Available online at www.elixirpublishers.com (Elixir International Journal)

Chemical Engineering

Elixir Chem. Engg. 52 (2012) 11478-11482

ISSN: 2229-712X

Controllability studies on unstable SOPTD systems with a zero

D. Krishna*, A.Aaron, K.Satyakala and Bh.V.Swetha

Department of Chemical Engineering, M.V.G.R. College of Engineering, Vizianagaram, India-535005.

ARTICLE INFO

Article history:

Received: 28 September 2012;

Received in revised form:

11 November 2012;

Accepted: 15 November 2012;

Keywords

PID controller,

Proposed method,

IMC method,

Synthesis method,

SOPTD systems with a zero.

Abstract

Design of proportional integral and derivative (PID) controllers for unstable SOPTDZ (Second Order Plus Time Delay with a Zero) system with a negative/positive zero is difficult. If zero is positive, it shows an inverse response. A simple method is proposed to design proportional integral and derivative controllers for such systems. The proposed controller is applied to the various unstable transfer function models of exothermic CSTR, an isothermal CSTR carrying out an autocatalytic reaction and crystallizer. Simulation results on linear model equations of exothermic CSTR, an isothermal CSTR carrying out an autocatalytic reaction and crystallizer are given to show the effectiveness of the proposed PID controller. The performance of proposed controller in terms of integral square error (ISE), integral absolute error (IAE) and integral time weighted absolute error (ITAE) is compared with the literature reported data.

Introduction

PID controllers give satisfactory performance for many of the control processes. Due to their simplicity and usefulness, PID controller has become a powerful solution to the control of a large number of industrial processes. The control systems performance is complicated by the numerator dynamics (presence of a zero) of the process. Several processes exhibit second order plus time delay system with a zero transfer function model. Examples for such processes are jacketed CSTR [1], distillation column [2], autocatalytic CSTR [3] and crystallizer [4]. Many recycle processes where energy and mass recycle takes place are represented by SOPTDZ transfer function model [2].

Very limited methods of designing PID controllers for unstable SOPTDZ systems are available in the literature. They are IMC method [5, 7], stability analysis method [7] and synthesis method [6]. In the synthesis method [6], closedloop transfer function is assumed. From the closedloop transfer function, controller transfer function is derived using process transfer function. Later controller transfer function is written as PID controller with a lead lag filter. In the IMC method [5], the PID controllers are designed for unstable FOPTD and SOPTD systems with and without a zero from IMC filter using Maclaurin series expansion.

However, methods of designing PID controllers for SOPTD system with positive/negative a zero are not discussed extensively and there is a need to propose a simple method. Therefore, the present work is directed to design PID controller for such systems by extending the method proposed by Chidambaram et al. [3]. The PID tuning parameters are given as function of process model parameters. Simulation results for various transfer function models are given to show the efficiency of the proposed controller.

Proposed method

Unstable Second Order Plus Time Delay System With A Positive/Negative zero.

The process transfer function model is given by

$$G_p(s) = \frac{K_p(1+ps)e^{-Ls}}{a_1 s^2 + a_2 s + 1} \qquad (1)$$

The closed loop transfer function model is assumed as follows

$$\frac{y}{y_r} = \frac{(1+ps)(1+\eta s)e^{-Ls}}{(\tau_1 s+1)^3(\tau_2 s+1)} \qquad (2)$$

From Eq (2), the following equation is obtained for the controller.

