Magnetic coercivity: misconceptions and misinterpretations in textbooks

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ABSTRACT

We describe various misconceptions and misinterpretations concerning presentation of the hysteresis loop for ferromagnets occurring in undergraduate and college textbooks. These problems originate from our teaching a Theoretical solid state / condensed matter physics (TSSP/CMP) course. A closer look at the definition of "coercivity" reveals two distinct notions referred to as the hysteresis loop: B vs. H or M vs. H, which can be easily confused and, in fact, are confused in several textbooks. The properties of the M vs. H type hysteresis loop are often ascribed to the B vs. H type loops, giving rise to various misconceptions. An extensive survey of textbooks at first in the TSSP/CMP area and later extended into the areas of general physics, materials science and magnetism / electromagnetism has been carried out. Relevant encyclopedias and physics dictionaries have also been consulted. The survey has revealed various other substantial misconceptions and/or misinterpretations than those originally identified in the TSSP/CMP area. The results are presented here to help in clarifying the misconceptions and misinterpretations in question. The physics education aspects arising from the textbook survey are also discussed. Additionally, analysis of the CMP examination results concerning questions pertinent to the hysteresis loop is provided.

Introduction

During years of studying and now teaching the theoretical solid state physics (TSSP), which more recently became condensed matter physics (CMP) course, we have realized that textbooks often contain not only common misprints but sometimes more serious misconceptions. The latter occur mostly when the authors attempt to present a more advanced topic in a simpler way using schematic diagrams. One such case concerns presentation of the magnetic hysteresis loop for ferromagnetic materials. Having identified some misconceptions existing in several textbooks currently being used for our TSSP/CMP course at DELSU, Abraka and UNIBEN, Benin City, and Teaching Physics at College of Education, Agbor by one of us (E. I); Delta State Polytechnic, Ozoro and Caleb University, Imota, Lagos State by (O.O.E.E.), we have embarked on an extensive literature survey. Search of physics education journals have revealed only a few articles dealing with magnetism, e.g. Enaroseha *et al* (2011), Enaroseha and Igherighe (2010), Hickey and Schibeci (1999), Hoon and Tanner (1985).

Interestingly, a review of middle school physical science texts by Hubisz (http://www.psrconline. org/curriculum/book.html), and analysis of hysteresis loop by workers like Sung and Rudowicz (2002) (www. arXiv.org) which has recently come to our attention, provides ample examples of various errors and misconceptions together with pertinent critical comments. However, none of these sources have provided clarifications of the problems in question. To find out the extent of these misconceptions existing in other physics areas, we have surveyed a large number of available textbooks pertinent for solid state / condensed matter, general physics, materials science, and magnetism / electromagnetism. Several pertinent encyclopedias and physics dictionaries have also been consulted. The survey has given us more than we bargained for, namely, it has revealed various other substantial misconceptions than those originally identified in the TSSP/CMP area. The results of this survey are presented here for the benefit of physics teachers (as well as researchers) and students. The textbooks, in which no relevant misconceptions and/or confusions were identified, are not quoted in text. In order to provide the counter examples for the misconceptions identified in the textbooks, we have reviewed a sample of recent scientific journals searching for real examples of the magnetic hysteresis loop, beyond the schematic diagrams found in most textbooks. To our surprise a number of general misconceptions concerning magnetism have been identified in this review.

The root of the problem appears to be the existence of two ways of presenting the hysteresis loop for ferromagnets: (i) B vs H curve or (ii) M vs H curve. In both cases, the *coercivity (coercive force)* is defined as the point on the negative H axis, often using an identical symbol, most commonly H_c. Yet it turns out that the two meanings of *coercivity* are not equivalent. In some textbooks the second notion of coercivity (M vs H) is distinguished from the first one (B vs H) as the *intrinsic* coercivity H_{ci}. An apparent identification of the two meanings of coercivity H_c (B vs H) and H_{ci} (M vs H) as well as of the properties of soft and hard magnetic materials have lead to misinterpretation of H_c as the point on the B vs H hysteresis loop where the magnetization is zero.

This is evident, for example, in the statements referring to H_c as the point at which *the sample is again unmagnetized* (Serway, 1990) or *the field required to demagnetize the sample* (Rogalski and Palmer, 2000). Minor problems concerning terminology and the drawbacks of using schematic diagrams are also discussed. Analysis of the condensed matter physics examination results concerning questions pertinent for the hysteresis loop is provided to illustrate some popular misconceptions in students' understanding.

