\lambda x_{i}}{\theta}\right)+\frac{n-k}{n}\exp\left(-\frac{x_{i}}{\theta}\right)\right]}$$

Let $l(\theta)=\frac{d}{d\theta}\ln\left(f(x|\theta)\right)$, then

$$l(\theta)=\frac{d}{d\theta}\left[\sum_{i=1}^{n}\ln\left(\frac{k\lambda}{n\theta}\exp\left(-\frac{\lambda x_{i}}{\theta}\right)+\frac{n-k}{n\theta}\exp\left(-\frac{x_{i}}{\theta}\right)\right)\right.$$

$$\left.-\sum_{i=1}^{n-1}\ln\left(\frac{k}{n}\exp\left(-\frac{\lambda x_{i}}{\theta}\right)+\frac{n-k}{n\theta}\exp\left(-\frac{x_{i}}{\theta}\right)\right)\right]$$

$$=\frac{1}{\theta^{2}}\sum_{i=1}^{n}\frac{k\lambda(\lambda x_{i}-\theta)\exp\left(-\frac{\lambda x_{i}}{\theta}\right)+(n-k)(x_{i}-\theta)\exp\left(-\frac{x_{i}}{\theta}\right)}{k\lambda\exp\left(-\frac{\lambda x_{i}}{\theta}\right)+(n-k)\exp\left(-\frac{x_{i}}{\theta}\right)}$$

$$-\frac{1}{\theta^{2}}\sum_{i=1}^{n}\frac{k\lambda x_{i}\exp\left(-\frac{\lambda x_{i}}{\theta}\right)+(n-k)x_{i}\exp\left(-\frac{x_{i}}{\theta}\right)}{k\exp\left(-\frac{\lambda x_{i}}{\theta}\right)+(n-k)\exp\left(-\frac{x_{i}}{\theta}\right)}\tag{13}$$

To estimate θ, consider $\frac{dl(\theta)}{d\theta}=0$. So

$$g(\theta)=\sum_{i=1}^{n}\frac{k\lambda(\lambda x_{i}-\theta)\exp\left(-\frac{\lambda x_{i}}{\theta}\right)+(n-k)(x_{i}-\theta)\exp\left(-\frac{x_{i}}{\theta}\right)}{k\lambda\exp\left(-\frac{\lambda x_{i}}{\theta}\right)+(n-k)\exp\left(-\frac{x_{i}}{\theta}\right)}\tag{14}$$

$$-\sum_{i=1}^{n}\frac{k\lambda x_{i}\exp\left(-\frac{\lambda x_{i}}{\theta}\right)+(n-k)x_{i}\exp\left(-\frac{x_{i}}{\theta}\right)}{k\exp\left(-\frac{\lambda x_{i}}{\theta}\right)+(n-k)\exp\left(-\frac{x_{i}}{\theta}\right)}$$

Then by using numerical methods, for example, Newton-Raphson

$$\theta_{i+1}=\theta_{i}-\frac{g'(\theta)}{g(\theta)}$$

where $g'(\theta)$ is derivation of θ

Bayes estimation θ based on records value with the presence of outliers

To obtain Bayes estimation of θ, consider $f(\underline{x}|\theta)$ as

$$f(\underline{x}|\theta)=\left[\frac{k\lambda}{n\theta}\exp\left(-\frac{\lambda x_{n}}{\theta}\right)+\frac{n-k}{n\theta}\exp\left(-\frac{x_{n}}{\theta}\right)\right]\prod_{i=1}^{n-1}\left[\frac{\frac{k\lambda}{n\theta}\exp\left(-\frac{\lambda x_{i}}{\theta}\right)+\frac{n-k}{n\theta}\exp\left(-\frac{x_{i}}{\theta}\right)}{\frac{k}{n}\exp\left(-\frac{\lambda x_{i}}{\theta}\right)+\frac{n-k}{n}\exp\left(-\frac{x_{i}}{\theta}\right)}\right]$$