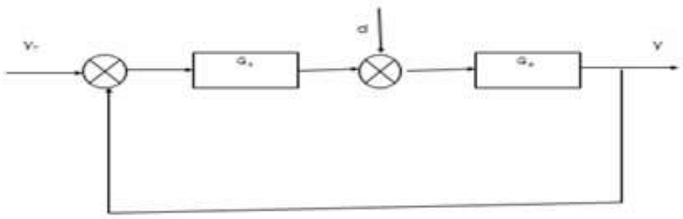

Fig. 1. Block diagram for simple feedback control system

$$G_C = \frac{\left(\frac{y}{y_r}\right)}{\left[G_P\left(1-\frac{y}{y_r}\right)\right]} \qquad (3)$$

Substitute Eq (2) and Eq (1) in Eq (3), results

$$G_C = \frac{(1+\eta s)(a_1 s^2 + a_2 s + a_3)(1+0.5Ls)}{[(1+0.5Ls)(\tau_1 s+1)^3(\tau_2 s+1) - (1+ps)(1+\eta s)(1-0.5Ls)]K_P} \qquad (4)$$

In deriving Eq (4), the approximation for exp($-\tau_d s$) as $\frac{1-0.5Ls}{1+0.5Ls}$ is used

Let

$$x_1 = 0.5L \qquad (5)$$

$$x_2 = \tau_1{}^3 \qquad (6)$$

Tele:

E-mail addresses: darapureddi.krishna@gmail.com

$$x_3 = 3\tau_1^{\ 2} \quad (7)$$

$$x_4 = 3\tau_1 \quad (8)$$

Eq (4) becomes

$$G_c = \frac{(1+\eta s)(a_1 s^2 + a_2 s + a_3)(1 + x_1 s)}{K_P D_2 s} \quad (9)$$

$$D_2 = s^4(x_1x_2\tau_2) + s^3(x_2\tau_2 + x_1x_2 + x_1x_3\tau_2) + s^2(x_2 + x_3\tau_2 + x_1x_3 + x_1x_4\tau_2 + p\eta x_1) + s(x_3 + x_4\tau_2 + x_4x_1 + x_1\tau_2 - p\eta + x_1\eta + px_1) + (x_4 + \tau_2 + x_1 - \eta - p + x_1) \quad (10)$$

Let

$$y_1 = x_4 + 2x_1 - p \quad (11)$$

$$y_2 = x_1x_2 \quad (12)$$

$$y_3 = x_1x_3 + x_2 \quad (13)$$

$$y_4 = x_1x_4 + x_3 \quad (14)$$

$$y_5 = x_1 + x_4 \quad (15)$$

$$y_7 = x_1 - p \quad (16)$$

$$y_8 = px_1 \quad (17)$$

Eq (9) becomes

$$G_c = \frac{(1+\eta s)(a_1 s^2 + a_2 s + a_3)(1 + x_1 s)}{K_P D_2' s(y_1 + \tau_2 - \eta)} \quad (18)$$

Eq (10) becomes

$$D_2 = D_2'(y_1 + \tau_2 - \eta) \quad (19)$$

$$D_2' = s^4\left(\frac{y_2\tau_2}{y_1 + \tau_2 - \eta}\right) + s^3\left(\frac{y_2 + y_3\tau_2}{y_1 + \tau_2 - \eta}\right) + s^2\left(\frac{y_3 + y_4\tau_2 + y_8\eta}{y_1 + \tau_2 - \eta}\right) + s\left(\frac{\tau_2 y_5 + y_4 + y_8 + y_7\eta}{y_1 + \tau_2 - \eta}\right) + 1 \quad (20)$$

Let

$$D_2' = (a_1 s^2 + a_2 s + a_3)(\alpha_1 s^2 + \alpha_2 s + 1) \quad (21)$$

Comparison of coefficients of s^4, s^3, s^2, s of LHS and RHS in Eq (21)

$$a_1\alpha_1 = \frac{y_2\tau_2}{y_1 + \tau_2 - \eta} \quad (22)$$

$$a_1\alpha_2 + a_2\alpha_1 = \frac{y_2 + y_3\tau_2}{y_1 + \tau_2 - \eta} \quad (23)$$

$$a_1 + a_2\alpha_2 + a_3\alpha_1 = \frac{y_3 + y_4\tau_2 + y_8\eta}{y_1 + \tau_2 - \eta} \quad (24)$$

$$a_2 + a_3\alpha_2 = \frac{\tau_2 y_5 + y_7\eta + y_4 + y_8}{y_1 + \tau_2 - \eta} \quad (25)$$