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Methodology and Two Notions of Coercivity

For a ferromagnetic material, the magnetic induction (or the magnetic field intensity) inside the sample, **B**, is defined as (see, e.g. any of the books listed in References):

$$B = H + 4\pi M \qquad \text{(CGS)}$$

$$B = \mu_0(H + 4M) \qquad \text{(SI)} \qquad (1)$$

Where M is the magnetization induced inside the sample by the applied magnetic field H. In the Free space: $M = 0$ and then in the SI units: $B \equiv \mu_0 H$, where μ_0 is the permeability of free space ($\mu_0 = 4\pi X 10^{-7}$ [10^{-7} [m kg A^{-2} sec^{-2}]; note that the units [Hm^{-1}] and [WbA^{-1}m^{-1}] are also in use). The standard SI units are: B [tesla] = [T], H and M [A/m], whereas B [Gauss] = [G], H [Oersted] = [Oe], and M [emu/cc] (see, e.g. Jiles, 1991; Anderson, 1989). Both the CGS units and the SI units are provided since the CGS unit system is in use in some textbooks surveyed and comparisons of values need to be made later.
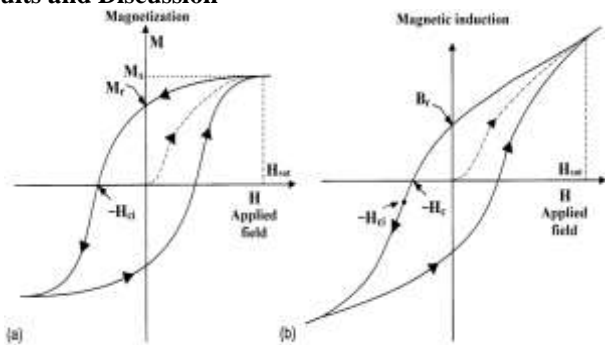
Results and Discussion

Fig. 1. Hysteresis curves for a ferromagnetic material: (a) *M vs H*: M_r is the remanent magnetization at H = 0; H_{ci} is the *intrinsic* coercivity, i.e. the reverse field that reduces M to zero; M_s is the saturation magnetization; (b) *B vs H*: Br is the remanent induction (or remanence) at H = 0; H_c is the coercivity, i.e. the reverse field required to reduce B to zero (adapted from Elliot, 1998).

In Fig. 1 we present schematically the hysteresis curves for a ferromagnetic material together with the definitions of the terms important for technological applications of magnetic materials.

The two meanings of *coercivity* H_{ci} and H_c as defined on the diagrams: (a) the magnetization M vs applied field H and (b) magnetic induction (or flux density) B vs H, respectively, are clearly distinguished. Both curves have a similar general characteristic, except for one crucial point. After the saturation point is reached, the M curve becomes a straight line with exactly zero slope, whereas the slope of the B curve reflects the constant magnetic susceptibility and depends on the scale and units used to plot B vs H (see above). In other words, the B vs H curve does not saturate by approaching a limiting value as in the case of the M vs H curve.

For an initially unmagnetized sample, i.e. M ≡ 0 at H = 0, as H increases from zero, M and B increases as shown by the dashed curves in Fig. 1 (a) and (b), respectively. This magnetization process is due to the motion and growth of the magnetic domains, i.e. othe areas with the same direction of the local magnetization. For a full discussion of the formation of hysteresis loop and the nature of magnetic domains inside a ferromagnetic sample one may refer to the specialized textbooks listed in the References, e.g. Kittel (1996), Elliott (1998),

Dalven (1990), Skomski and Coey (1999). Here we provide only a brief description of these aspects. A distinction must be made at this point between the magnetically isotropic materials [e.g using Heseinberg Model Enaroseha and Andikara (2010), Enaroseha and Igherighe, (2011)], for which the magnetization process does not depend on the orientation of the sample in the applied field H, and the anisotropic ones (e.g. Anderson or Hubbard Model), which are magnetized first in the easy direction at the lower values of H. In the former case, as each domain magnetization tends to rotate to the direction of the applied field, Kittel (1996), the domain wall displacements occur, resulting in the growth of the volume of domains favorably oriented (i.e. parallel) to the applied field and the decrease of the unfavorably oriented domains, Kittel (1996). In the latter case, only after the magnetic anisotropy [for definition, see, e.g. Kittel (1996), Elliott (1998), Dalven (1990), Skomski and Coey (1999), Jiles (1991)] is overcome the sample is fully magnetized with the direction of M along H. In either case, when this *saturation point* is reached, the magnetization curve no longer retraces the original dashed curve when H is reduced. This is due to the irreversibility of the domain wall displacements. When the applied field H reaches again zero, the sample still retains some magnetization due to the existence of domains still aligned in the original direction of the applied field Dalven (1990). The respective values at H = 0 are defined [see, e.g. Kittel (1996), Elliott (1998), Dalven (1990), Skomski and Coey (1999), Jiles (1991)] as the remnant magnetization M_r, Fig. 1 (a), and the remnant induction B_r, Fig. 1 (b). To reduce the magnetization M and magnetic induction B to zero, a reverse field is required known as the coercive force or coercivity. The soft and hard magnetic materials are distinguished by their small and large area of the hysteresis loop, respectively.