$$=\frac{\prod_{i=1}^{n}\left[\frac{k\lambda}{n\theta}\exp\left(-\frac{\lambda x_{i}}{\theta}\right)+\frac{n-k}{n\theta}\exp\left(-\frac{x_{i}}{\theta}\right)\right]}{\prod_{i=1}^{n-1}\left[\frac{k}{n}\exp\left(-\frac{\lambda x_{i}}{\theta}\right)+\frac{n-k}{n}\exp\left(-\frac{x_{i}}{\theta}\right)\right]}$$

$$=\frac{1}{n\theta^n}\frac{\prod_{i=1}^{n}\left[k\lambda\exp\left(-\frac{\lambda x_i}{\theta}\right)+(n-k)\exp\left(-\frac{x_i}{\theta}\right)\right]}{\prod_{i=1}^{n-1}\left[k\exp\left(-\frac{\lambda x_i}{\theta}\right)+(n-k)\exp\left(-\frac{x_i}{\theta}\right)\right]}$$

$$=\frac{1}{n\theta^n}\frac{\prod_{i=1}^{n}A_i}{\prod_{i=1}^{n-1}B_i} \qquad (15)$$

where

$$A_i=k\lambda\exp\left(-\frac{\lambda x_i}{\theta}\right)+(n-k)\exp\left(-\frac{x_i}{\theta}\right)$$

$$B_i=k\exp\left(-\frac{\lambda x_i}{\theta}\right)+(n-k)\exp\left(-\frac{x_i}{\theta}\right)$$

For $k=0$ it is given by Asgharzadeh (2009).

By using (5) and (15), the joint pdf of $\underline{x}$ and θ is given by

$$\prod(\theta\ ,\underline{x})=\frac{n\beta^{\alpha}}{\Gamma(\alpha)}\frac{\exp\left(-\frac{\beta}{\theta}\right)}{\theta^{n+\alpha-1}}\frac{\prod_{i=1}^{n}A_i}{\prod_{i=1}^{n-1}B_i} \qquad (16)$$

Then

$$\prod(\theta\ ,\underline{x})=\frac{\frac{\exp\left(-\frac{\beta}{\theta}\right)}{\theta^{n+\alpha-1}}\frac{\prod_{i=1}^{n}A_i}{\prod_{i=1}^{n-1}B_i}}{\int_0^{\infty}\frac{\exp\left(-\frac{\beta}{\theta}\right)}{\theta^{n+\alpha-1}}\frac{\prod_{i=1}^{n}A_i}{\prod_{i=1}^{n-1}B_i}\,d\theta} \qquad (17)$$

Hence

$$E\left(\theta^r\,|\,\underline{x}\right)=\frac{I_r}{I_0} \qquad (18)$$

where

$$I_r=\int_0^{\infty}\theta^r\,\frac{\exp\left(-\frac{\beta}{\theta}\right)}{\theta^{n+\alpha-1}}\frac{\prod_{i=1}^{n}A_i}{\prod_{i=1}^{n-1}B_i}$$

Numerical Analysis

To get an idea about the Bias and mean square error (MSE) of three type of estimators, we have generated a sample of size n=5, 6,..., 50, k=1, 2, 3, α =0.5, 2, 5, β =0.3, 3, θ =0.4, 2, 5, λ =2, 3. The results are derived by using R software based on 1000 independent replication (Figure 1-4). In each graph, red color is for Bayes estimator of θ with outliers, green color is for estimation of θ based on records value with the presence of outliers and blue color is to show the behavior of Bayes estimator of θ based on records value in the presence of outliers.

From the graphs, it is seen that the estimator of θ based on records value in the presence of outliers is unbiased and consistent. It has also a better behavior among the other types of estimator for all values of n and the parameters α, β, and λ.

