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# On πgb-Separation Axioms in Bitopological Spaces

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**ABSTRACT**

In this paper, we introduce and study some new separation axioms using the $(1, 2)^*$-πgb-open sets in bitopological spaces.



## 1.Introduction

Levine [8,9] introduced the concept of generalized closed sets and semi open sets in topological space and a class of topological spaces called $T_{½}$ spaces. The investigation of generalized closed sets leads to several new separation axioms. Andrijevic [2] introduced a new class of generalized open sets in a topological space, the so-called b-open sets. This type of sets was discussed by Ekici and Caldas [6] under the name of γ-open sets. Ashish Kar and Bhattacharya [1], in 1990, continued their work on pre-open sets and offered another set of separation axioms analogous to the semi separation axioms defined by Maheshwari and Prasad [10]. Caldas [11] defined a new class of sets called semi-Difference (briefly sD) sets by using the semi-open sets [7], and introduced the semi-$D_i$ spaces for i= 0, 1, 2.S. Athisaya ponmani and M. Lellis Thivagar [3] discussed pre $T_k$ spaces and pD sets in bitopological spaces.

In this paper, we have introduced a new generalized axiom called πgb-separation axioms in bitopological spaces. We have incorporated $(1,2)^*$-πgb-$T_i$, $(1,2)^*$-πgb-$D_i$, where i = 0,1,2, and$(1,2)^*$-πgb-$R_i$,i=0,1.

## 2. Preliminaries

Let us recall the following definitions which we shall require later.

**Definition 2.1:** A subset A of a space (X)is called

(1) a regular open set if A= int (cl(A)) and a regular closed set if A= cl(int (A));

(2) b-open [2] or sp-open [4], γ –open [6] if A⊂cl(int(A)) ∪ int (cl(A)).

The complement of a b-open set is said to be b-closed [2]. The intersection of all b-closed sets of X containing A is called the b-closure of A and is denoted by bCl(A). The union of all b-open sets of X contained in A is called b-interior of A and is denoted by bInt(A). The family of all b-open (resp. α-open, semi-open, preopen, β-open, b-closed, preclosed) subsets of a space X is denoted by bO(X)(resp. αO(X), SO(X), PO(X), βO(X), bC(X), PC(X)) and the collection of all b-open subsets of X containing a fixed point x is denoted by bO (X, x). The sets SO(X, x), αO(X, x), PO(X, x), βO(X, x) are defined analogously.

**Definition 2.2:** A subset A of a space (X, τ) is called πg-closed [5] if cl(A) ⊂ U whenever A⊂ U and U is π-open.

**Definition 2.3:** A subset A of a space X is called πgb-closed [13] if bcl(A)⊂ U whenever A⊂U and U is π-open in (X, τ).

By πGBC(τ) we mean the family of all πgb- closed subsets of the space(X, τ).

**Definition 2.4[14]:** A subset A of a bitopological space $(X, \tau_1, \tau_2)$ is called $(1, 2)^*$-π generalized b-closed (briefly $(1, 2)^*$-πgb-closed) if $\tau_1\tau_2$-bcl(A)⊂U whenever A⊂U and U is $\tau_1\tau_2$ -π-open in X.

**Definition 2.5[14]:** A subset A of a bitopological space $(X, \tau_1, \tau_2)$ is called $(1, 2)^*$-b-open [14] if A ⊂ $\tau_1\tau_2$- cl($\tau_1\tau_2$-int(A))∪ $\tau_1\tau_2$-int ($\tau_1\tau_2$- cl(A)).

**Definition 2.6[14]:**A function f: $(X, \tau_1,\tau_2) \rightarrow (Y, \sigma_1,\sigma_2)$ is called

1) $(1,2)^*$- πgb- continuous [13] if every $f^{-1}(V)$ is $(1,2)^*$- πgb-closed in X for every closed set V of Y.

2) $(1,2)^*$- πgb- irresolute [13] if $f^{-1}(V)$ is $(1,2)^*$- πgb- closed in X for every πgb-closed set V in Y.

**Definition 2.7[14]:** A map f: $(X, \tau_1,\tau_2) \rightarrow (Y, \sigma_1,\sigma_2)$ is said to be$(1, 2)^*$-πgb-open if for every $(1, 2)^*$-open set F of X, f(F) is $(1, 2)^*$-πgb-open in Y.

**Definition 2.8[15] :**A map f: $(X, \tau_1,\tau_2) \rightarrow (Y, \sigma_1,\sigma_2)$ is said to be M-$(1, 2)^*$-πgb-open if for every πgb-open set F of X, f(F) is πgb-open in Y.

**Definition 2.8[12].**A bitopological space X is said to be

(i) Ultra α-$R_0$ (resp. ultra semi-$R_0$) if (1, 2)αcl ({x}) ⊆U(resp. (1, 2)scl({x})⊆U) whenever x ∈U ∈(1, 2)αO(X) (resp. x ∈U ∈ (1, 2)SO(X)).

(ii) Ultra α-$R_1$ (resp. ultra semi-$R_1$) if for x, y ∈X such that x ∉(1, 2)αcl ({y}) (resp. (1, 2)scl ({y})), there exist disjoint (1, 2)α-open (resp. (1, 2) semiopen)sets U, V in X such that x ∈U and y ∈V.

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**Definition 2.9:** A bitopological space X is (1,2)*-$T_0$ if for each pair of distinct points x, y of X, there exists a (1,2)*-open set containing one of the points but not the other.

Complement of (1, 2)*-b-open is called (1, 2)*-b-closed.

Throughout the following sections by X and Y we mean bitopological spaces (X, $\tau_1,\tau_2$) and (Y, $\sigma_1,\sigma_2$) respectively.

**(1,2)*-πgb-$T_i$-Spaces**

**Definition 3.1:** A bitopological space X is (1,2)*-πgb-$T_0$ if for each pair of distinct points x, y of X, there exists a (1,2)*-πgb-open set containing one of the points but not the other.

**Lemma 3.2:** If for some x∈X, {x} is(1,2)*-πgb-open, then x∉(1,2)*-πgbcl({y}) for all y≠x.

**Proof:** If {x} is (1,2)*-πgb-open for some x∈X,then X-{x} is (1,2)*-πgb-closed and x∉X-{x}.If x∈(1,2)*-πgbcl{y} for some y≠x, then x,y both are in all the (1,2)*-πgb-closed sets containing y.This implies x∈ X-{x} which is not true. Hence x∉ (1,2)*-πgbcl({y}).

**Theorem 33:** In a space X, distinct points have distinct (1,2)*-πgb-closures.

**Proof:** Letx,y ∈X,x ≠y.Take A={x}$^c$.

Case(i): If$\tau_1$ $\tau_2$-cl(A)=A .Then A is $\tau_{1,2}$-closed.This implies A is (1,2)*-πgb-closed. Then X-A={x} is (1,2)*-πgb-open not containing y. Then by previous lemma 3.2, x∉ (1,2)*-πgb-cl({y}) and y ∈ (1,2)*-πgb-cl({y}).Thus (1,2)*-πgb-cl({x}) and (1,2)*-πgbcl({y}) are distinct.

Case(ii): If $\tau_1$ $\tau_2$-cl(A)=X. Then A is (1,2)*-πgb-open and {x} is (1,2)*-πgb-closed. This implies (1,2)*-πgb-cl({x})={x} which is not equal to (1,2)*-πgb-cl({y}).

**Theorem3.4:** A bitopological space X is (1,2)*-πgb -$T_0$ iff for each pair of distinct points x, y of X,(1,2)*-πgb-cl(x) ≠ (1,2)*-πgb-cl(y ).

**Proof: Necessity:** Let Xbe a (1,2)*-πgb-$T_o$ space. Let x, y∈X such that x ≠y. Then there exists a (1,2)*-πgb-open set V containing one of the points but not the other, say x∈V and y∉V. Then V$^c$ is a (1,2)*-πgb -closed set containing y but not x. But (1,2)*-πgb-cl(y) is the smallest (1,2)*-πgb-closed set containing y. Therefore (1,2)*-πgb-cl(y) ⊂V$^c$ and hence x∉ (1,2)*-πgb-cl(y).Thus (1,2)*-πgb-cl(x) ≠ (1,2)*-πgb-cl(y).