By definition, the coercive force (coercivity) defined in Fig. 1 (a), and that in Fig. 1 (b) are two different notions, although their values may be very close for some materials. In order to distinguish them, some authors define either the related coercivity (Kittel, 1996) or the intrinsic coercivity (Elliott, 1998; Jiles, 1991) H_{ci} as the reverse field required to reduce the magnetization M from the remnant magnetization M_r again to zero as shown in Fig. 1 (a), whereas reserve the symbol H_c and the name coercivity (coercive force) to denote the reverse field required to reduce the magnetic induction in the sample B to zero as shown in Fig. 1 (b), as done, e.g. by Kittel (1996). Hence, the confusion between the two notions of coercivity referred to the curve B vs H and the curve M vs H can be avoided. Since a clear distinction between H_c and H_{ci}, is often not the case in a number of textbooks, a question arises under what conditions and for which magnetic systems, if any, H_c and H_{ci} can be considered as equivalent quantities. If it was the case, the point H_c on the B vs H curve would also correspond to the magnetization M ≡ 0 as in the case of H_{ci} on the M vs H curve. Only in one of the books surveyed such approximation is explicitly considered. Dalven (1990) shows that, in general, the values of B and M are much larger than H in both curves in Fig. 1. Hence, if H can be neglected in Eq. 1, then B ≈ μ_oM. This turns to be valid only for low values of H and the narrow hysteresis loop pertinent for the soft magnetic materials. In other words, the value of H_c and H_{ci} are indeed very close, so not identical, *for the soft magnetic materials only*. In this case Hci in Fig. 1 (a) and Hc in Fig. 1 (b) can be considered as two equivalent points and hence M ≈ 0 at H_c as well.

The real examples of the magnetic hysteresis loop, identified in the review of Sung and Rudowicz (2002) of a sample of recent scientific journals, indicate that Hc and Hci turn out to be significantly non-equivalent *for the hard magnetic materials*. In the article by Sung and Rudowicz (2002) where the values of H_{ci}, H_c, and B_r are compiled for several commercially available permanent magnetic materials revealed by our recent Internet search. These data indicate that although H_c and H_{ci} are of the same order of magnitude, in a number of cases H_{ci} is substantially larger than H_c. Hence, in general, it is necessary to distinguish between H_c and H_{ci}. Moreover, as a consequence of $H_{ci} \neq H_c$, the magnetization does not reach zero at the point .Hc on the B vs H curve but at a larger value of H_{ci} indicated schematically in Fig. 1 (b). However, in the early investigations of magnetic materials, before the present day very strong permanent magnets become available, the values of Hc and Hci were in most cases not distinguishable. As the advances in the magnet technology progressed, more and more hard magnetic materials have been developed, for which the distinction between H_{ci} and H_c is quite pronounced [see Table 1 in Sung and Rudowicz (2002)]. The presentation in most textbooks reflects the time lag it takes for new materials or ideas to filter from scientific journals into the textbooks as 'schematically presented established knowledge'

Results of textbooks survey

In our survey of the presentation of the hysteresis loop for ferromagnetic materials, in total about 300 textbooks in the area of theoretical solid state / condensed matter, general physics, materials science, magnetism / electromagnetism as well as several encyclopedias and physics dictionaries available in Caleb University Lagos, University of Benin, Delta State University, Delta State Polytechnic Ozoro and College of Education Agbor library were examined.