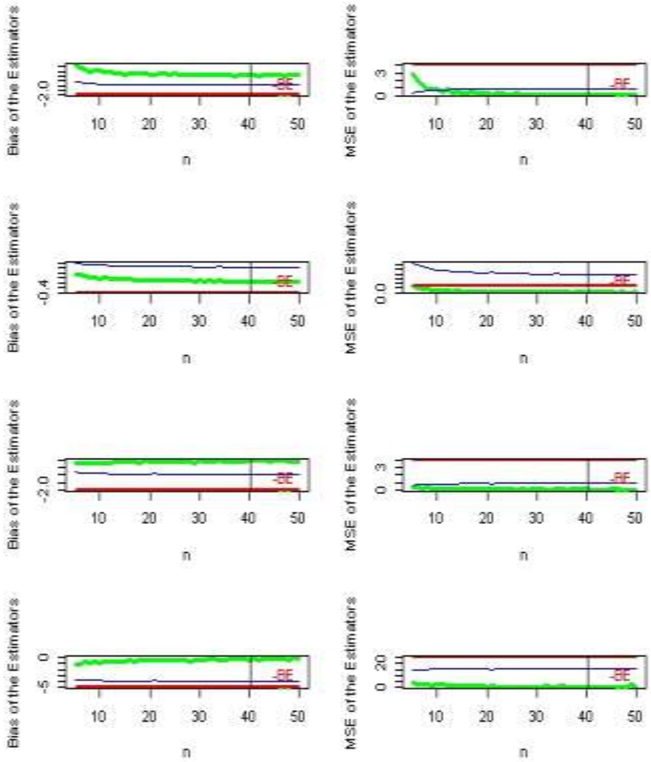

Figure 1. Comparison Bias and MSE of the estimators for k=1, ($\alpha,\beta,\lambda,\theta$)=(0.5,0.3,2,0.4), (0.5,3,3,2), (0.5,0.3,3,5) and (2,0.3,3,0.4), respectively.

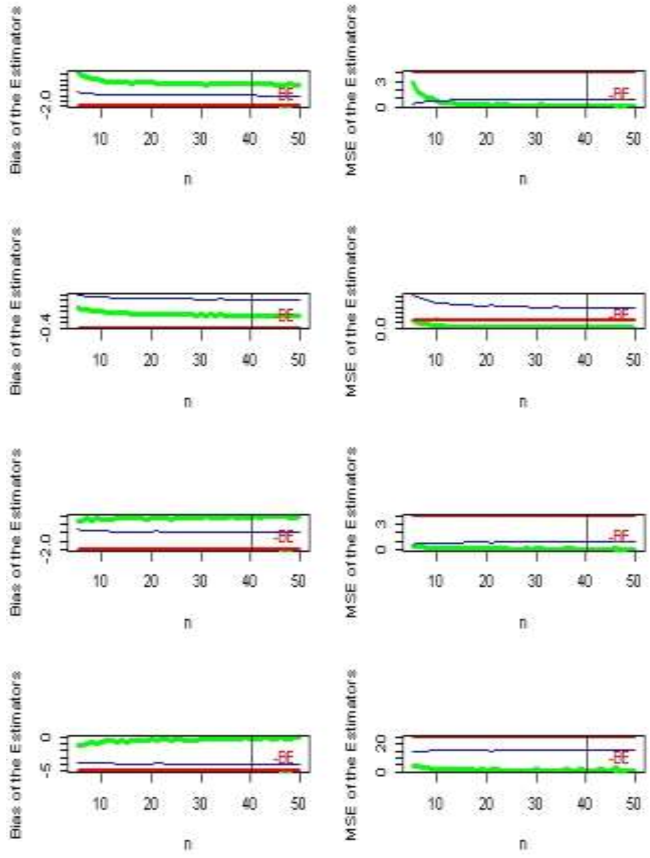

Figure 2. Comparison Bias and MSE of the estimators for k=1, ($\alpha,\beta,\lambda,\theta$)=(2,3,2,2), (5,3,3,0.4), (5,3,3,2) and (5,3,2,5), respectively.