**Sufficiency:** Suppose x, y∈X, x ≠y and (1,2)*-πgb-cl(x) ≠(1,2)*-πgb-cl(y).Let z∈X such that z∈ (1,2)*-πgb-cl({ x}) but z∉ (1,2)*-πgb-cl({y}).If x∈ (1,2)*-πgb-cl({y}), then (1,2)*-πgb-cl(x)⊂(1,2)*-πgb-cl(y) and hence z∈ (1,2)*-πgb-cl({y }).This is a contradiction. Therefore x∉(1,2)*-πgb-cl ({y}).That implies x∈ ((1,2)*-πgb-cl({y}).Therefore ((1,2)*-πgb-cl({y}) $^c$ is a (1,2)*-πgb -open set containing x but not y.Hence X is (1,2)*-πgb-$T_0$ .

**Theorem 3.5:** Every bitopological space is (1,2)*-πgb-$T_0$.

**Proof:** Follows from Previous two theorems 3.3 and 3.4.

**Theorem 3.6:** Let f:X→Y be a bijection, M-(1,2)*-πgb-open map and X is (1,2)*-πgb-$T_0$ space, then Y is also (1,2)*-πgb-$T_0$ space.

**Proof:** Let $y_1$ ,$y_2$∈Y with $y_1$≠$y_2$.Since f is a bijection, there exists $x_1$,$x_2$∈X with $x_1$≠$x_2$ such that f($x_1$)= $y_1$and f($x_2$)= $y_2$.Since X is (1,2)*-πgb- $T_0$ space, there exists a (1,2)*-πgb-open set M in X such that $x_1$∈M and $x_2$∉M.Since f is M-(1,2)*-πgb-open map, f(M) is a (1,2)*-πgb-open set in Y.Now we have $x_1$∈M⇒ f($x_1$)∈f(M)⇒ $y_1$∈f(M). $x_2$∉M⇒ f($x_2$)∉ f(M)⇒ $y_2$∉f(M). Hence for any two distinct points $y_1$, $y_2$ in Y, there exists (1,2)*-πgb-open set f(M) in Y such that $y_1$∈ f(M) and $y_2$∉ f(M).Hence Y is a (1,2)*-πgb-$T_0$ space.

**Theorem 3.7:** Let f: X→Y be a bijection, (1,2)*-πgb-open map and X is (1,2)*-$T_0$ space, then Y is a (1,2)*-πgb-$T_0$ space.

**Proof:** Let $y_1$ ,$y_2$∈Y with $y_1$≠$y_2$.Since f is a bijection, there exists $x_1$,$x_2$∈X with $x_1$≠$x_2$ such that f($x_1$)= $y_1$and f($x_2$)= $y_2$.Since X is (1,2)*-$T_0$ space, there exists a (1,2)*-open set M in X such that $x_1$∈M and $x_2$∉M.Since f is (1,2)*-πgb-open map, f(M) is a (1,2)*-πgb-open set in Y. Now we have $x_1$∈M⇒ f($x_1$)∈f(M)⇒ $y_1$∈f(M). $x_2$∉M⇒ f($x_2$)∉ f(M)⇒ $y_2$∉f(M). Hence for any two distinct points $y_1$, $y_2$ in Y,there exists (1,2)*-πgb-open set f(M) in Y such that $y_1$∈ f(M) and $y_2$∉ f(M).Hence Y is a (1,2)*-πgb-$T_0$ space.

**Theorem 3.8:** Let f:X→Y be a bijection,(1,2)*-πgb-irresolute map and Y is (1,2)*-πgb-$T_0$ space, then X is also (1,2)*-πgb-$T_0$ space.

**Proof:** Let $x_1$ ,$x_2$∈X with $x_1$≠$x_2$.Since f is a bijection, there exists $y_1$,$y_2$∈X with $y_1$≠$y_2$ such that f($x_1$)= $y_1$and f($x_2$)= $y_2$⇒$x_1$= $f^{-1}$($y_1$) and $x_2$= $f^{-1}$($y_2$).Since Y is a (1,2)*-πgb-$T_0$ space, there exists a (1,2)*-πgb-open set M in Y such that $y_1$∈M and $y_2$∉M.Since f is (1,2)*-πgb-irresolute,$f^{-1}$(M) is (1,2)*-πgb-open set in X.Now we have $y_1$∈M⇒ $f^{-1}$($y_1$)∈$f^{-1}$(M)⇒$x_1$∈$f^{-1}$(M). $y_2$∉M⇒ $f^{-1}$($y_2$)∉ $f^{-1}$(M)⇒$x_2$∉$f^{-1}$(M). Hence for any two distinct points $x_1$, $x_2$ in X,there exists (1,2)*-πgb-open set $f^{-1}$(M) in X such that $x_1$∈ $f^{-1}$(M) and $x_2$∉ $f^{-1}$(M).Hence X is a (1,2)*-πgb-$T_0$ space.

**Theorem 3.9:** Let f: X→Y be a bijection, (1,2)*-πgb-continuous and Y is (1,2)*-$T_0$ space, then X is a (1,2)*-πgb-$T_0$ space.

**Proof:** Let f: X→Y be a bijection, (1,2)*-πgb-continuous map and Y is a (1,2)*-$T_0$space.Let$x_1$ ,$x_2$∈X with $x_1$≠$x_2$.Since f is a bijection, there exists $y_1$,$y_2$∈Y with $y_1$≠$y_2$ such that f($x_1$)= $y_1$and f($x_2$)= $y_2$⇒$x_1$= $f^{-1}$($y_1$) and $x_2$= $f^{-1}$($y_2$).Since Y is (1,2)*-$T_0$ space, there exists a (1,2)*-open set M in X such that $y_1$∈M and $y_2$∉M.Since f is (1,2)*-πgb-continuous map, $f^{-1}$(M) is a (1,2)*-πgb-open set in Y.Now we have $y_1$∈M⇒ $f^{-1}$($y_1$)∈$f^{-1}$(M)⇒$x_1$∈$f^{-1}$(M)and$x_2$∉$f^{-1}$(M). Hence for any two distinct points $y_1$,$y_2$ in Y,there exists (1,2)*-πgb-open set $f^{-1}$(M) in Y such that $x_1$∈ $f^{-1}$(M) and $x_2$∉$f^{-1}$(M).Hence Y is a (1,2)*-πgb-$T_0$ space.

**Definition 3.10:** A bitopological space Xis (1,2)*-πgb-symmetric if for x and y in X, x ∈ (1,2)*-πgb-cl({y})⇒ y∈ (1,2)*-πgb-cl({x}).

**Theorem 3.11:** X is (1,2)*-πgb-symmetric iff {x} is (1,2)*-πgb-closed for x ∈X.

**Proof:** Assume that x ∈ (1,2)*-πgb-cl({y}) but y∉ (1,2)*-πgb-cl({x}).This implies ( (1,2)*-πgb-cl({x})$^c$ contains y.Hence the set {y} is a subset of ( (1,2)*-πgb-cl({x})$^c$ .This implies (1,2)*-πgb-cl({y}) is a subset of ( (1,2)*-πgb-cl({x})$^c$. Now ( (1,2)*-πgb-cl({x})$^c$ contains x which is a contradiction.

Conversely, Suppose that $\{x\} \subset E \in$ (1,2)*-πgbO(X)but (1,2)*-πgb-cl({y}) which is a subset of $E^c$ and $x \notin E$. But this is a contradiction.

**Definition 3.12:** A space X is (1,2)*-πgb-$T_1$ if for any pair of distinct points x, y of X, there is a (1,2)*-πgb -open set U in X such that $x\in U$ and $y\notin U$ and there is a (1,2)*-πgb -open set V in X such that $y\in U$ and $x\notin V$.

**Remark 3.13 :** Every (1,2)*-πgb-$T_1$ space is (1,2)*-πgb-$T_0$ space.

**Theorem3.14 :** In a space X, the following are equivalent

(1) X is (1,2)*-πgb-$T_1$

(2) For every $x\in X$, {x} is (1,2)*-πgb-closed in X.

