



## Comparisons of different beamforming algorithm for beamformer

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### ABSTRACT

A linear adaptive beamforming structure consists of an antenna array and a digital signal processor to adjust adaptively its weight according to the particular criterion and adaptive algorithm. There are many criteria and adaptive algorithms to minimum co-channel interference (or multi-access interference). Different Algorithms (S.Stergiopoulos, 2001& Joseph C. Liberti, et al, 1999) are employed to determine the complex weights of the signal and to produce narrow beams toward intended users and deep nulls in the direction of interference.

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### Introduction

A linear adaptive beamforming structure consists of an antenna array and a digital signal processor to adjust adaptively its weight according to the particular criterion and adaptive algorithm. There are many criteria and adaptive algorithms to minimum co-channel interference (or multi-access interference). Different Algorithms (S.Stergiopoulos, 2001& Joseph C. Liberti, et al, 1999) are employed to determine the complex weights of the signal and to produce narrow beams toward intended users and deep nulls in the direction of interference:

1. Least Mean Square (LMS) Algorithm
2. Minimum Variance Distortion-less response (MVDR) Algorithm
3. Recursive Least Square (RLS) Algorithm
4. Constant Modulus (CMA) Algorithm
5. Maximum Directivity (MD) Algorithm

#### Least Mean Square Algorithm

Application of least mean square algorithm to estimate optimal weights of an array is widespread and its study has been of considerable interest for some time. The algorithm is referred to as the constrained LMS algorithm when the weights are subjected to constraints at each iteration, whereas it is referred to as unconstrained algorithm when weights are not constrained at each iteration. The latter is applicable mainly when weights are updated by reference signals and no knowledge of the direction of the signal is utilized, as is the case for the constrained case.

The algorithms update the weights at each iteration by estimating the gradient of the quadratic Mean Square Error (MSE) surface, and then moving the weights in the negative direction of the gradient by a small amount. The constant that determines this amount is referred to as the step size. When the step size is small enough, the process leads these estimated weights to the optimal weights. The convergence and transient behavior of these weights along with their covariance characterize the LMS algorithm, and the way the step size and the process of gradient estimation affect these parameters are of great practical importance.

A real time unconstrained LMS algorithm for determining optimal  $W_{MSE}$  of the system using the reference signal is given

$$(n+1) = w(n) - \mu g(w(n)) \quad (1.1)$$

where  $w(n+1)$  denotes the new weights computed at the  $(n+1)^{th}$  iteration,  $\mu$  is a positive scalar and  $g(w(n))$  is an unbiased estimate of the MSE gradient. For a given  $w(n)$ , the MSE is given by

$$\xi(w(n)) = E \left[ |r(n+1)|^2 \right] + w^H(n) R w(n) - w^H(n) z - z^H w(n) \quad (1.2)$$

The MSE gradient at the  $n$ th iteration is obtained by differentiating the above equation with respect to  $w$ , yielding

$$\nabla_w \xi(w) = 2Rw(n) - 2z \quad (1.3)$$

At the  $(n+1)$  the iteration, the array is operating with weights  $w(n)$  computed at the previous iteration, however the array signal vector  $x(n+1)$ , the reference signal sample is  $r(n+1)$  and the array output is

$$y(w(n)) = w^H(n) x(n+1) \quad (1.4)$$

LMS determines the optimum weight vectors sample by sample in time domain and takes a long time to converge [S.F. Shaikat, et al, 2009]. LMS is best for beamforming towards desired user but they have limitation towards interference rejection.