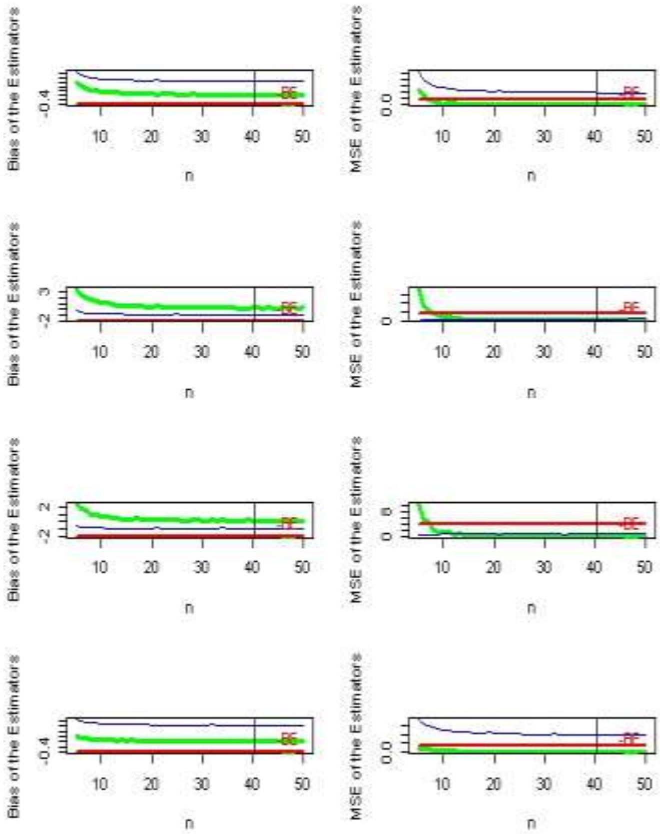

Figure 3. Comparison Bias and MSE of the estimators for k=2, $(\alpha,\beta,\lambda,\theta)$=(0.5,0.3,2,0.4), (0.5,3,3,2), (0.5,0.3,3,5) and (2,0.3,3,0.4), respectively

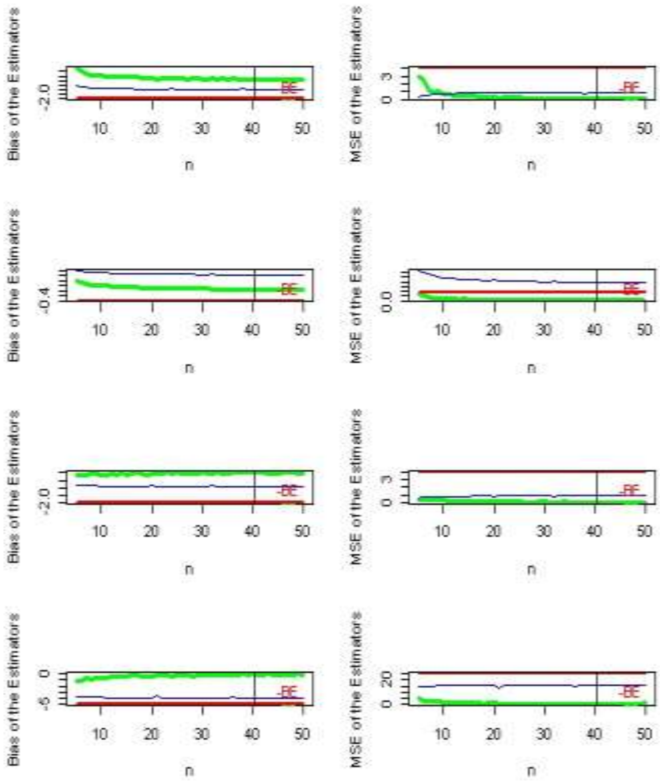

Figure 4. Comparison Bias and MSE of the estimators for k=2, $(\alpha,\beta,\lambda,\theta)$=(2,3,2,2), (5,3,3,0.4), (5,3,3,2) and (5,3,2,5), respectively.