Let

$$y_9 = y_5 - a_2 \quad (26)$$

$$y_{10} = y_7 + a_2 \quad (27)$$

$$y_{11} = y_4 + y_8 - a_2 y_1 \quad (28)$$

$$y_{12} = \frac{a_1}{a_3} \quad (29)$$

$$y_{13} = \frac{a_2}{a_1} \quad (30)$$

$$y_{14} = y_3 - y_{12}y_9 - y_{13}y_2 \quad (31)$$

$$y_{15} = y_2 - y_{12}y_{11} \quad (32)$$

$$y_{16} = \frac{a_2}{a_3} \quad (33)$$

$$y_{17} = y_4 - a_1 - y_{16}y_9 - \frac{y_2}{y_{12}} \quad (34)$$

$$y_{18} = y_3 - a_1 y_1 - y_{16}y_{11} \quad (35)$$

$$y_{19} = -a_1 + y_{16}y_{10} - y_8 \quad (36)$$

Solving equations (22) to(25), the following equations are obtained

$$\tau_2 = \frac{y_{10}y_{12}y_{18} - y_{15}y_{19}}{y_{14}y_{19} - y_{10}y_{12}y_{17}} \quad (37)$$

$$\eta = \frac{\tau_2 y_{17} + y_{18}}{y_{19}} \quad (38)$$

$$\alpha_1 = \frac{y_2\tau_2}{a_1(y_1 + \tau_2 - \eta)} \quad (39)$$

$$\alpha_2 = \frac{\tau_2 y_9 + \eta y_{10} + y_{11}}{a_3(y_1 + \tau_2 - \eta)} \quad (40)$$

Eq (9) can be rearranged in the following form

$$G_c = \frac{(1+\eta s)(1 + x_1 s)}{K_P s(y_1 + \tau_2 - \eta)(\alpha_1 s^2 + \alpha_2 s + 1)} \quad (41)$$

Above equation is arranged in to the following form

$$G_c = \left(\frac{\eta + x_1}{K_P(y_1 + \tau_2 - \eta)}\right)\frac{\left(1 + \left(\frac{x_1\eta}{x_1 + \eta}\right)s + \frac{1}{(x_1 + \eta)s}\right)}{(\alpha_1 s^2 + \alpha_2 s + 1)} \quad (42)$$

It is in the form of PID controller with second order filter as follows

$$G_c = \frac{K_c\left(1 + \tau_d s + \frac{1}{\tau_I s}\right)}{\alpha_1 s^2 + \alpha_2 s + 1} \quad (43)$$

Eq (43) is compared with eq (42), the conventional PID settings are obtained as

$$K_c = \frac{\eta + x_1}{K_P(y_1 + \tau_2 - \eta)} \quad (44)$$

$$\tau_I = x_1 + \eta \quad (45)$$

$$\tau_d = \frac{x_1\eta}{x_1 + \eta} \quad (46)$$

$$\alpha_1 = \frac{y_2\tau_2}{a_1(y_1 + \tau_2 - \eta)} \quad (47)$$

$$\alpha_2 = \frac{\tau_2 y_9 + \eta y_{10} + y_{11}}{a_3(y_1 + \tau_2 - \eta)} \quad (48)$$

Simulation Results

In this section, PID controller design by the proposed method is applied to various CSTR transfer function models and crystallizer to show the efficiency of the proposed controller. The performance of the proposed controller is compared with that of the controller designed by IMC method [3] and synthesis method [3].

Case study-1

The transfer function model of the autocatalytic CSTR [3] is given by $G_P = \dfrac{-0.2679(1-41.667s)e^{-10s}}{279.03s^2 - 2.978s + 1}$. The unstable complex conjugate poles of the system are located at $0.005356 \pm 0.0596i$. The proposed method, IMC method and synthesis method PID controller parameters are given in Table 1. The performance comparison in terms ISE, IAE and ITAE for linear model for the servo and regulatory response is given in Table 2. The servo and regulatory response with these controllers are shown in Fig 1 and Fig 2 respectively. The servo and regulatory performance of proposed PID controller is better than IMC method [3] and Synthesis method [3] (refer to Table 2).

