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Some properties of Q-Intuitionistic L-Fuzzy subnearrings of a nearring

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ABSTRACT

In this paper, we study some of the properties of Q-intuitionistic L-fuzzy subnearring of a nearring and prove some results on these.

Introduction

After the introduction of fuzzy sets by L.A.Zadeh[15], several researchers explored on the generalization of the notion of fuzzy set. The concept of intuitionistic L-fuzzy subset was introduced by K.T.Atanassov[4,5], as a generalization of the notion of fuzzy set. Azriel Rosenfeld[6] defined a fuzzy groups. Asok Kumer Ray[3] defined a product of fuzzy subgroups and A.Solairaju and R.Nagarajan[13,14] have introduced and defined a new algebraic structure called Q-fuzzy subgroups. We introduce the concept of Q-intuitionistic L-fuzzy subnearring of a nearring and established some results.

1.Preliminaries:

1.1 Definition: Let X be a non-empty set and L = (L, $\leq$) be a lattice with least element 0 and greatest element 1 and Q be a non-empty set. A (Q, L)-fuzzy subset A of X is a function A : XxQ $\rightarrow$ L.

1.2 Definition: Let (L, $\leq$) be a complete lattice with an involutive order reversing operation N : L $\rightarrow$ L and Q be a non-empty set. A Q-intuitionistic L-fuzzy subset (QILFS) A in X is defined as an object of the form A={< (x, q), μ_A(x, q), ν_A(x, q) > / x in X and q in Q }, where μ_A : XxQ $\rightarrow$ L and ν_A : XxQ $\rightarrow$ L define the degree of membership and the degree of non-membership of the element x$\in$X respectively and for every x$\in$X satisfying $\mu_A(x) \leq N(\nu_A(x))$.

1.3 Definition: Let (R, +, .) be a nearring. A Q-intuitionistic L-fuzzy subset A of R is said to be a Q-intuitionistic L-fuzzy subnearring(QILFSNR) of R if it satisfies the following axioms:

(i) $\mu_A(x-y, q) \geq \mu_A(x, q) \wedge \mu_A(y, q)$

(ii) $\mu_A(xy, q) \geq \mu_A(x, q) \wedge \mu_A(y, q)$

(iii) $\nu_A(x-y, q) \leq \nu_A(x, q) \vee \nu_A(y, q)$

(iv) $\nu_A(xy, q) \leq \nu_A(x, q) \vee \nu_A(y, q)$, for all x and y in R and q in Q.

1.4 Definition: Let A and B be any two Q-intuitionistic L-fuzzy subnearrings of nearrings R_1 and R_2 respectively. The product of A and B denoted by AxB is defined as AxB ={ $\langle$ ((x, y), q), μ_{AxB}((x, y), q), ν_{AxB}((x, y), q) $\rangle$ / for all x in R_1 and y in R_2 and q in Q }, where $\mu_{AxB}((x, y), q) = \mu_A(x, q) \wedge \mu_B(y, q)$ and $\nu_{AxB}((x, y), q) = \nu_A(x, q) \vee \nu_B(y, q)$.

1.5 Definition: Let A be a Q-intuitionistic L-fuzzy subset in a set S, the strongest Q-intuitionistic L-fuzzy relation on S, that is a Q-intuitionistic L-fuzzy relation on A is V given by $\mu_V((x, y), q) = \mu_A(x, q) \wedge \mu_A(y, q)$ and $\nu_V((x, y), q) = \nu_A(x, q) \vee \nu_A(y, q)$, for all x and y in S and q in Q.

2. Some properties of q-intuitionistic l-fuzzy subnearrings of a nearring

2.1 Theorem: Intersection of any two Q-intuitionistic L-fuzzy subnearrings of a nearring R is a Q-intuitionistic L-fuzzy subnearring of R.