We have identified around 130 books dealing with the hysteresis loop. In order to save the space an additional list of the books surveyed (37 items), which deal with the hysteresis loop in a correct way but are not quoted in the References, is available from the authors upon request. It appears that from the points of view under investigation, generally, the encyclopedias and physics dictionaries contain no explicit misconceptions. This is mainly due to the fact that the hysteresis loop is usually presented at a rather low level of sophistication [see, e.g. Lapedes (1978), Lord (1986), Meyers (1990), Besancon (1985), Parker (1993)]. However, in a few instances in the same source book both types of hysteresis loop (B vs H and M vs H) are discussed in separate articles written by different authors without clarifying the distinct notions, which may also lead to confusion. Examples include, e.g. (a) Anderson and Blotzer (1999) and Vermariën *et al* (1999), and (b) Arrott (1983), Donoho (1983), and Rhyne (1983). Hence, these authoritative sources could not help us to clarify the intricacies we have encountered. This have been achieved by consulting more advanced books on the topic, e.g., Kittel (1996), Dalven (1990), Skomski and Coey (1999), and/or regular scientific journals [for references, see, Sung and Rudowicz (2002)]. Only a small number of books surveyed contain both types of the curves: B vs H and M vs H as well as provide clarification of the terminology concerning H_c and H_{ci} - Kittel (1996), Elliott (1998), Dalven (1990), Skomski and Coey (1999), Jiles (1991), Arrott (1983), Donoho (1983), Rhyne (1983), Levy (1968), Anderson and Blotzer (1999), Vermariën *et al.* (1999). Barger and Olsson (1987) provide both graphs but terminology is only referred to

the B vs H graph. Most books deal only with one type of the hysteresis loop. The B vs H curve, which is more prone to misinterpretations, has been used more often in the surveyed books in all areas.

A few books deal with the M vs H curve and provide, with a few exceptions, correct description and graphs [see, e.g. Lovell *et al* (1981); Aharoni (1996); Wert and Thomson, 1970; Elwell and Pointon, 1979]. On the other hand, the M vs H curve is dominant in research papers surveyed Sung and Rudowicz (2002). Surprisingly, while most of the textbooks surveyed attempt to adhere to the SI units, all but a few research articles reviewed still use the CGS units.

This in itself is a worrying factor Sung and Rudowicz (2002). The various misconceptions and/or misinterpretations identified in the course of our comprehensive survey of textbooks can be classified into three categories. Below we provide a systematic review of the books with respect to the problems in each category.

Misinterpretation of the coercivity H_c on the B vs H curve as the point at which M=0.

This was the original problem which has triggered the textbook survey. Various examples of this misinterpretation, consisting in ascribing zero magnetization to the point H_c on the B vs H hysteresis loop, are listed below with the nature of the problem indicated by the pertinent sentences quoted.

Theoretical Solid state / condensed matter physics books

• *The magnetic field has to be reversed and raised to a value H_c (called the coercive force) in order to push domain walls over the barriers so that we regain zero magnetization.* Wilson (1979)

• *The point at which B=0 is the coercive field and is usually designated as H_c. It represents the magnetic field required to demagnetize the specimen* (Pollock, 1990)

• *The reverse field required to demagnetize the material is called the coercive force, H_c.* Pollock (1985)

• *To remove all magnetization from a specimen then requires the application of a field in the opposite direction termed the coercive field.* Elliott and Gibson (1978)

• *H at c is called the coercive force and is a measure of the field required to demagnetize the sample.* Rogalski and Palmer (2000).

General physics books

• *The coercive force is a measure of the magnitude of the external field in the opposite direction needed to reduce the residual magnetization to zero.* Ouseph, (1986)

• *In order to demagnetize the rod completely, H must be reversed in direction and increased to H_d, the coercive force.* Beiser (1986)

• *If the external field is reversed in direction and increased in strength by reversing the current, the domains reorient until the sample is again unmagnetized at point c, where B=0.* Serway (1990)

• *The magnetization does not return to zero, but remains (D) not far below its saturation value; and an appreciable reverse field has to be applied before it is much reduced again (E).. [where E corresponds to H_c in Fig. 1 (b), and later ."the field required to reverse the magnetization (point E on the graph) varies."* Akril *et al*, (1982)

Materials science and magnetism / electromagnetism books

• *In order to destroy the magnetization, it is then necessary to apply a reversed field equal to the coercive force H_c.* Anderson *et al* (1990)

- *"To reduce the magnetisation, B, to zero the direction of the applied magnetic field must be reversed and its magnitude increased to a value H_c."* John (1983) Note here the symbol B is confusingly used for the magnetization as discussed later.
- *"If the H field is now reversed, the graph continues down to R in the saturated case. This represents the H field required to make the magnetization zero within a saturation loop and is termed the coercivity of the material."* (Compton, 1986)
- *" the value of H when B=0 is called the coercivity, H_c; . It follows that the coercivity H_c is a measure of the field required to reduce M to zero."* Dugdale (1993)
- *Note that an external field of strength H_c, called the coercive field, is needed to obtain a microstructure with an equal volume fraction of domains aligned parallel and antiparallel to the external field (i.e., B = 0)..* Schaffer *et al* (1999)