#### Minimum variance distortion less Response (MVDR)

The goal is to optimize the beam former response so that the output contains minimal contributions due to noise and signal arriving from directions other than the desired signal direction. For this optimization procedure, it is desired to find a linear filter vector;  $w(f, \theta)$  is a solution to the constrained minimization problem that allows signals from the look direction to pass with a specified gain:

Minimize:

$$\sigma_{MV}^2 = w^*(f_i, \theta) R(f_i)(n) w(f_i, \theta),$$

subject

to

$$\bar{w}(f_i, \theta) \bar{D}(f_i, \theta) = 1 \tag{1.5}$$

where  $\bar{D}(f_i, \theta)$  is the conventional steering vector. The solution is given by

$$\bar{w}(f_i, \theta) = \frac{R^{-1}(f_i) \bar{D}(f_i, \theta)}{\bar{D}^*(f_i, \theta) R^{-1}(f_i) \bar{D}(f_i, \theta)} \tag{1.6}$$

It provides the adaptive steering vectors for beam forming of the received signals by the N hydrophone line array. Then in the frequency domain, the adaptive beam at a steering  $\theta_s$  is defined by

$$B(f_i, \theta) = \bar{w}(f_i, \theta) \bar{x}(f_i) \tag{1.7}$$

MVDR has fast convergence ability to operate in the complicated interference environment that is suitable for wireless communication.

**Recursive Least Mean Square Algorithm (RLS)**

The convergence speed of the LMS algorithm depends on the eigen values of the array correlation matrix. In an environment yielding an array correlation matrix with large eigen values spread the algorithm converges with a slow speed. This problem is solved with the RLS algorithm by replacing the gradient step size  $\mu$  with a gain matrix  $\hat{R}^{-1}(n)$  at the nth iteration, producing the weight update equation.

$$w(n) = w(n+1) - R^{-1}(n)x(n)\varepsilon^*(w(n-1)) \tag{1.8}$$

where  $R^{-1}$  is given by

$$\begin{aligned} \hat{R}(n) &= \delta_0 \hat{R}(n-1) + x(n)x^H(n) \\ &= \sum_{k=0}^n \delta_0^{n-k} x(k)x^H(k) \end{aligned} \tag{1.9}$$

with  $\delta_0$  denoting a real scalar less than but close to 1. The  $\delta_0$  is used for exponential weighting of past data and is referred to as the forgetting factor as the update equation tends to de-emphasize the old samples. The quantity  $1/(1-\delta_0)$  is normally referred to as the algorithm memory. Thus, for  $\delta_0=0.99$ , the algorithm memory is close to 100 samples. The RLS algorithm updates the required inverse using the previous samples and the present samples as

$$\hat{R}^{-1}(n) = \frac{1}{\delta_0} [\hat{R}^{-1}(n-1) - \frac{\hat{R}^{-1}(n-1)x(n)x^H(n)\hat{R}^{-1}(n-1)}{\delta_0 + x^H(n)\hat{R}^{-1}(n-1)x(n)}] \tag{1.10}$$

The matrix is initialized as

$$\hat{R}^{-1}(0) = \frac{1}{\varepsilon_0} I_e, \varepsilon_0 > 0 \tag{1.11}$$

A comparison of the convergence speed of the LMS, RLS using quantized or clipped data indicates that RLS is the most efficient and LMS is the slowest. RLS algorithm involves more computations than LMS and it provides safe towards main lobe and have better response towards co-channel interference. RLS algorithm is found to have minimum BER (Bit Error Rate).

**Constant Modulus Algorithm (CMA)**

Adaptive beam former using LMS algorithm generally minimizes array output power subject to constraint. These beam formers have two major deficiencies. First, they have a tendency to cancel the signal of interest along with the interference when the two signals are mutually correlated. This has prevented the use in multipath environments. Second they are extremely sensitive to array imperfections.

The CM Array does not suffer from either of the above alignments. Since it does not use output power minimization to adopt the array weights, it is unaffected by correlated source problems. Similarly, it does not rely upon array geometry nor gain characteristic to set a constraint is thus unaffected by array imperfection. These two advantages make the CM array as an attractive alternative to conventional adaptive beam formers.