Proof: Let A and B be any two Q-intuitionistic L-fuzzy subnearrings of a nearring R and x and y in R and q in Q. Let A = { ((x, q), μ_A(x, q), ν_A(x, q)) / x$\in$R and q in Q } and B = { ((x, q), μ_B(x, q), ν_B(x, q)) / x$\in$R and q in Q } and also let C = A$\cap$B = { ((x, q), μ_C(x, q), ν_C(x, q)) / x$\in$R and q in Q }, where $\mu_A(x, q) \wedge \mu_B(x, q) = \mu_C(x, q)$ and $\nu_A(x, q) \vee \nu_B(x, q) = \nu_C(x, q)$. Now, $\mu_C(x-y, q) = \mu_A(x-y, q) \wedge \mu_B(x-y, q) \geq [\mu_A(x, q) \wedge \mu_A(y, q)] \wedge [\mu_B(x, q) \wedge \mu_B(y, q)] = [\mu_A(x, q) \wedge \mu_B(x, q)] \wedge [\mu_A(y, q) \wedge \mu_B(y, q)] = \mu_C(x, q) \wedge \mu_C(y, q)$. Therefore, $\mu_C(x-y, q) \geq \mu_C(x, q) \wedge \mu_C(y, q)$, for all x and y in R and q in Q. And, $\mu_C(xy, q) = \mu_A(xy, q) \wedge \mu_B(xy, q) \geq [\mu_A(x, q) \wedge \mu_A(y, q)] \wedge [\mu_B(x, q) \wedge \mu_B(y, q)] = [\mu_A(x, q) \wedge \mu_B(x, q)] \wedge [\mu_A(y, q) \wedge \mu_B(y, q)] = \mu_C(x, q) \wedge \mu_C(y, q)$. Therefore, $\mu_C(xy, q) \geq \mu_C(x, q) \wedge \mu_C(y, q)$, for all x and y in R and q in Q. Also, $\nu_C(x-y, q) = \nu_A(x-y, q) \vee \nu_B(x-y, q) \leq [\nu_A(x, q) \vee \nu_A(y, q)] \vee [\nu_B(x, q) \vee \nu_B(y, q)]=[\nu_A(x, q)\vee\nu_B(x, q)] \vee [\nu_A(y, q) \vee \nu_B(y, q)] = \nu_C(x, q) \vee \nu_C(y, q)$. Therefore, $\nu_C(x-y, q) \leq \nu_C(x, q) \vee \nu_C(y, q)$, for all x and y in R and q in Q. And, $\nu_C(xy, q) = \nu_A(xy, q) \vee \nu_B(xy, q) \leq [\nu_A(x, q) \vee \nu_A(y, q)] \vee [\nu_B(x, q) \vee \nu_B(y, q)] = [\nu_A(x, q) \vee \nu_B(x, q)] \vee [\nu_A(y, q) \vee \nu_B(y, q)] = \nu_C(x, q) \vee \nu_C(y, q)$.

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Therefore, $\nu_C(xy, q) \le \nu_C(x, q) \vee \nu_C(y, q)$, for all x and y in R and q in Q. Therefore, C is a Q-intuitionistic L-fuzzy subnearring of a nearring R. Hence, intersection of any two Q-intuitionistic L-fuzzy subnearrings of a nearring R is a Q-intuitionistic L-fuzzy subnearring of R.

2.2 Theorem: Let (R, +, .) is a nearring. The intersection of a family of Q-intuitionistic L-fuzzy subnearrings of R is a Q-intuitionistic L-fuzzy subnearring of R.