(3) Each subset A of X is the intersection of all (1,2)*-πgb -open sets containing x.

(4) The intersection of all (1,2)*-πgb-open sets containing the point x in X is {x}.

**Proof:** (1) ⇒ (2) Suppose X is (1,2)*-πgb-$T_1$. Let $x\in X$ and $y\in\{x\}^c$ .Then $x\neq y$ and so there exists a (1,2)*-πgb-open set $U_y$ such that $y\in U_y$ but $x\notin U_y$. Therefore $y\in U_y\subset\{x\}^c$.That is, $\{x\}^c=\cup\{U_y / y\in\{x\}^c\}$ is (1,2)*-πgb-open. Hence {x} is (1,2)*-πgb -closed.

(2)⇒ (3) Let $A\subset X$ and $y\notin A$. Then $A\subset\{y\}^c$ and $\{y\}^c$ is (1,2)*-πgb-open in X and $A=\cap\{\{y\}^c: y\in A^c\}$ which is the intersection of all (1,2)*-πgb-open sets containing A.

(3)⇒ (4) is obvious

(4)⇒ (1) Let x, y ∈X, x≠y.By assumption, there exists a (1,2)*-πgb-open set containing x but not y and the (1,2)*-πgb-open set V containing y but not x. Hence X is (1,2)*-πgb -$T_1$.

**Theorem 3.15:** X is (1,2)*-πgb-symmetric iff {x} is (1,2)*-πgb-closed for x∈ X.

**Proof:** Assume that $x \in$ (1,2)*-πgb-cl({y}) but $y \notin$ (1,2)*-πgb-cl({x}).This implies (1,2)*- $(\pi gb\text{-}cl(\{x\})^c$ contains y.Hence the set {y} is a subset of (1,2)*- $(\pi gb\text{-}cl(\{x\})^c$ .This implies (1,2)*-πgb-cl({y}) is a subset of(1,2)*-$(\pi gb\text{-}cl(\{x\})^c$. Now (1,2)*-$(\pi gb\text{-}cl(\{x\})^c$ contains x which is a contradiction.

Conversely, Suppose that $\{x\} \subset E \in$ (1,2)*-πGBO(X) but (1,2)*-πgb-cl({y}) which is a subset of $E^c$ and $x \notin E$. But this is a contradiction.

**Theorem 3.16 :** A bitopological space X is a (1,2)*-πgb-$T_1$iff the singletons are (1,2)*-πgb-closed sets.

**Proof:** LetXbe (1,2)*-πgb-$T_1$ and x be any point of X. Suppose $y \in\{X\}^c$. Then $x \neq y$ and so there exists a (1,2)*-πgb-open set U such that $y\in U$ but $x \notin U$. Consequently, $y \in U \subset (\{x\})^c$. That is $(\{x\})^c=\cup\{U \mid y\in (\{x\})^c\}$ which is (1,2)*-πgb-open.

Conversely suppose {x} is (1,2)*-πgb-closed for every x∈X. Let x, y∈X with x≠y. Then $x\neq y \Rightarrow y \in(\{x\})^c$. Hence $(\{x\})^c$ is a (1,2)*-πgb-open set containing y but not x. Similarly $(\{y\})^c$ is a (1,2)*-πgb-open set containing x but not y. Hence X is (1,2)*-πgb-$T_1$-space.

**Remark 3.17:** If X is (1,2)*-πgb-$T_i$, then X is (1,2)*-πgb-$T_{i-1}$ ;i=1,2.

**Corollary 3.18 :** If X is (1,2)*-πgb-$T_1$, then it is (1,2)*-πgb-symmetric.

Proof: In a (1,2)*-πgb-$T_1$ space, singleton sets are (1,2)*-πgb-closed. By theorem 3.17, and by theorem 3.16, the space is (1,2)*-πgb-symmetric.

**Corollary 3.19:** The following statements are equivalent

(i)X is (1,2)*-πgb-symmetric and (1,2)*-πgb-$T_0$

(ii)X is (1,2)*-πgb-$T_1$.

**Proof:** By corollary 3.18 and remark 3.17 ,it suffices to prove (1) ⇒ (2).Let x≠y and by (1,2)*-πgb-$T_0$ ,assume that $x\in G_1\subset (\{y\})^c$ for some $G_1\in$ (1,2)*-πGBO(X).Then $x\notin$ (1,2)*-πgb-cl({y}) and hence $y\notin$(1,2)*-πgb-cl({x}).There exists a $G_2 \in$ (1,2)*-πGBO(X)such that $y\in G_2\subset (\{x\})^c$. Hence Xis a (1,2)*-πgb-$T_1$ space.

**Definition3.20:**A space X is (1,2)*-πgb-$T_2$ if for each pair of distinct points x and y in X, there exists a (1,2)*-πgb -open set U and a (1,2)*-πgb -open set V in X such that x∈U, y∈Vand U∩V =Φ.

**Remark 3.21:** Every (1,2)*-πgb-$T_2$ space is (1,2)*-πgb-$T_1$.

**Definition 3.22:** Let X be a bitopological space. Let x be a point of X and G be a subset of X. Then G is called an (1,2)*-πgb-neighbourhood of x (briefly(1,2)*-πgb-nbd of x) if there exists an (1,2)*-πgb-open set U of X such that x∈U⊂G.

**Theorem 3.23:**For a bitopological space X the following are equivalent:

(1) X is (1,2)*-πgb-$T_2$

(2)If x∈X, then for each y≠x, then there is an (1,2)*-πgb-nbd N(x) of x such that y∉(1,2)*-πgbcl N(x)

(3) For each x∈{(1,2)*-πgbcl(N): N is an (1,2)*- πgb -nhd of x}={x}.

(2) If x∈X, then for each y ≠ x, there is a (1,2)*-πgb-open set U containing x such that y∉(1,2)*-πgb - cl(U)

**Proof:** (1) ⇒ (2):Let x∈X. If y∈X is such that y≠x there exists disjoint (1,2)*-πgb open sets U and V such that x∈U and y∈V. Then x ∈U⊂X-V which implies X-V is (1,2)*-πgb-nbd of x. Also N(x)=X-V. Therefore y∉ (1,2)*-πgb-cl N(x).

(2)⇒(3) Obvious

(3)⇒ (1): Let x, y∈X and x≠y. By (2), there exists a (1,2)*-πgb-nbd N of x such that y∉(1,2)*-πgb-cl(N).Therefore y∈X-((1,2)*-πgb-cl(N)). X-((1,2)*-πgb-cl(N)) is (1,2)*-πgb-open. Since N is (1,2)*-πgb-nbd of x, there exists U∈(1,2)*-πGBO(X)such that x∈U⊂N and U∩ X-( (1,2)*-πgb-cl(N))=Φ. Hence X is (1,2)*-πgb-$T_2$.

**Theorem 3.24:** Iff : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is injective, (1,2)*-πgb-irresolute open map and Y is (1,2)*- πgb-$T_2$,then X is (1,2)*- πgb-$T_2$.

**Proof:** Let x, y∈X and x≠y. Since f is injective,f(x) ≠f(y) in Y and there exists disjoint (1,2)*-πgb-open sets U and V such that f(x) ∈U and f(y) ∈V.Let $G=f^{-1}(U)$ and $H= f^{-1}(V)$.Then $G\cap H=f^{-1}(U) \cap f^{-1}(V)=f^{-1}(U\cap V)= \Phi$. Hence X is (1,2)*- πgb-$T_2$.

4. (1,2)*-πgb-D-sets and associated separation axioms

**Definition 4.1:** A subset A of a bitopological space X is called (1, 2)*-D-set if there are two U,V∈( (1,2)*-O(X)such that U≠X and A=U-V.

**Definition 4.2 :** A space X is said to be

(i) (1,2)*-$D_0$ if for any pair of distinct points x and y of X , there exist a (1,2)*-D-set in X containing x but not y (or) a (1,2)*-D-set in X containing y but not x.

(ii) (1,2)*-$D_1$ if for any pair of distinct points x and y in X,there exists a (1,2)*-D-set of X containing x but not y and a (1,2)*-D-set in X containing y but not x.
(iii) (1,2)*-$D_2$ if for any pair of distinct points x and y of X, there exists disjoint ((1,2)*-D-sets G and H in X containing x and y respectively.