CMA is gradient based on works on the premise that existing interference causes fluctuations in the amplitude of array output that otherwise has constant modules. It updates the weights by minimizing the cost functions.

$$J(n) = \frac{1}{2} E [ (|y(n)|^2 - y_0^2)^2 ] \tag{1.12}$$

Using the following equation

$$w(n+1) = w(n) - \mu g(w(n)) \tag{1.13}$$

where  $y(n) = w^H(n)x(n+1)$  is the array output after the n<sup>th</sup> iteration,  $y_0$  is the desired amplitude in the absence of imperfection,  $g(w(n))$  denotes an estimate of the cost function gradient. Similar to the LMS algorithm, the CMA uses an estimate of the gradient by replacing the true gradient with an instant value given by

$$g(w(n)) = 2\varepsilon(n)x(n+1) \tag{1.14}$$

Where

$$\varepsilon(n) = (|y(n)|^2 - y_0^2) y(n) \tag{1.15}$$

The weight update equation of this case becomes

$$w(n+1) = w(n) - 2\mu\varepsilon(n)x(n+1) \tag{1.16}$$

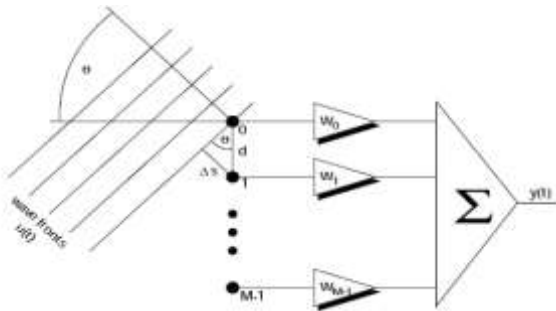
In appearance, this is similar to the LMS algorithm with reference signal where

$$\varepsilon(n) = r(n) - y(n)$$

CMA is useful for eliminating correlated arrivals is an effective constant modulated enveloped signal. CMA doesn't use any reference signal but automatically selects one or several of the multipaths as the desired signal. CMA and LMS give maximum BER when user interference is quite close to each other which are not affordable in practical Base Station installations. The performance is satisfactory for minimum number of elements. Hence it may not be suited for a large array.

**Maximum Directivity beam forming Algorithm**

A new constrained beam former (Thomas kuhwald, et al 1996) has been presented which allows the suppression of an arbitrary number of interfering signals while simultaneously providing a maximum directivity. An ideal linear antenna array consists of M identical half wavelength separated elements



**Figure 1.1. Arriving wave fronts at M-element linear array**  
**Linear Array Model**

Consider a set of L waveforms

$$u_l(t) = a_l(t)e^{j(w_c t + \phi_l(t))} = s_l(t)e^{jw_c t} \quad 1 \leq l \leq L \tag{1.17}$$

with carrier  $e^{jw_c t}$  and complex base band signal  $s_l(t) = a_l(t)e^{j\phi_l(t)}$  impinging on the array of M spatially distributed identical sensors. The output of the M<sup>th</sup> sensor is given as

$$x_m(t) = \sum_{l=1}^L s_l(t)e^{jw_c(t + \tau_m(\psi_l, \theta_l))} + n_m(t) \quad 0 \leq m \leq M \tag{1.18}$$

With  $n_m(t)$  being a noise component of the m-th element with zero mean and variance  $\sigma_n^2$ . Without losing generality, we'll confine the signals to the plane defined by  $\psi = 0$ . It is convenient to introduce the array steering vector  $\alpha(\theta)$  which describes the array response to a signal of frequency  $f_c = w_c / 2\pi$  from angle of arrival  $\theta$ :

$$X(t) = \sum_{l=1}^{M-1} s_l(t)e^{jw_c t} \alpha(\theta_l) \tag{1.19}$$

$$\alpha(\theta) = [e^{jw_c \tau_0(\theta)}, e^{jw_c \tau_1(\theta)}, \dots, e^{jw_c \tau_{M-1}(\theta)}]^T \tag{1.20}$$

These signals are weighted and combined in order to produce the output signal

$$y(t) = \sum_{m=0}^{M-1} w_m x_m(t) = W^T X(t) \tag{1.21}$$

Clearly, for a fixed set of weights,  $y(t)$  is a function of the incident angle  $\theta$ .

Figure 1.1 shows a uniform linear array (ULA) of M elements with spacing  $d = \lambda/2$ .