Proof: Let { $V_i : i \in I$ } be a family of Q-intuitionistic L-fuzzy subnearrings of a nearring R and let A = = $\bigcap_{i\in I} V_i$. Let x and y in R and q in Q. Then, $\mu_A(x - y, q) = \inf_{i\in I} \mu_{V_i}(x - y, q) \ge \inf_{i\in I} [\mu_{V_i}(x, q) \wedge \mu_{V_i}(y, q)] = \inf_{i\in I} \mu_{V_i}(x, q) \wedge \inf_{i\in I} \mu_{V_i}(y, q) = \mu_A(x, q) \wedge \mu_A(y, q)$. Therefore, $\mu_A(x- y, q) \ge \mu_A(x, q) \wedge \mu_A(y, q)$, for all x and y in R and q in Q. And, $\mu_A(xy, q) = \inf_{i\in I} \mu_{V_i}(xy, q) \ge \inf_{i\in I} [\mu_{V_i}(x, q) \wedge \mu_{V_i}(y, q)] = \inf_{i\in I} \mu_{V_i}(x, q) \wedge \inf_{i\in I} \mu_{V_i}(y, q) = \mu_A(x, q) \wedge \mu_A(y, q)$. Therefore, $\mu_A(xy, q) \ge \mu_A(x, q) \wedge \mu_A(y, q)$, for all x and y in R and q in Q. Also, $\nu_A(x - y, q) = \sup_{i\in I} \nu_{V_i}(x-y, q) \le \sup_{i\in I} [\nu_{V_i}(x, q) \vee \nu_{V_i}(y, q)] = \sup_{i\in I} \nu_{V_i}(x, q) \vee \sup_{i\in I} \nu_{V_i}(y,q) = \nu_A(x, q) \vee \nu_A(y, q)$. Therefore, $\nu_A(x -y, q) \le \nu_A(x, q) \vee \nu_A(y, q)$, for all x and y in R and q in Q. And, $\nu_A(xy, q) = \sup_{i\in I} \nu_{V_i}(xy, q) \le \sup_{i\in I} [\nu_{V_i}(x, q) \vee \nu_{V_i}(y, q)] = \sup_{i\in I} \nu_{V_i}(x, q) \vee \sup_{i\in I} \nu_{V_i}(y, q) = \nu_A(x, q) \vee \nu_A(y, q)$. Therefore, $\nu_A(xy, q) \le \nu_A(x, q) \vee \nu_A(y, q)$, for all x and y in R and q in Q. That is, A is a Q-intuitionistic L-fuzzy subnearring of a nearring R. Hence, the intersection of a family of Q-intuitionistic L-fuzzy subnearrings of R is a Q-intuitionistic L-fuzzy subnearring of R.

2.3 Theorem: If A and B are any two Q-intuitionistic L-fuzzy subnearrings of the nearrings R_1 and R_2 respectively, then AxB is a Q-intuitionistic L-fuzzy subnearring of R_1xR_2.