**Definition 4.3:** A bitopological space X is said to be (1,2)*-D-connected if X cannot be expressed as the union of two disjoint non-empty (1,2)*-D-sets.

**Definition 4.4:** A bitopological space X is said to be (1,2)*- D-compact if every cover of X by (1,2)*-D-sets has a finite subcover.

**Definition 4.5:** A subset A of a bitopological space X is called (1,2)*-πgb-D-set if there are two U,V∈( (1,2)*-πGBO(X)such that U≠X and A=U-V.

Clearly every (1,2)*-πgb-open set U different from X is a (1,2)*-πgb-D set if A=U and V=Φ.

**Example 4.6 :** Let X={a,b,c} and $\tau_1$={Φ,{a},{b},{a,b},X}, $\tau_2$={Φ,{a,b},X}..Then {c} is a (1,2)*-πgb-D-set but not (1,2)*-πgb-open. Since (1,2)*-πGBO(X)={ Φ,{a},{b},{b,c},{a,c},{a,b},X}.Then U={b,c}≠X and V={a,b} are (1,2)*-πgb-open sets in X. For U and V, since U-V={b,c}-{a,b}={c},then we have S={c} is a (1,2)*-πgb-D-set but not (1,2)*-πgb-open.

**Definition4.7:** A space X is said to be
(iv) (1,2)*-πgb-$D_0$ if for any pair of distinct points x and y of X ,there exist a (1,2)*-πgb-D-set in X containing x but not y (or) a (1,2)*-πgb-D-set in X containing y but not x.
(v) (1,2)*-πgb-$D_1$ if for any pair of distinct points x and y in X,there exists a (1,2)*-πgb-D-set of X containing x but not y and a (1,2)*-πgb-D-set in X containing y but not x.
(vi) (1,2)*-πgb-$D_2$ if for any pair of distinct points x and y of X, there exists disjoint (1,2)*-πgb-D-sets G and H in X containing x and y respectively.

**Example 4.8:** Let X={a,b,c,d} and $\tau_1$={Φ,{a},{a,b},{c,d},{a,c,d},X}, and $\tau_2$={Φ,{a},{c,d},{a,c,d},X}, then X is (1,2)*-πgb-$D_i$, i=0,1,2.

**Remark 4.9:**
(i)If X is (1,2)*-πgb-$T_i$, then Xis (1,2)*-πgb-$D_i$;i=0,1,2.
(ii) If Xis (1,2)*-πgb-$D_i$,then it is (1,2)*-πgb-$D_{i-1}$;i=1,2.

**Theorem4.10**: For a bitopological space X, the following statements hold.
(i) X is (1,2)*-πgb-$D_0$iff it is (1,2)*-πgb-$T_0$
(ii) X is (1,2)*-πgb-$D_1$iff it is (1,2)*-πgb-$D_2$.

**Proof: (1)**The sufficiency is stated in remark 4.9 (i)

Let Xbe (1,2)*-πgb-$D_0$.Then for any two distinct points x, y ∈X, atleast one of x,y say x belongs to (1,2)*-πgb-D-set G where y∉G. Let G=$U_1$-$U_2$ where $U_1$≠X and $U_1$ and $U_2$∈ (1,2)*-πGBO(X).Then x∈$U_1$.For y∉G we have two cases.(a)y ∉ U1 (b)y∈$U_1$ and y ∈ $U_2$.

In case (a), x∈$U_1$ but y∉$U_1$;In case (b); y ∈ $U_2$ and x∉$U_2$.Hence X is (1,2)*-πgb-T0.

(2) Sufficiency: Remark 4.9 (ii).

**Necessity:** Suppose X is (1,2)*-πgb-$D_1$.Then for each distinct pair x,y∈X ,we have (1,2)*-πgb-D-sets $G_1$ and $G_2$ such that x∈$G_1$ and y∉$G_1$; x∉$G_2$ and y∈$G_2$. Let $G_1$ =$U_1$-$U_2$ and $G_2$ =$U_3$ –$U_4$. By x∉$G_2$, it follows that either x∉U3 or x∈ $U_3$ and x ∈ $U_4$

Now we have two cases(i)x ∉U3.By y ∉G, we have two subcases (a)y ∉ $U_1$.By x ∈ $U_1$-$U_2$,it follows that x ∈$U_1$-($U_2$∪ $U_3$) and by y ∈ $U_3$-$U_4$,we have y ∈ $U_3$-($U_1$ ∪$U_4$).Hence ($U_1$-($U_3$∪ $U_4$)) ∩ $U_3$-($U_1$∪ $U_4$)= Φ.(b)y∈U1 and y∈$U_2$ ,we have x∈ $U_1$-$U_2$ ;y∈U2.⇒($U_1$-$U_2$)∩$U_2$=Φ..

(ii)x∈$U_3$ and x∈$U_4$.We have y ∈$U_3$-$U_4$; x∈$U_4$⇒($U_3$-$U_4$) ∩$U_4$=Φ. We get $U_1$-$U_2$ and $U_2$ are disjoint(1,2)*-πgb-D sets containing x and y respectively Thus X is (1,2)*-πgb-$D_2$.

**Theorem4.11:** If X is (1,2)*-πgb-$D_1$, then it is (1,2)*-πgb-$T_0$.

**Proof:** Remark 4.9 and theorem 4.10.

**Theorem 4.12:** If f : X→Y is a (1,2)*-πgb-continuous surjective function and S is a (1,2)*-D-set of Y,then the inverse image of S is a (1,2)*-πgb-D-set of X.

**Proof:** Let $U_1$ and $U_2$ be two open sets of Y.Let S=$U_1$-$U_2$ be a (1,2)*-D-set and $U_1$≠Y.We have $f^{-1}(U_1)$ ∈ (1,2)*-πGBO(X) and $f^{-1}(U_2)$ ∈ (1,2)*-πGBO(X)and $f^{-1}(U_1)$ ≠X.Hence$f^{-1}(S)=f^{-1}(U_1-U_2)=f^{-1}(U_1)-f^{-1}(U_2)$.Hence $f^{-1}(S)$ is a (1,2)*-πgb-D-set.

**Theorem 4.13 J:** If f: X→ Y is a (1,2)*-πgb-irresolute surjection and E is a (1,2)*-πgb-D-set in Y,then the inverse image of E is an (1,2)*-πgb-D-set in X.

Proof: Let E be a (1,2)*-πgb-D-set in Y.Then there are (1,2)*-πgb-open sets $U_1$ and $U_2$ in Y such that E= $U_1$-$U_2$ and $\cup_1$≠Y.Since f is (1,2)*-πgb-irresolute,$f^{-1}(U_1)$ and $f^{-1}(U_2)$ (1,2)*-πgb-open in X.Since $U_1$ ≠Y,we have $f^{-1}(U_1)$ ≠X. Hence $f^{-1}(E) = f^{-1}(U_1-U_2) = f^{-1}(U_1)- f^{-1}(U_2)$ is a (1,2)*-πgb-D-set.

**Definition 4.14:** A point x∈X which has X as a (1,2)*-πgb-neighbourhood is called (1,2)*-πgb-neat point.

**Example4.15:**LetX={a,b,c}.$\tau_1$={Φ,{a},{b},{a,b},X}.$\tau_2$={Φ,{a} ,{a,b},X}.
(1,2)*πgbO(X)={Φ,{a},{b},{a,b},{b,c},{a,c},X}.The point {c} is a (1,2)*-πgb-neat point.

**Theorem 4.16:** For a (1,2)*-πgb-$T_0$ bitopological spaceX, the following are equivalent.
(i) X is a (1,2)*-πgb-$D_1$
(ii)X has no (1,2)*-πgb-neat point.

**Proof:** (i) ⇒ (ii).Since X is a (1,2)*-πgb-$D_1$,then each point x of X is contained in an (1,2)*-πgb-D-set O=U-V and hence in U.By definition, U≠X.This implies x is not a (1,2)*-πgb-neat point.