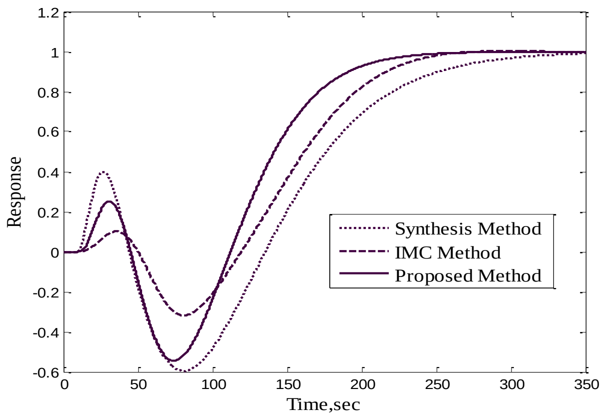

Fig 1. Servo response of an isothermal CSTR carrying out autocatalytic reaction in case study-1.

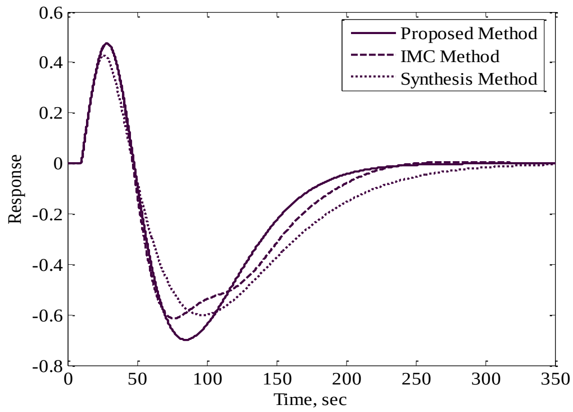

Fig 2. Regulatory response of an isothermal CSTR carrying out autocatalytic reaction in case study-1

Case study-2

The transfer function model of crystallizer [4] is given by $G_P(s) = \dfrac{0.03(1+s)e^{-0.1s}}{0.516s^2 - 0.0945s + 1}$. The unstable complex conjugate poles of the system are located at $0.0916 \pm 1.389i$. The present method, IMC method and synthesis method PID controller parameters are given in Table 1. The performance comparison in terms ISE, IAE and ITAE for linear model for the servo and regulatory response is given in Table 2. The servo and regulatory response with these controllers are shown in Fig 3 and Fig 4 respectively. The servo and regulatory performance of proposed PID controller is better than IMC method [3] and Synthesis method [3] (refer to Table 2).