Proof: Let A and B be two Q-intuitionistic L-fuzzy subnearrings of the nearrings R_1 and R_2 respectively. Let x_1 and x_2 be in R_1, y_1 and y_2 be in R_2. Then (x_1, y_1) and (x_2, y_2) in R_1xR_2 and q in Q. Now, $\mu_{AxB} [(x_1, y_1) - (x_2, y_2), q] = \mu_{AxB} ((x_1- x_2, y_1- y_2), q) = \mu_A(x_1- x_2, q) \wedge \mu_B(y_1- y_2, q) \ge [\mu_A(x_1, q) \wedge \mu_A(x_2, q)] \wedge [\mu_B(y_1, q) \wedge \mu_B(y_2, q)] = [\mu_A(x_1, q) \wedge \mu_B(y_1, q)] \wedge [\mu_A(x_2, q) \wedge \mu_B(y_2, q)] = \mu_{AxB} ((x_1, y_1), q) \wedge \mu_{AxB} ((x_2, y_2), q)$. Therefore, $\mu_{AxB} [(x_1, y_1) - (x_2, y_2), q] \ge \mu_{AxB} ((x_1, y_1), q) \wedge \mu_{AxB} ((x_2, y_2), q)$, for all (x_1, y_1) and (x_2, y_2) in R_1xR_2 and q in Q. Also, $\mu_{AxB} [(x_1, y_1)(x_2, y_2), q] = \mu_{AxB} ((x_1x_2, y_1y_2), q) = \mu_A(x_1x_2, q) \wedge \mu_B(y_1y_2, q) \ge [\mu_A(x_1, q) \wedge \mu_A(x_2, q)] \wedge [\mu_B(y_1, q) \wedge \mu_B(y_2, q)] = [\mu_A(x_1, q) \wedge \mu_B(y_1, q)] \wedge [\mu_A(x_2, q) \wedge \mu_B(y_2, q)] = \mu_{AxB}((x_1, y_1), q) \wedge \mu_{AxB}((x_2, y_2), q)$. Therefore, $\mu_{AxB}[(x_1, y_1)(x_2, y_2), q] \ge \mu_{AxB}((x_1, y_1), q) \wedge \mu_{AxB} ((x_2, y_2), q)$, for all (x_1, y_1) and (x_2, y_2) in R_1xR_2 and q in Q. And, $\nu_{AxB}[(x_1, y_1)-(x_2, y_2), q] = \nu_{AxB}((x_1- x_2, y_1- y_2), q) = \nu_A(x_1- x_2, q) \vee \nu_B(y_1- y_2, q) \le [\nu_A(x_1, q) \vee \nu_A(x_2, q)] \vee [\nu_B(y_1, q) \vee \nu_B(y_2, q)] = [\nu_A(x_1, q) \vee \nu_B(y_1, q)] \vee [\nu_A(x_2, q) \vee \nu_B(y_2, q)] = \nu_{AxB} ((x_1, y_1), q) \vee \nu_{AxB} ((x_2, y_2), q)$. Therefore, $\nu_{AxB} [(x_1, y_1) - (x_2, y_2), q] \le \nu_{AxB} ((x_1, y_1), q) \vee \nu_{AxB} ((x_2, y_2), q)$, for all (x_1, y_1) and (x_2, y_2) in R_1xR_2 and q in Q. Also, $\nu_{AxB} [(x_1, y_1)(x_2, y_2), q] = \nu_{AxB} ((x_1x_2, y_1y_2), q) = \nu_A(x_1x_2, q) \vee \nu_B(y_1y_2, q) \le [\nu_A(x_1, q) \vee \nu_A(x_2, q)] \vee [\nu_B(y_1, q) \vee \nu_B(y_2, q)] = [\nu_A(x_1, q) \vee \nu_B(y_1, q)] \vee [\nu_A(x_2, q) \vee \nu_B(y_2, q)] = \nu_{AxB} ((x_1, y_1), q) \vee \nu_{AxB} ((x_2, y_2), q)$. Therefore, $\nu_{AxB} [(x_1, y_1)(x_2, y_2), q] \le \nu_{AxB} ((x_1, y_1), q) \vee \nu_{AxB} ((x_2, y_2), q)$, for all (x_1, y_1) and (x_2, y_2) in R_1xR_2 and q in Q. Hence AxB is a Q-intuitionistic L-fuzzy subnearring of R_1xR_2.

2.4 Theorem: Let A and B be Q-intuitionistic L-fuzzy subnearrings of the nearrings R_1 and R_2 respectively. Suppose that e and e' are the identity element of R_1 and R_2 respectively. If AxB is a Q-intuitionistic L-fuzzy subnearring of R_1xR_2, then at least one of the following two statements must hold.

(i) $\mu_B(e', q) \ge \mu_A(x, q)$ and $\nu_B(e', q) \le \nu_A(x, q)$, for all x in R_1 and q in Q,

(ii) $\mu_A(e, q) \ge \mu_B(y, q)$ and $\nu_A(e, q) \le \nu_B(y, q)$, for all y in R_2 and q in Q.

Proof: Let AxB be a Q-intuitionistic L-fuzzy subnearring of R_1xR_2. By contraposition, suppose that none of the statements (i) and (ii) holds. Then we can find a in R_1 and b in R_2 such that $\mu_A(a, q) > \mu_B(e', q)$, $\nu_A(a, q) < \nu_B(e', q)$ and $\mu_B(b, q) > \mu_A(e, q)$, $\nu_B(b, q) < \nu_A(e, q)$. We have, $\mu_{AxB} ((a, b), q) = \mu_A(a, q) \wedge \mu_B(b, q) > \mu_B(e', q) \wedge \mu_A(e, q) = \mu_A(e, q) \wedge \mu_B(e', q) = \mu_{AxB} ((e, e'), q)$. And, $\nu_{AxB} ((a, b), q) = \nu_A(a, q) \vee \nu_B(b, q) < \nu_B(e', q) \vee \nu_A(e, q) = \nu_A(e, q) \vee \nu_B(e', q) = \nu_{AxB} ((e, e'), q)$. Thus AxB is not a Q-intuitionistic L-fuzzy subnearring of R_1xR_2. Hence either $\mu_B(e', q) \ge \mu_A(x, q)$ and $\nu_B(e', q) \le \nu_A(x, q)$, for all x in R_1 and q in Q or $\mu_A(e, q) \ge \mu_B(y, q)$ and $\nu_A(e, q) \le \nu_B(y, q)$, for all y in R_2 and q in Q.