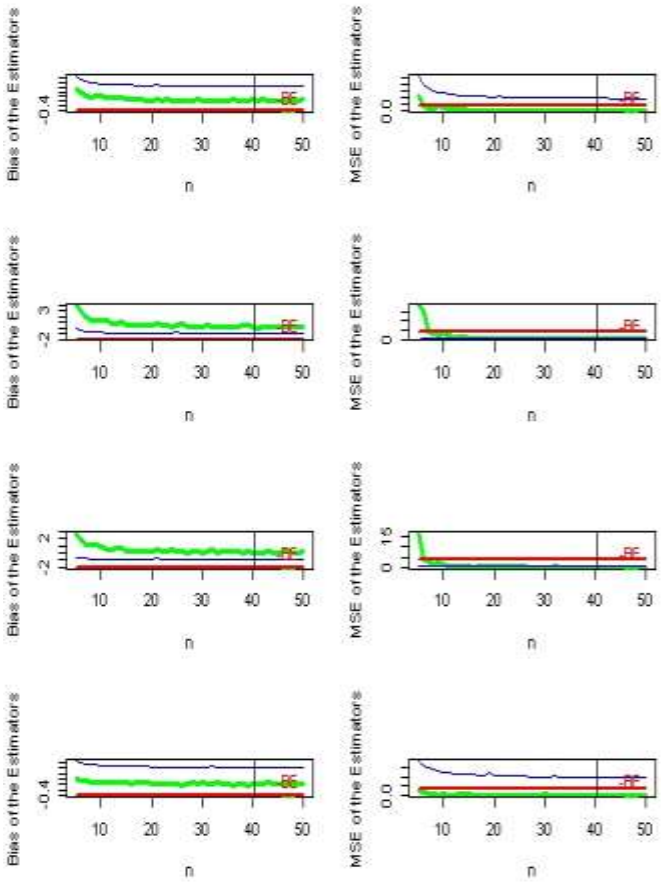

Figure 5. Comparison Bias and MSE of the estimators for k=3, $(\alpha,\beta,\lambda,\theta)$=(0.5,0.3,2,0.4), (0.5,3,3,2), (0.5,0.3,3,5) and (2,0.3,3,0.4), respectively.

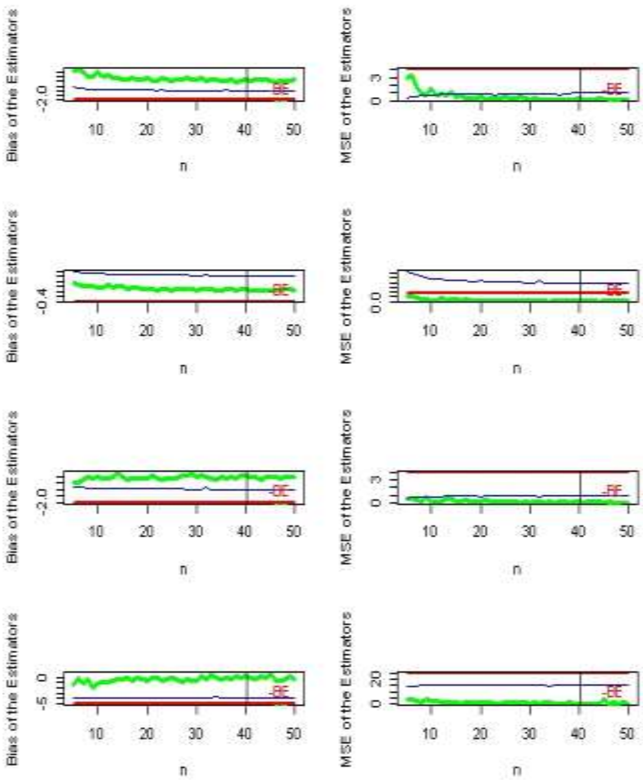

Figure 6. Comparison Bias and MSE of the estimators for k=3, $(\alpha,\beta,\lambda,\theta)$=(2,3,2,2), (5,3,3,0.4), (5,3,3,2) and (5,3,2,5), respectively.

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