(ii)⇒(i) If X is (1,2)*-πgb-$T_0$,then for each distinct points x, y∈X, atleast one of them say(x) has a (1,2)*-πgb-neighbourhood U containing x and not y. Thus U≠X is a (1,2)*-πgb-D-set. If X has no (1,2)*-πgb-neat point, then y is not a (1,2)*-πgb-neat point. That is there exists (1,2)*-πgb-neighbourhood V of y such that V ≠X. Thus y ∈ (V-U) but not x and V-U is a (1,2)*-πgb-D-set. Hence X is (1,2)*-πgb-$D_1$.

**Remark 4.17:** It is clear that an (1,2)*-πgb-$T_0$bitopological space X is not a (1,2)*-πgb-$D_1$iff there is a (1,2)*-πgb-neat point in X. It is unique because x and y are both (1,2)*-πgb-neat point in X,then atleast one of them say x has an (1,2)*-πgb-neighbourhood U containing x but not y.This is a contradiction since U ≠X.

**Theorem 4.18:** If Yis a (1,2)*-$D_1$ space and f:X→ Y is a (1,2)*-πgb-continuous bijective function,then Xis a (1,2)*-πgb-$D_1$-space.

**Proof:** Suppose Y is a (1,2)*-$D_1$space.Let x and y be any pair of distinct points in X,Since f is injective and Y is a (1,2)*-$D_1$space,yhen there exists (1,2)*-D-sets $S_x$ and $S_y$ of Y containing f(x) and f(y) respectively that f(x) ∉$S_y$ and f(y) ∉$S_x$. By theorem 4.1 $f^{-1}(S_x)$ and $f^{-1}(S_y)$ are (1,2)*-πgb-D-sets in X containing x and y respectively such that x∉$f^{-1}(S_y)$ and y∉$f^{-1}(S_x)$.Hence X is a (1,2)*-πgb-$D_1$-space.

**Theorem 4.19:** If Y is (1,2)*-πgb-$D_1$ andf: X→ Y is (1,2)*-πgb-irresolute and bijective, then X is (1,2)*-πgb-$D_1$.

**Proof:** Suppose Y is (1,2)*-πgb-$D_1$ and f is bijective, (1,2)*-πgb-irresolute. Let x,y be any pair of distinct points of X. Since f is injective and Y is (1,2)*-πgb-$D_1$,there exists (1,2)*-πgb-D-sets $G_x$ and $G_y$of Y containing f(x) and f(y) respectively such that f(y) $\notin G_z$ and f(x) $\notin G_y$. By theorem 4.9 ,$f^{-1}(G_z)$ and $f^{-1}(G_y)$ are (1,2)*-πgb-D-sets in X containing x and y respectively. Hence X is (1,2)*-πgb-$D_1$.

**Theorem 4.20:** A topological space Xis a (1,2)*-πgb-$D_1$ if for each pair of distinct points x,y ∈ X, there exists a (1,2)*-πgb-continuous surjective function f: X → Y where Y is a (1,2)*-$D_1$ space such that f(x) and f(y) are distinct.

**Proof:** Let x and y be any pair of distinct points in X,By hypothesis, there exists a (1,2)*-πgb-continuous surjective function f of a space X onto a (1,2)*-$D_1$-space Y such that f(x)≠f(y).Hence there exists disjoint (1,2)*-D-sets $S_x$and$S_y$ in Y such that f(x) $\in S_x$ and f(y) $\in S_y$. Since f is (1,2)*-πgb-continuous and surjective, by theorem 4.8, $f^{-1}(S_x)$ and $f^{-1}(S_y)$are disjoint (1,2)*-πgb-D-sets in X containing x and y respectively. HenceX is a (1,2)*-πgb-$D_1$-set.

**Theorem 4.21:** X is (1,2)*-πgb-$D_1$ iff for each pair of distinct points x, y∈X ,there exists a (1,2)*-πgb-irresolute surjective function f: X → Y,where Y is (1,2)*-πgb-$D_1$ space such that f(x) and f(y) are distinct.

**Proof: Necessity:** For every pair of distinct points x, y ∈X, it suffices to take the identity function on X.

Sufficiency: Let x ≠y ∈X.By hypothesis ,there exists a (1,2)*-πgb-irresolute, surjective function from X onto a (1,2)*-πgb-$D_1$ space such that f(x) ≠f(y).Hence there exists disjoint (1,2)*-πgb-Dsets $G_x$, $G_y \subset Y$ such that f(x) $\in G_x$ and f(y) $\in G_y$. Since f is (1,2)*-πgb-irresolute and surjective, by theorem 4.2 , $f^{-1}(G_x)$ and $f^{-1}(G_y)$ are disjoint (1,2)*-πgb-D-sets in X containing x and y respectively. Therefore X is (1,2)*-πgb-$D_1$ space.

**Definition 4.22:** A bitopological space X is said to be (1,2)*-πgb-D-connected if Xcannot be expressed as the union of two disjoint non-empty (1,2)*-πgb-D-sets.

**Theorem 4.23:** If X→ Y is (1,2)*-πgb-continuous surjection and X is (1,2)*-πgb-D-connected, then Y is (1,2)*-D-connected.

Proof: Suppose Y is not (1,2)*-D-connected. Let Y=A∪B where A and B are two disjoint non empty (1,2)*-D sets in Y. Since f is (1,2)*-πgb-continuous and onto, X=$f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty (1,2)*-πgb-D-sets in X. This contradicts the fact that X is (1,2)*-πgb-D-connected. Hence Y is (1,2)*-D-connected.

**Theorem 4.24:** If X → Yis (1,2)*-πgb-irresolute surjection and Xis (1,2)*-πgb-D-connected, then Y is (1,2)*-πgb-D-connected.

**Proof:** Suppose Y is not (1,2)*-πgb-D-connected.Let Y=A∪B where A and B are two disjoint non empty (1,2)*-πgb-D-sets in Y.Since f is (1,2)*-πgb-irreesolute and onto,X=$f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty (1,2)*-πgb-D-sets in X.This contradicts the fact that X is (1,2)*-πgb-D-connected. Hence Y is (1,2)*-πgb-D-connected.

**Definition 4.25:** A bitopological space X is said to be (1,2)*-πgb- D-compact if every cover of X by (1,2)*-πgb-D-sets has a finite subcover.

**Theorem 4.26:** If a function f: (X,τ) → (Y,σ) is (1,2)*-πgb-continuous surjection and (X,τ) is (1,2)*-πgb-D-compact then Yis (1,2)*-D-compact.

**Proof:** Let f: X→ Y is (1,2)*-πgb-continuous surjection.Let $\{A_i:i\in \wedge\}$ be a cover of Y by (1,2)*-D-set.Then $\{f^{-1}(A_i):i\in \wedge\}$ is a cover of X by (1,2)*-πgb-D-set.Since X is (1,2)*-πgb-D-compact,every cover of X by (1,2)*-πgb-D set has a finite subcover, say$\{f^{-1}(A_1),f^{-1}(A_2)\ldots.f^{-1}(A_n)\}$. Since f is onto, $\{A_1,A_2,\ldots,A_n)$ is a cover of Y by (1,2)*-D-set has a finite subcover.Therefore Y is (1,2)*-D-compact.

**Theorem 4.27:** If a function f: X→ Y is (1,2)*-πgb-irresolute surjection and Xis (1,2)*-πgb-D-compact then Yis (1,2)*-πgb-D-compact.

Proof:Let f: X→ Y is (1,2)*-πgb-irresolutesurjection.Let $\{A_i:i\in \wedge\}$ be a cover of Y by (1,2)*-πgb-D-set.Hence Y=$\bigcup_i A_i$Then X=$f^{-1}(Y)=f^{-1}(\bigcup_i A_i)= \bigcup_i f^{-1}(A_i)$.Since f is (1,2)*-πgb-irresolute,for each i∈∧,$\{f^{-1}(A_i):i\in \wedge\}$ is a cover of X by (1,2)*-πgb-D-set.Since X is (1,2)*-πgb-D-compact,every cover of X by (1,2)*-πgb-D set has a finite subcover ,say$\{f^{-1}(A_1), f^{-1}(A_2)\ldots. f^{-1}(A_n)\}$.Since f is onto,$\{A_1,A_2,\ldots,A_n)$ is a cover of Y by (1,2)*-πgb-D-set has a finite subcover.Therefore Y is (1,2)*-πgb-D-compact.