2.5 Theorem: Let A and B be two Q-intuitionistic L-fuzzy subsets of the nearrings R_1 and R_2 respectively and AxB is a Q-intuitionistic L-fuzzy subnearring of R_1xR_2. Then the following are true :

(i) if $\mu_A(x, q) \le \mu_B(e', q)$ and $\nu_A(x, q) \ge \nu_B(e', q)$, then A is a Q-intuitionistic L-fuzzy subnearring of R_1.

(ii) if $\mu_B(x, q) \le \mu_A(e, q)$ and $\nu_B(x, q) \ge \nu_A(e, q)$, then B is a Q-intuitionistic L-fuzzy subnearring of R_2.

(iii) either A is a Q-intuitionistic L-fuzzy subnearring of R_1 or B is a Q-intuitionistic L-fuzzy subnearring of R_2.

Proof: Let AxB be a Q-intuitionistic L-fuzzy subnearring of R_1xR_2, x and y in R_1 and e' in R_2. Then (x, e') and (y, e') are in R_1xR_2. Now, using the property that $\mu_A(x, q) \le \mu_B(e', q)$ and $\nu_A(x, q) \ge \nu_B(e', q)$, for all x in R_1 and q in Q, we get, $\mu_A(x-y, q) = \mu_A(x-y, q) \wedge \mu_B(e' + e', q) = \mu_{AxB} [((x - y), (e' + e')), q] = \mu_{AxB} [(x, e')+(-y, e'), q] \ge \mu_{AxB}((x, e'), q) \wedge \mu_{AxB}((-y, e'), q) = [\mu_A(x, q) \wedge \mu_B(e', q)] \wedge [\mu_A(-y, q) \wedge \mu_B(e', q)] = \mu_A(x, q) \wedge \mu_A(-y, q) \ge \mu_A(x, q) \wedge \mu_A(y, q)$. Therefore, $\mu_A(x-y, q) \ge \mu_A(x, q) \wedge \mu_A(y, q)$, for all x and y in R_1 and q in Q. Also, $\mu_A(xy, q) = \mu_A(xy, q) \wedge \mu_B(e'e', q) = \mu_{AxB} [((xy), (e'e')), q] = \mu_{AxB} [(x, e') (y, e'), q] \ge \mu_{AxB}((x, e'), q) \wedge \mu_{AxB}((y, e'), q) = [\mu_A(x, q) \wedge \mu_B(e', q)] \wedge [\mu_A(y, q) \wedge \mu_B(e', q)] = \mu_A(x, q) \wedge \mu_A(y, q)$. Therefore, $\mu_A(xy, q) \ge \mu_A(x, q) \wedge \mu_A(y, q)$, for all x and y in R_1 and q in Q. And, $\nu_A(x-y, q) = \nu_A(x-y, q) \vee \nu_B(e' + e', q) = \nu_{AxB} [((x-y), (e' + e')), q] = \nu_{AxB} [(x, e')+ (-y, e'), q] \le \nu_{AxB}((x, e'), q) \vee \nu_{AxB}((-y, e'), q) = [\nu_A(x, q) \vee \nu_B(e', q)] \vee [\nu_A(-y, q) \vee \nu_B(e', q)] = \nu_A(x, q) \vee \nu_A(-y, q) \le \nu_A(x, q) \vee \nu_A(y, q)$. Therefore, $\nu_A(x-y, q) \le \nu_A(x, q) \vee \nu_A(y, q)$, for all x and y in R_1 and q in Q. Also, $\nu_A(xy, q) = \nu_A(xy, q) \vee \nu_B(e'e', q) = \nu_{AxB}[((xy), (e'e')), q] = \nu_{AxB}[(x, e')(y, e'), q] \le \nu_{AxB}((x, e'), q) \vee \nu_{AxB}((y, e'), q) = [\nu_A(x, q) \vee \nu_B(e', q)] \vee [\nu_A(y, q) \vee \nu_B(e', q)] = \nu_A(x, q) \vee \nu_A(y, q)$. Therefore, $\nu_A(xy, q) \le \nu_A(x, q) \vee \nu_A(y, q)$, for all x and y in R_1 and q in Q. Hence A is a Q-intuitionistic L-fuzzy subnearring of R_1. Thus (i) is proved.