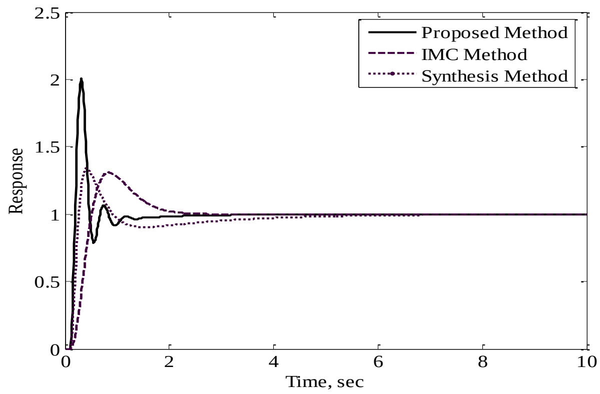

Fig 3. Servo response of crystallizer in case study-2

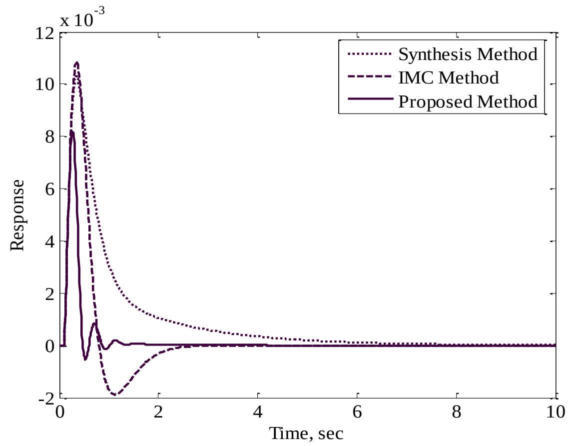

Fig 4. Regulatory response of crystallizer in case study-2

Case study-3

The transfer function model of CSTR [8] is given by $G_p(s) = \dfrac{3.87(1+0.5283s)e^{-0.1s}}{0.4769s^2 - 0.348s + 1}$. The unstable complex conjugate poles of the system are located at $0.3649 \pm 1.4013i$. The proposed method, IMC method and synthesis method PID controller parameters are given in table 1. The performance comparison in terms ISE, IAE and ITAE for linear model for the servo and regulatory response is given in Table 2. The servo and regulatory response with these controllers are shown in Fig 5 and Fig 6 respectively. The servo and regulatory performance of proposed PID controller is better than Synthesis method [3] (refer to Table 2).

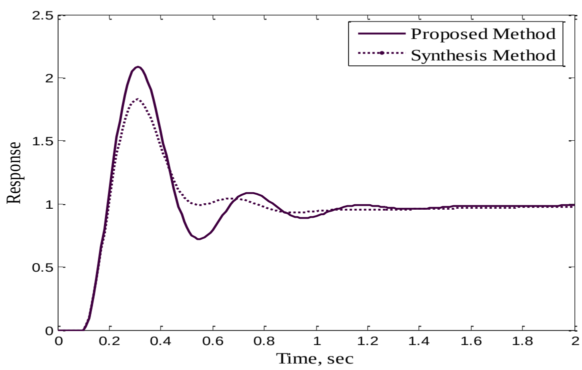

Fig 5. Servo response of CSTR in case study-3

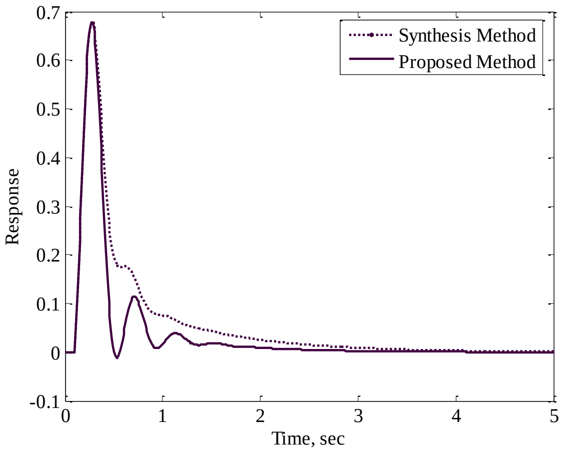

Fig 6. Regulatory response of CSTR in case study-3.

Conclusions

A simple method is proposed for unstable SOPTD systems with a zero transfer function model. Simple tuning formulae are given for PID settings. The zero may be a positive or a negative. The proposed PID controller is applied to control number of an isothermal CSTR carrying out autocatalytic reaction, an exothermic CSTR and crystallizers. The performance of the proposed method is compared with that of literature reported methods. The performance of the controller designed by proposed method is better than the literature reported methods in terms of the least ISE, IAE and ITAE. Simulation results on linear model equations of an isothermal CSTR carrying out autocatalytic reaction, an exothermic CSTR and crystallizer show that the proposed PID controller performs better than controller designed by IMC method [3] and synthesis method [3].

Acknowledgement

We are thankful to Dr. R.Padma Sree, Professor, Department of chemical engineering, A.U.College of Engineering (A), Visakhapatnam, Andhra Pradesh, who did support for doing this type of work & also thankful to Prof.K.V.L.Raju, Principal, M.V.G.R.College of Engg, Vizianagaram, Andhra Pradesh, India, who helped a lot to do this type of work in our college.