**MD Algorithm:**

In the present work, we use (Gerd Grosshop, et al 2002, 2003 &) Maximum Directivity (MD) algorithm for adaptive beam formation in mobile communication. Adaptive beam forming is achieved by amplitude and phase control on the individual signals feeding the array antenna elements. These amplitude and phase weights are calculated using Maximum Directivity beamforming algorithm. Generally, beam forming is performed by complex weighting and combining the individual antenna signals.

The advantage of this algorithm is that it delivers the optimal solution simply by solving a Linear Equations System (LES). In this algorithm, we calculate the complex weights of the antenna array elements using unit response in the look direction, which will maximize the output Signal to Noise Ratio

(SNR). So this algorithm steers the main lobe maximum in the look direction and side lobes minima in the interference directions while maintaining maximum directivity. This results in the array factor  $G(\theta)$  which describes the spatial radiation characteristic of the antenna array. This way, the angular distribution of the radiation strength in the transmit case and the sensitivity in the receive case can be adapted to the respective requirements. The output of the beam former is given by

$$G(\theta) = \sum_{m=1}^M w_m^* e^{j(m-1)\pi \sin(\theta)} \tag{1.22}$$

where  $w_m^*$  is the complex weight that needs to be adjusted to optimize the radiation pattern, M is the number of the array antenna elements. Generally the number of nulls can vary from 0 to M-1. Eq. (1.24) results in the more general expression

$$G(\theta) = \sum_{m=1}^M w_m^* g_m(\theta) \tag{1.23}$$

where  $g_m(\theta)$  is the radiation characteristics of array elements. The expression for the complex weight factor can be written as

$$w_m^* = b_m e^{-j(m-1)\Omega_a} \tag{1.24}$$

where  $\Omega_a = -\pi \sin \psi$  is the normalized wave number in azimuth direction and  $\psi$  is the azimuth angle and  $b_m$  is the amplitude of the signal. In our special case of eight antenna elements we have one look direction  $\theta_{LD}$  and K null directions  $\theta_{0k}$ ,  $1 \leq k \leq K$  with K between 1 and 7. The LES can be written as

$$\begin{pmatrix} G(\theta_{LD}) \\ G(\theta_{01}) \\ \vdots \\ G(\theta_{0K}) \end{pmatrix}_{8 \times 1} = \begin{pmatrix} g_1(\theta_{LD}) & g_2(\theta_{LD}) & \dots & g_8(\theta_{LD}) \\ g_1(\theta_{01}) & g_2(\theta_{01}) & \dots & g_8(\theta_{01}) \\ \vdots & \vdots & \ddots & \vdots \\ g_1(\theta_{0K}) & g_2(\theta_{0K}) & \dots & g_8(\theta_{0K}) \end{pmatrix}_{8 \times 8} \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_8 \end{pmatrix}_{8 \times 1} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}_{8 \times 1} \tag{1.25}$$

Each row of the LES describes the constraints in the corresponding angular direction. Because of the variation of null directions, the LES has a varying number of rows. Due to the maximum directivity, the constructed beam pattern provides optimal solution for the uplink channel of mobile communication systems. The following table 1.1 depicts the comparisons between different adaptive beamforming algorithms.

**Table 1.1. Comparison of algorithms [L.C.Godara, 1997 & A.P.Kabilan et al, 2007 & S.F.Shaukat, 2009]**

Algorithm m	Amplitude response	Reference signal	BER	Scan Sector (°)	Convergence rate/computation
LMS	Maxi signal strength in the user direction	Required	High	-50 to -50	Slow/less computation
MVDR	Not available	Required	High	+ 52 to -52	Faster
RLS	Interference rejection better	Required	Low	Not available	Faster/ more computation
CMA	Interference rejection good	Not required	High	- 55 to +55	Not available
MD	Maximum Directivity	Not required	Minimum	-60 to +60	Not available /less computation-solving LES

**Conclusion:**

The significances of MD algorithm are observed as: This algorithm gives the maximum directivity which results in a minimum bit error rate for the reverse link of a CDMA based mobile communication system. The algorithm provides an optimum solution for arbitrary spaced constraints while maintaining a maximum possible directivity and doesn't require additional reference signal. The weights can efficiently be calculated solving a set of LES.

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