Now, using the property that $\mu_B(x, q) \le \mu_A(e, q)$ and $\nu_B(x, q) \ge \nu_A(e, q)$, for all x in R_2 and q in Q. Let x and y in R_2 and e in R_1.

Then (e, x) and (e, y) are in R_1xR_2. We get, $\mu_B(x - y, q) = \mu_B(x - y, q) \wedge \mu_A(e + e, q) = \mu_A(e + e, q) \wedge \mu_B(x - y, q) = \mu_{AxB}[((e+e), (x-y)), q]= \mu_{AxB}[(e, x) + (e, - y), q] \geq \mu_{AxB}((e, x), q) \wedge \mu_{AxB}((e, -y), q) = [\mu_A(e, q) \wedge \mu_B(x, q)] \wedge [\mu_A(e, q) \wedge \mu_B(-y, q)] = \mu_B(x, q)\wedge\mu_B(-y, q) \geq \mu_B(x, q)\wedge\mu_B(y, q)$.

Therefore, $\mu_B(x-y, q) \geq \mu_B(x, q) \wedge \mu_B(y, q)$, for all x and y in R_2 and q in Q. Also, $\mu_B(xy, q)= \mu_B(xy, q)\wedge \mu_A(ee, q) = \mu_A(ee, q)\wedge \mu_B(xy, q) = \mu_{AxB} [((ee), (xy)), q] = \mu_{AxB} [(e, x)(e, y), q] \geq \mu_{AxB}((e, x), q)\wedge\mu_{AxB}((e, y), q) = [\mu_A(e, q)\wedge\mu_B(x, q)]\wedge[\mu_A(e, q) \wedge \mu_B(y, q)] = \mu_B(x, q)\wedge\mu_B(y, q)$. Therefore, $\mu_B(xy, q) \geq \mu_B(x, q) \wedge \mu_B(y, q)$, for all x and y in R_2 and q in Q. And, $\nu_B(x-y, q) = \nu_B(x-y, q) \vee \nu_A(e+e, q) = \nu_A(e+e, q) \vee\nu_B(x-y, q) = \nu_{AxB} [((e+e), (x -y)), q] = \nu_{AxB} [(e, x)+(e, -y), q]\leq \nu_{AxB}((e, x), q) \vee\nu_{AxB}((e, -y), q) = [\nu_A(e, q) \vee \nu_B(x, q)] \vee [\nu_A(e, q) \vee \nu_B(-y, q)] = \nu_B(x, q) \vee \nu_B(-y, q) \leq \nu_B(x, q) \vee \nu_B(y, q)$. Therefore, $\nu_B(x-y, q) \leq \nu_B(x, q) \vee \nu_B(y, q)$, for all x and y in R_2 and q in Q. Also, $\nu_B(xy, q) = \nu_B(xy, q) \vee \nu_A(ee, q) = \nu_A(ee, q) \vee \nu_B(xy, q) = \nu_{AxB} [((ee), (xy)), q] = \nu_{AxB} [(e, x)(e, y), q] \leq \nu_{AxB}((e, x), q) \vee \nu_{AxB}((e, y), q)=[\nu_A(e, q)\vee\nu_B(x, q)]\vee[\nu_A(e, q) \vee \nu_B(y, q)] = \nu_B(x, q) \vee \nu_B(y, q)$. Therefore, $\nu_B(xy, q) \leq \nu_B(x, q) \vee \nu_B(y, q)$, for all x and y in R_2 and q in Q. Hence B is a Q-intuitionistic L-fuzzy subnearring of a nearring R_2. Thus (ii) is proved. (iii) is clear.