Table 1. PID settings for different methods $G_C = K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)\frac{1}{\left(1+\alpha_2 s+\alpha_1 s^2\right)}$.

Transfer function model	Controller	K_C	τ_I	τ_D	α_0	α_1	α_2
Case study-1	Proposed Method	0.7764	-34.8074	5.7982		13.59	5.76
	IMC Method	0.5786	-27.82	33.55		63.59	19.86
	Synthesis Method	1.1256	-64.86	5.39		0	2.4537
Case study-2	Proposed Method	187.169	0.3603	0.0431		0	0.0173
	IMC Method	281.3	0.76	0.373	0.05	0.0344	1.0344
	Synthesis Method	111.783	1.0457	0.0457	---	---	0.0225
Case study-3	Proposed Method	2.5269	0.4956	0.0450		0	0.0164
	Synthesis Method	2.32	0.704	0.0464			0.013

Table 2. Performance comparison in terms of ISE, IAE, ITAE for linear models

Transfer function model	Controller	Servo problem			Regulatory problem		
		ISE	IAE	ITAE	ISE	IAE	ITAE
Case study-1	Proposed Method	179.37	167.39	1.328×10^4	31.76	67.33	6.34×10^3
	IMC Method	174.99	179.57	1.588×10^4	18.14	36.95	2.32×10^3
	Synthesis Method	235.32	215.13	2.167×10^4	31.34	77.05	9.06×10^3
Case study-2	Proposed Method	0.2836	0.4182	0.1965	1.07×10^{-5}	0.002	8.73×10^{-4}
	IMC Method	0.3359	0.5853	0.3340	3.57×10^{-5}	0.005	0.0037
	Synthesis Method	0.2160	0.5547	0.7826	4.73×10^{-5}	0.009	0.0123
Case study-3	Proposed Method	0.310	0.4365	0.1784	0.0772	0.1969	0.1063
	Synthesis Method	0.2396	0.3821	0.1939	0.0974	0.3034	0.2366

References

[1] Bequette, B.W. 2003. "*Process control, Modeling, design and simulation"*. Prentice Hall India. New Delhi.
[2] Jacobsen, E.W. 1999. On the dynamics of integrated plants Non-minimum phase behavior. *J Process Contr,* 9: 439-451.
[3] Padmasree, R. and Chidambaram, M. 2006. "*Control of unstable systems"*, Narosa publishers. New Delhi.
[4] Kausthubh, S.M. and Chidambaram, M. 2000. Control of Isothermal Crystallizers, *Proc. CHEMCON, Calcutta*., December 18th to 21st .2000.
[5] Lee, Y. Lee, J. and Park, S. 2000. PID controller tuning for integrating and unstable processes with time delay. *Chem Eng Sci*, 55: 3481-3493.
[6] Wang, L.W and Hwang, S.H. 2005. Identification and control for unstable processes of three dynamics, *Chem Eng Comm*, 192: 34-61.
[7] Krishna, D., Suryanarayana, K., Aparna, G., and Padma Sree, R. 2012. Tuning of PID Controllers for Unstable Continuous Stirred Tank Reactors, *Int.J.Appl.Sci.Eng.*, 10:1-18.
[8] Marlin, T.E. 1995. Process control; Designing processes and control systems for dynamic performance, McGraw-Hill, New York.

Nomenclature

G_c	Control transfer function
G_p	Process transfer function
K_c	Controller gain
τ_I	Integral time
K_p	Process gain
L	Time delay
P	Numerator time constant
t	Time
τ_D	Derivative time
α_0, α_1 α_2	Filter time constants
τ_1, τ_2	Denominator constants
ISE	Integral square error
IAE	Integral absolute error
ITAE	Integral time absolute error
PID	Proportional Integral and Derivative Controller
IMC	Internal model control