5.(1,2)*-πgb-$R_0$ spaces and (1,2)*-πgb-$R_1$ spaces

**Definition 5.1:** Let X be a bitopological space then the (1,2)*-πgb-closure of A denoted by (1,2)*-πgb-cl (A) is defined by (1,2)*-πgb-cl(A) = ∩ { F | F ∈ (1,2)*-πgbC (X, τ ) and F⊃A}.

**Definition 5.2:** Let x be a point of bitopological space X.Then (1,2)*-πgb-Kernel of x is defined and denoted by Ker(1,2)*-πgb{x}=∩{U:U∈(1,2)*-πgbO(X) and x∈U}.

**Definition 5.3:** Let F be a subset of a bitopological space X .Then (1,2)*-πgb-Kernel of F is defined and denoted by Ker(1,2)*-πgb(F)=∩{U:U∈(1,2)*-πgbO(X) and F⊂U}.

**Lemma 5.4:** Let Xbe a bitopological space and x ∈X.Then Ker(1,2)*-πgb(A) = {x ∈X| (1,2)*-πgb-cl({x})∩A ≠Φ}.

**Proof:** Let x ∈Ker(1,2)*-πgb(A) and (1,2)*-πgb-cl({x})∩A = Φ. Hence x ∉ X−(1,2)*-πgb-cl({x}) which is an (1,2)*-πgb-open set containing A. This is impossible, since x ∈Ker(1,2)*-πgb (A).

Consequently, (1,2)*-πgb- cl({x})∩A ≠Φ Let (1,2)*-πgb-cl({x})∩ A ≠Φ and x ∉ Ker (1,2)*-πgb (A). Then there exists an (1,2)*-πgb- open set G containing A and x ∉ G. Let y ∈ (1,2)*-πgb-cl({x})∩A. Hence G is an (1,2)*-πgb- neighbourhood of y where x ∉ G. By this contradiction, x ∈ Ker (1,2)*-πgb(A).

**Lemma 5.5:** Let X be a bitopological space and x ∈ X. Then y ∈Ker(1,2)*-πgb({x}) if and only if x ∈ (1,2)*-πgb-Cl({y}).

Proof: Suppose that y ∉ Ker (1,2)*-πgb({x}). Then there exists an (1,2)*-πgb-open set V containing x such that y ∉V. Therefore we have x ∉ (1,2)*-πgb-cl({y}). Converse part is similar.

**Lemma 5.6:** The following statements are equivalent for any two points x and y in abitopological space X

(1) Ker (1,2)*-πgb ({x}) ≠Ker (1,2)*-πgb({y});

(2) (1,2)*-πgb- cl({x}) ≠ (1,2)*-πgb-cl({y}).

**Proof:** (1) ⇒ (2): Suppose that Ker (1,2)*-πgb({x}) ≠ Ker (1,2)*-πgb({y}) then there exists a point z in X such that z ∈X such that z∈Ker (1,2)*-πgb({x}) and z ∉ Ker (1,2)*-πgb({y}). It follows from z ∈ Ker (1,2)*-πgb({x}) that {x} ∩ (1,2)*-πgb-cl({z}) ≠Φ .This implies that x ∈ (1,2)*-πgb-cl({z}). By z ∉ Ker

(1,2)*-πgb({y}), we have {y}∩(1,2)*-πgb- cl({z}) = Φ. Since x ∈ (1,2)*-πgb-cl({z}) , (1,2)*-πgb-cl({x}) ⊂ (1,2)*-πgb-cl({z}) and {y} ∩ (1,2)*-πgb-cl({z}) = Φ. Therefore, (1,2)*-πgb-cl({x}) ≠ (1,2)*-πgb-cl({y}). Now Ker (1,2)*-πgb({x})≠ Ker (1,2)*-πgb({y}) implies that (1,2)*-πgb-cl({x}) ≠ (1,2)*-πgb-cl({y}).

(2) ⇒ (1): Suppose that (1,2)*-πgb-cl({x}) ≠ (1,2)*-πgb-cl({y}). Then there exists a point z ∈ X such that z ∈ (1,2)*-πgb-cl({x}) and z ∉ (1,2)*-πgb-cl({y}). Then, there exists an (1,2)*-πgb-open set containing z and hence containing x but not y, i.e., y ∉ Ker({x}). Hence Ker({x}) ≠Ker({y}).

**Definition 5.7:** A bitopological space X is said to be (1,2)*-πgb-$R_0$iff (1,2)*-πgb-cl{x} ⊆ G whenever x∈G∈ (1,2)*-πGBO(X).

**Definition 5.8:** A bitopological space X is said to be (1,2)*-πgb-$R_1$ if for any x,y in X with (1,2)*-πgb-cl({x})≠(1,2)*-πgb-cl({y}),there exists disjoint (1,2)*-πgb-open sets U and V such that (1,2)*-πgb-cl({x}) ⊆ U and (1,2)*-πgb-cl({y}) ⊆ V.

**Definition 5.9:** A bitopological space X is said to be (1,2)*-weakly $\pi$ gb-$R_0$ if ∩ X ∈X (1,2)*-πgb-cl({x})= Φ .

**Example5.10:**LetX={a,b,c,d}.$\tau_1$={Φ,{b},{a,b},{b,c},{a,b,c},X},$\tau_2$={Φ,{a,b},X}.(1,2)*-πgbO(X,τ)=P(X),Then X is (1,2)*-πgb-$R_0$, weakly (1,2)*-πgb-$R_0$.

**Remark 5.11:** Every (1,2)*-πgb-$R_1$ space is (1,2)*-πgb-$R_0$ space.

Let U be a (1,2)*-πgb-open set such that x∈U. If y∉U,then since x∉(1,2)*-πgb-cl({y}),(1,2)*-πgb-cl({x})≠(1,2)*-πgb-cl({y}). Hence there exists an (1,2)*-πgb-open set V such that y∈V such that (1,2)*-πgb-cl({y}) ⊂ V and x∉V ⇒ y∉ (1,2)*-πgb-cl({x}).Hence (1,2)*-πgb-cl({x}) ⊆ U. Hence X is (1,2)*-πgb-$R_0$.

**Theorem 5.12 :** X is (1,2)*-πgb-$R_0$ iff given x≠y; (1,2)*-πgb-cl{x}≠ (1,2)*-πgb-cl{y}.

Proof: Let X be (1,2)*-πgb-$R_0$ and let x≠y∈X. Suppose U is a (1,2)*-πgb-open set containing x but not y, then y∈ (1,2)*-πgb-cl{y}⊂X-U and hence x∉ (1,2)*-πgb-cl{y}.Hence (1,2)*-πgb-cl{x}≠ (1,2)*-πgb-cl{y}.

Conversely, let x≠y∈X such that (1,2)*-πgb-cl{x}≠ (1,2)*-πgb-cl{y},.This implies (1,2)*-πgb-cl{x}⊂X- (1,2)*-πgb-cl{y}=U(say),a (1,2)*-πgb-open set in X. This is true for every (1,2)*-πgb-cl{x}.Thus ∩ (1,2)*-πgb-cl{x} ⊆ U where x∈ (1,2)*-πgb-cl{x}⊂U∈ (1,2)*-πGBO(X).This implies ∩ (1,2)*-πgb-cl{x} ⊆ U where x∈ U∈ (1,2)*-πGBO(X).Hence X is (1,2)*-πgb-$R_0$.

**Theorem 5.13 :** The following statements are equivalent

(i)X is (1,2)*-πgb-$R_0$-space

(ii)For each x∈X, (1,2)*-πgb-cl{x}⊂Ker (1,2)*-πgb{x}

(iii)For any (1,2)*-πgb-closed set F and a point x ∉ F, there exists U∈ (1,2)*-πgbO(X) such that x ∉ U and F ⊂ U,

(iv) Each (1,2)*-πgb-closed F can be expressed as F=∩{G:G is (1,2)*-πgb-open and F ⊂ G}

(v) ) Each (1,2)*-πgb-open G can be expressed as G= ∪ {A:A is (1,2)*-πgb-closed and A ⊂ G}

(vi) For each (1,2)*-πgb-closed set,x∉F implies (1,2)*-πgb-cl{x}∩F=Φ.