2.6 Theorem: Let A be a Q-intuitionistic L-fuzzy subset of a nearring R and V be the strongest Q-intuitionistic L-fuzzy relation of R. Then A is a Q-intuitionistic L-fuzzy subnearring of R if and only if V is a Q-intuitionistic L-fuzzy subnearring of RxR.

Proof: Suppose that A is a Q-intuitionistic L-fuzzy subnearring of a nearring R. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in RxR and q in Q. We have, $\mu_V (x-y, q) = \mu_V[(x_1, x_2) - (y_1, y_2), q] = \mu_V[(x_1-y_1 , x_2-y_2), q] = \mu_A((x_1-y_1), q) \wedge \mu_A((x_2-y_2), q) \geq [\mu_A(x_1, q) \wedge \mu_A(y_1, q)] \wedge [\mu_A(x_2, q) \wedge \mu_A(y_2, q)] = [\mu_A(x_1, q) \wedge \mu_A(x_2, q)] \wedge [\mu_A(y_1, q) \wedge \mu_A(y_2, q)] = \mu_V ((x_1, x_2), q) \wedge \mu_V ((y_1, y_2), q) = \mu_V (x, q)\wedge\mu_V (y, q)$. Therefore, $\mu_V (x-y, q) \geq \mu_V (x, q) \wedge \mu_V (y, q)$, for all x and y in RxR and q in Q. And, $\mu_V (xy, q) = \mu_V [(x_1, x_2) (y_1, y_2), q] = \mu_V[(x_1y_1, x_2y_2), q] = \mu_A(x_1y_1, q)\wedge \mu_A(x_2y_2, q) \geq [\mu_A(x_1, q) \wedge \mu_A(y_1, q)] \wedge [\mu_A(x_2, q) \wedge\mu_A(y_2, q)] = [\mu_A(x_1, q) \wedge \mu_A(x_2, q)] \wedge [\mu_A(y_1, q) \wedge \mu_A(y_2, q)] = \mu_V((x_1, x_2), q) \wedge \mu_V((y_1, y_2), q) = \mu_V (x, q) \wedge \mu_V (y, q)$. Therefore, $\mu_V (xy, q) \geq \mu_V (x, q) \wedge \mu_V (y, q)$, for all x and y in RxR and q in Q. Also we have, $\nu_V (x-y, q) = \nu_V [(x_1, x_2) -(y_1, y_2), q] = \nu_V[(x_1-y_1 , x_2-y_2), q] = \nu_A(x_1-y_1, q) \vee \nu_A(x_2-y_2, q) \leq [\nu_A(x_1, q)\vee\nu_A(y_1, q)] \vee[\nu_A(x_2, q) \vee \nu_A(y_2, q)] = [\nu_A(x_1, q) \vee \nu_A(x_2, q)]\vee[\nu_A(y_1, q) \vee \nu_A(y_2, q)] = \nu_V ((x_1, x_2), q) \vee \nu_V ((y_1, y_2), q) = \nu_V (x, q) \vee \nu_V (y, q)$. Therefore, $\nu_V (x-y, q) \leq \nu_V (x, q) \vee \nu_V (y, q)$, for all x and y in RxR and q in Q. And, $\nu_V (xy, q) = \nu_V[(x_1, x_2) (y_1, y_2), q] = \nu_V((x_1y_1 , x_2y_2), q) = \nu_A(x_1y_1, q) \vee\nu_A(x_2y_2, q) \leq [\nu_A(x_1, q) \vee \nu_A(y_1, q)] \vee [\nu_A(x_2, q) \vee \nu_A(y_2, q)] = [\nu_A(x_1, q) \vee \nu_A(x_2, q)] \vee [\nu_A(y_1, q) \vee \nu_A(y_2, q)] = \nu_V ((x_1, x_2), q)\vee\nu_V ((y_1, y_2), q) = \nu_V (x, q) \vee \nu_V (y, q)$. Therefore, $\nu_V (xy, q) \leq \nu_V (x, q) \vee \nu_V(y, q)$, for all x and y in RxR and q in Q. This proves that V is a Q-intuitionistic L-fuzzy subnearring of RxR. Conversely assume that V is a Q-intuitionistic L-fuzzy subnearring of RxR, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in RxR and q in Q, we have $\mu_A(x_1- y_1, q) \wedge \mu_A(x_2- y_2, q)= \mu_V((x_1- y_1 , x_2- y_2), q) = \mu_V [(x_1, x_2) -(y_1, y_2), q] = \mu_V (x - y, q) \geq \mu_V (x, q) \wedge \mu_V (y, q) = \mu_V ((x_1, x_2), q) \wedge \mu_V ((y_1, y_2), q) = [\mu_A(x_1, q) \wedge \mu_A(x_2, q)] \wedge [\mu_A(y_1, q) \wedge \mu_A(y_2, q)]$. If we put $x_2 = y_2 = 0$, we get, $\mu_A(x_1- y_1, q) \geq \mu_A(x_1, q) \wedge \mu_A(y_1, q)$, for all x_1 and y_1 in R and q in Q. And, $\mu_A(x_1y_1, q) \wedge \mu_A(x_2y_2, q) = \mu_V((x_1y_1 , x_2y_2), q) = \mu_V[(x_1, x_2) (y_1, y_2), q] = \mu_V (xy, q)\geq \mu_V (x, q) \wedge \mu_V(y, q) = \mu_V((x_1, x_2), q) \wedge\mu_V ((y_1, y_2), q)= [\mu_A(x_1, q)\wedge\mu_A(x_2, q)]\wedge[\mu_A(y_1, q)\wedge \mu_A(y_2, q)]$. If we put $x_2 = y_2 = 0$, we get, $\mu_A(x_1y_1, q) \geq \mu_A(x_1, q) \wedge \mu_A(y_1, q)$, for all x_1 and y_1 in R and q in Q. Also we have, $\nu_A(x_1-y_1, q)\vee\nu_A(x_2- y_2, q) = \nu_V((x_1-y_1 , x_2- y_2), q) = \nu_V [(x_1, x_2) - (y_1, y_2), q] = \nu_V (x - y, q) \leq\nu_V (x, q)\vee\nu_V (y, q) = \nu_V ((x_1, x_2), q)\vee\nu_V ((y_1, y_2), q) = [\nu_A(x_1, q)\vee\nu_A(x_2, q)] \vee [\nu_A(y_1, q) \vee \nu_A(y_2, q)]$. If we put $x_2 = y_2 = 0$, we get, $\nu_A(x_1- y_1, q) \leq \nu_A(x_1, q)\vee\nu_A(y_1, q)$, for all x_1 and y_1 in R and q in Q. And, $\nu_A(x_1y_1, q) \vee \nu_A(x_2y_2, q) = \nu_V((x_1y_1 , x_2y_2), q) = \nu_V [(x_1, x_2) (y_1, y_2), q] = \nu_V (xy, q) \leq \nu_V (x, q) \vee \nu_V (y, q) = \nu_V ((x_1, x_2), q) \vee \nu_V ((y_1, y_2), q) = [\nu_A(x_1, q) \vee \nu_A(x_2, q)] \vee [\nu_A(y_1, q) \vee \nu_A(y_2, q)]$. If we put $x_2 = y_2 = 0$, we get, $\nu_A(x_1y_1, q) \leq \nu_A(x_1, q) \vee \nu_A(y_1, q)$, for all x_1 and y_1 in R and q in Q. Hence A is a A is a Q-intuitionistic L-fuzzy subnearring of a nearring R.

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