**Proof:** (i) ⇒ (ii): For any x ∈X, we have Ker (1,2)*-πgb{x}= ∩{U:U∈ (1,2)*-πGBO(X). Since X is (1,2)*-πgb-$R_0$,there exists (1,2)*-πgb-open set containing x contains (1,2)*-πgb-cl{x}.Hence (1,2)*-πgb- cl{x} ⊂ Ker (1,2)*-πgb{x}.

(ii) ⇒ (iii): Let x ∉ F∈ (1,2)*-πGBC(X). Then for any y ∈F, (1,2)*-πgb-cl{y} ⊂ F and so x ∉ (1,2)*-πgb- cl{y} ⇒ y ∉ (1,2)*-πgb- cl{x}.That is there exists $U_y$∈ (1,2)*-πgbO(X) such that y∈$U_y$and x ∉ $U_y$for all y∈F. Let U= ∪ {$U_y$∈ (1,2)*-πGBO(X) such that y∈$U_y$and x ∉ $U_y$}.Then U is (1,2)*-πgb-open such that x ∉ U and F ⊂ U.

(iii) ⇒ (iv): Let F be any (1,2)*-πgb-closed set and N=∩{G:G is (1,2)*-πgb-open and F ⊂ G}.Then F ⊂ N---(1).Let x ∉ F,then by (iii) there exists G∈ (1,2)*-πGBO(X) such that x ∉ G and F ⊂ G, hence x ∉ N which implies x∈N ⇒ x∈F. Hence N ⊂ F.---(2).From (1) and (2),each (1,2)*-πgb-closed F=∩{G:G is (1,2)*-πgb-open and F ⊂ G}.

(iv) ⇒ (v) Obvious.

(v) ⇒ (vi) Let x ∉ F∈ (1,2)*-πgbC(X).Then X-F=G is a (1,2)*-πgb-open set containing x.Then by (v),G can be expressed as the union of (1,2)*-πgb-closed sets A ⊆ G and so there is an M∈ (1,2)*-πgbC(X) such that x∈M ⊂ G and hence (1,2)*-πgb- cl{x} ⊂ G implies (1,2)*-πgb- cl{x}∩F=Φ.

(vi) ⇒ (i) Let x ∈G∈ (1,2)*-πgbO(X).Then x ∉ (X-G) which is (1,2)*-πgb-closed set. By (vi) (1,2)*-πgb- cl{x}.∩(X-G)=Φ. ⇒ (1,2)*-πgb-cl{x} ⊂ G.Thus X is (1,2)*-πgb-$R_0$-space.

**Theorem 5.14 :**A bitopological space X is an (1,2)*-πgb-$R_0$ space if and only if for any x and y in X, (1,2)*-πgb-cl({x})≠ (1,2)*-πgb-cl({y}) implies (1,2)*-πgb-cl({x}) ∩ (1,2)*-πgb-cl({y}) =Φ.

Proof: Necessity. Suppose that X is (1,2)*-πgb-$R_0$ and x, y ∈ X such that (1,2)*-πgb-cl({x}) ≠ (1,2)*-πgb-cl({y}). Then, there exist z ∈ (1,2)*-πgb-cl({x}) such that z∉ (1,2)*-πgb-cl({y}) (or z ∈cl({y})) such that z ∉ (1,2)*-πgb-cl({x}). There exists V ∈ (1,2)*-πGBO(X) such that y ∉ V and z ∈V .Hence x ∈ V . Therefore, we have x ∉ (1,2)*-πgb-cl({y}). Thus x ∈ ( (1,2)*-πgb-cl({y}) $)^c$∈ (1,2)*-πGBO(X ), which implies (1,2)*-πgb-cl({x}) ⊂ ( (1,2)*-πgb-cl({y}) $)^c$and (1,2)*-πgb-cl({x}) ∩ (1,2)*-πgb-cl({y}) =Φ.

Sufficiency. Let V ∈ (1,2)*-πGBO(X) and let x ∈V .To show that (1,2)*-πgb-cl_({x}) ⊂ V. Let y ∉V , i.e., y ∈$V^c$. Then x ≠y and x ∉ (1,2)*-πgb-cl({y}). This showsthat (1,2)*-πgb-cl({x}) ≠(1,2)*-πgb-cl({y}). By assumption, (1,2)*-πgb-cl({x}) ∩ (1,2)*-πgb-cl({y}) =Φ. Hencey ∉ (1,2)*-πgb-cl({x}) and therefore (1,2)*-πgb-cl({x}) ⊂ V .

**Theorem 5.15 :** A bitopological space X is an (1,2)*-πgb- $R_0$ space if and only if for anypoints x and y in X , Ker (1,2)*-πgb({x}) ≠Ker (1,2)*-πgb({y}) implies Ker (1,2)*-πgb({x}) ∩Ker (1,2)*-πgb({y})=Φ.

**Proof.** Suppose that X is an (1,2)*-πgb- $R_0$ space. Thus by Lemma 5.6, for any points x and y in X if Ker (1,2)*-πgb({x})≠Ker (1,2)*-πgb({y}) then (1,2)*-πgb-cl({x})≠ (1,2)*-πgb-cl({y}).Now to prove that Ker (1,2)*-πgb({x})∩Ker (1,2)*-πgb({y}) =Φ. Assume that z ∈Ker (1,2)*-πgb({x})∩Ker (1,2)*-πgb({y}). By z ∈ Ker (1,2)*-πgb({x}) and Lemma 5.5, it follows that x∈ (1,2)*-πgb-cl({z}). Since x∈ (1,2)*-πgb-cl({z}); (1,2)*-πgb-cl({x})= (1,2)*-πgb-cl({z}). Similarly, we have (1,2)*-πgb-cl({y}) = (1,2)*-πgb-cl({z})= (1,2)*-πgb-cl({x}). This is a contradiction. Therefore, we have Ker (1,2)*-πgb({x}) ∩Ker (1,2)*-πgb({y})=Φ

Conversely, let X be a topological space such that for any points x and y inX such that (1,2)*-πgb-cl{x}≠ (1,2)*-πgb-cl{y}.Ker (1,2)*-πgb({x}) ≠Ker (1,2)*-πgb({y}) implies Ker (1,2)*-πgb({x}) ∩Ker (1,2)*-πgb({y})=Φ. Since z∈ (1,2)*-πgb-cl{x} ⇒ x∈Ker (1,2)*-πgb({z}) and therefore Ker (1,2)*-πgb({x}) ∩Ker (1,2)*-πgb({y})≠Φ.By hypothesis,we have Ker (1,2)*-πgb({x})=Ker (1,2)*-πgb({z}). Then z ∈ (1,2)*-πgb-cl({x}) ∩ (1,2)*-πgb-cl({y}) implies that Ker (1,2)*-πgb({x}) = Ker (1,2)*-πgb({z})=Ker (1,2)*-πgb({y}). This is a contradiction. Hence (1,2)*-πgb-cl({x})∩ (1,2)*-πgb-cl({y})=Φ;By theorem 5.14, X is an (1,2)*-πgb-$R_0$ space.

**Theorem 5.16:** For abitopological space X, the following properties are equivalent.

(1) X is an (1,2)*-πgb-$R_0$ space

(2) x∈ (1,2)*-πgb-cl({y}) if and only if y∈ (1,2)*-πgb-cl({x}), for any points x and y in X.

Proof: (1) ⇒ (2): Assume that X is (1,2)*-πgb- $R_0$. Let x ∈ (1,2)*-πgb-cl({y}) and G be any (1,2)*-πgb- open setsuch that y∈G. Now by hypothesis, x∈G. Therefore, every (1,2)*-πgb-openset containing y contains x. Hence y ∈ (1,2)*-πgb- Cl({x}).

(2) ⇒ (1) : Let U be an (1,2)*-πgb- open set and x∈U.If y ∉U, then x ∉ (1,2)*-πgb-cl({y}) and hence y∉ (1,2)*-πgb-cl({x}). This implies that (1,2)*-πgb-cl({x}) ⊂U. Hence Xis (1,2)*-πgb-$R_0$.

**Theorem 5.17:** For a bitopological space X, the following properties are equivalent:

(1) X is an (1,2)*-πgb-$R_0$ space;

(2) (1,2)*-πgb-cl({x}) = Ker (1,2)*-πgb({x}) for all x∈ X.

Proof: (1)⇒(2) : Suppose that X is an (1,2)*-πgb-$R_0$ space. By theorem 5.13, (1,2)*-πgb-cl({x}) ⊂ Ker (1,2)*-πgb({x}) for each x∈X. Let y ∈Ker (1,2)*-πgb({x}), then x∈ (1,2)*-πgb-cl({y})and so (1,2)*-πgb-cl({x}) = (1,2)*-πgb-cl({y}). Therefore, y ∈ (1,2)*-πgb-cl({x}) and hence Ker (1,2)*-πgb({x})⊂(1,2)*-πgb-cl({x}). This shows that (1,2)*-πgb-cl({x}) = Ker (1,2)*-πgb({x}).

(ii) ⇒ (i) Obvious from theorem5.15.

**Theorem 5.18:** For a bitopological space X ,the following are equivalent.

(i) X is a (1,2)*-πgb-$R_0$ space.

(ii) If F is (1,2)*-πgb-closed, then F=Ker (1,2)*-πgb(F).

(iii) If F is (1,2)*-πgb-closed, and x∈F, then Ker({x})⊂F.

(iv) If x∈X, then Ker (1,2)*-πgb({x})⊂ (1,2)*-πgb-cl({x}).

Proof:(i)⇒ (ii) Let F be a (1,2)*-πgb-closed and x∉F. Then X-F is (1,2)*-πgb-open and contains x. Since (X,τ) is a (1,2)*-πgb-$R_0$, (1,2)*-πgb-cl({x})⊆X-F.

Thus (1,2)*-πgb-cl({x})∩F=Φ. And by lemma 5.4, x∉ (1,2)*-πgb-Ker(F).Therefore (1,2)*-πgb-Ker(F)=F.

(ii) ⇒ (iii) If A⊆B, then Ker (1,2)*-πgb(A) ⊆Ker (1,2)*-πgb(B).

From (ii),it follows that Ker (1,2)*-πgb({x})⊆Ker (1,2)*-πgb(F).

(iii)⇒ (iv) Since x∈(1,2)*-πgb-cl({x}) and (1,2)*-πgb-cl({x}) is (1,2)*-πgb-closed. By(iii),Ker (1,2)*-πgb({x})⊂ (1,2)*-πgb-cl({x}).

(iv)⇒ (i)We prove the result using theorem 5.13.Let x ∈ (1,2)*-πgb-cl({y}) and by theorem 5.14,y ∈ Ker (1,2)*-πgb({x}).Since x ∈ (1,2)*-πgb-cl({x}) and (1,2)*-πgb-cl({x}) is (1,2)*-πgb-closed,then by (iv) we get y ∈ Ker (1,2)*-πgb({x})⊆ (1,2)*-πgb-cl({x}),Therefore x ∈ (1,2)*-πgb-cl({y}) ⇒ y ∈ (1,2)*-πgb-cl({x}).

Conversely, let y∈ (1,2)*-πgb-cl({x}).By lemma 5.5, x ∈Ker (1,2)*-πgb({y}).Since y∈ (1,2)*-πgb-cl({y})and (1,2)*-πgb-cl({y}) is (1,2)*-πgb-closed,then by (iv) we get x∈Ker (1,2)*-πgb({y})⊆(1,2)*-πgb-cl({y}). Thus y ∈ (1,2)*-πgb-cl({x})⇒ x∈ (1,2)*-πgb-cl({y}).By theorem 5.14, we prove that X is (1,2)*-πgb-$R_0$ space.

**Theorem 5.19:** A bitopological space X is (1,2)*-πgb-$R_1$ iff for x,y∈X, Ker (1,2)*-πgb({x})≠ (1,2)*-πgb-cl({y}),there exists disjoint (1,2)*-πgb-open sets U and V such that (1,2)*-πgb-cl({x})⊂ U and (1,2)*-πgb-cl({y}) ⊂V.

**Proof:** It follows from lemma 5.5.

**Theorem 5.20:** Abitopological space X is (1,2)*-πgb-$T_2$ if and only if it is(1,2)*-πgb-$T_1$ and(1,2)*-πgb-$R_1$.

**Proof:** If X is (1,2)*-πgb-$T_2$,then it is (1,2)*-πgb-$T_1$.If x,y∈X such that (1,2)*-πgb-cl({x})≠(1,2)*-πgb-cl({y}),then x≠y. Hence there exists disjoint (1,2)*-πgb open sets U and V such that x∈U and y∉U; y∈V and x∉V. This implies (1,2)*-πgb-cl({x})⊆U and (1,2)*-πgb-cl({y})⊆V. Hence X is (1,2)*-πgb-$R_1$.

Converse: If X is (1,2)*-πgb-$T_1$ and (1,2)*-πgb-$R_1$ and x,y∈X such that (1,2)*-πgb-cl({x})≠(1,2)*-πgb-cl({y}),there exist disjoint (1,2)*-πgb open sets U and V such that (1,2)*-πgb-cl({x})⊆U and 1,2)*-πgb-cl({y})⊆V. Since X is (1,2)*-πgb-$T_1$, (1,2)*-πgb-cl({x})={x} and(1,2)*-πgb-cl({y})={y}.This implies x∉U and y∉U, y∈V and x∉V, U∩V= Φ. This implies X is (1,2)*-πgb-$T_0$.

**Theorem 5.21:** A bitopological space X is said to be weakly (1,2)*-πgb-$R_0$ if (1,2)*-$Ker_{\pi gb}$({x})≠X for every x ∈X.

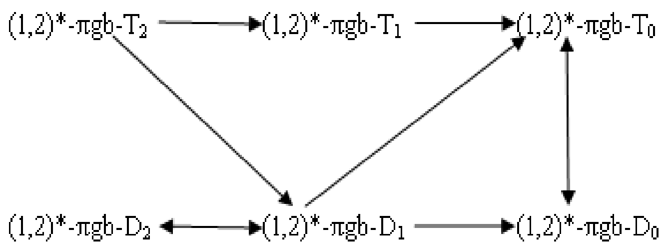
**Proof:** Suppose that the space X is weakly (1,2)*-πgb-$R_0$.Assume that there is a point y in X such that(1,2)*-$Ker_{\pi gb}$({y})=X. Then y∉O where O is some proper (1,2)*-πgb-open subset of X. This implies y∈$\cap_{x\in X}$ (1,2)*-πgb-cl({x}) which is a contradiction.

Conversely, Assume (1,2)*-$Ker_{\pi gb}$({x})≠X for every x∈X. If there is a point y∈X such that y ∈$\cap_{x\in X}$ (1,2)*-πgb-cl({x}),then every (1,2)*-πgb-open set containing y must contain every point of X. This implies the unique (1,2)*-πgb-open set containing yis X. Hence (1,2)*-$Ker_{\pi gb}$({y})=X, which is a contradiction. Thus X is weakly (1,2)*-πgb-$R_0$.

**Example5.22:**LetX={a,b,c,d}.$\tau_1$={Φ,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c},X}and$\tau_2$={Φ,{a},X}.(1,2)*πgbO(X,τ)={Φ,{a},{b},{c},{a,b},{a,c},{b,c},{a,d},{b,d},{c,d},{a,b,c},{b,c,d},{a,c,d},{a,b,d},X, Then X is (1,2)*-πgb-$R_0$ but not ultra-α-$R_0$.

**Conclusion:**

A study on new separation axioms called πgb-separation axioms using the $(1,2)^*$-πgb-open sets in bitopological spaces have been done. Also some results of (1,2)*-πgb-$T_i$, (1,2)*-πgb-$D_i$, where i = 0,1,2, and(1,2)*-πgb-$R_i$,i=0,1 are studied in this paper.

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