Apparently, all the above quotes refer to the intrinsic coercivity H_{ci} as defined on the M vs H curve, whereas the B vs H curve was, in fact, used to explain the properties of the hysteresis loop. Neither a proper explanation about the validity of the approximation $H_c \approx H_{ci}$ nor information on the type of ferromagnetic materials described by a given schematic hysteresis loop was provided in all the quoted cases. Hence, such statements constitute misconceptions, which could be avoided if the authors defined the term *coercive force / coercivity* as the reverse field required to demagnetize (M = 0) the ferromagnetic material sample with a reference to the M vs H curve. Otherwise, when referring to the B vs H curve, the quantity H_c should rather be defined as the field required to bring the magnetic induction, instead of the magnetization, to zero. The description in the text and the curve used in the books cited above, simply imply that both B and M were equal to zero at the same value of H, i.e. H_c. However, since B $=\mu_o$ (H + M) , when B = 0, M is equal to $-H_c$. Only when H_c is very small, as it is the case for soft magnetic materials, the approximation M ≈ 0 at B $=0$ and $H_c \approx H_{ci}$ holds. Without explicitly stating the necessary conditions for the validity of such approximation, the presentations of the hysteresis loop expressed in the above quotes convey an incorrect concept of the zero magnetization at them point $-H_c$ on the B vs H curve as applicable to any kind of ferromagnetic materials. To predict the value of H on the B vs H curve for which in fact M = 0, we consider M = B/μ_o - H .

In the second quadrant of the hysteresis loop (see Fig. 1), we have $H_c \leq H \leq 0$, and hence M diminishes from M = B_r/ μ_o at H = 0 to the nonzero value at $-H_c$, i.e. M = $-H_c$. This means that the direction of the magnetization is still opposite to that of the applied field. Further increase of the negative H_c in the third quadrant on the B vs H curve yields M = 0 at H = H_{ci}. This is why the value of H_{ci} on the M vs H curve is always greater than that of Hc on the B vs H curve. This relationship is indicated schematically by a dot (the point $-H_{ci}$) in Fig. 1 (b). The values in Table 1 in Sung and Rudowicz (2002) illustrate that for strong permanent magnets H_{ci} is substantially larger in magnitude than H_c.

Conclusions and Suggestions

It appears that the two possible ways of presenting the hysteresis loop for ferromagnetic materials, B vs H and M vs H, are, to a certain extent, confused with each other in several textbooks. This leads to various misconceptions concerning the meaning of the physical quantities as well as the characteristic features of the hysteresis loop for the soft and hard magnetic materials. We suggest that the name *coercive force* (or *coercivity*) and the symbol H_c correctly defined for the B vs H

curve, should not be used if referred to the M vs H curve. Using in the latter case the adjective *"intrinsic"* and the symbol H_{ci} is strongly recommended. It may help avoiding the misconceptions discussed above and reduce the present confusion widely spread in the textbooks. Hence the authors and editors should pay more attention to proper definitions of the terms involved.

Interestingly, among the books by Beiser (1986, 1991, 1992), the book (1986) belongs to the *misinterpretation sample*, while the two later books (1991, 1992) are correct in this aspect. It is hoped that by bringing the problems in questions to the attention of physics teachers and students, the correct interpretation of the hysteresis loop will prevail in future.

Our survey of textbooks reveals several deeper pedagogical issues related to the presentation of the hysteresis loop, which may apply to various other topics as well. One is the distinction between the exact and approximate quantities and the related description of a physical situation. In the present case we have considered the approximation H small as compared with M, leading to B $\approx \mu_o$M and $H_c \approx H_{ci}$ for soft magnetic materials. If the conditions for which a given approximation is valid are not clearly stated, the approximate picture may be implicitly taken as a representation of the exact situation. The consequences of such misleading approach may be wide-ranging - from imprinting misconceptions, i.e. false images, in the students minds to misinterpretation of the properties of one class of materials (here, soft magnets) as being equivalent to those of another class (here, hard magnets). The inherent danger in using *schematic* diagrams for presentation of the dependencies between various physical quantities is another important issue. Having no units and values provided for the y- and x-axis constitutes a detachment from a real physical situation. It may not only hamper students understanding of the underlying physics, but also lead to false impressions about the relationships between the quantities involved and, in consequence, create misconceptions.

The drawbacks of schematic representation of each hysteresis loops are compounded by the space saving. and using a combined diagram, which implies the same limits and values are applicable for both types of magnetic materials. As we amply illustrated above this is far from the true situation. Schematic diagrams which do not reflect correctly the underlying physical situation become a piece of graphic art only. Providing neither symbols nor description of the quantities on the x- and y-axis of a graph [see, e.g. Fig. 15.9 in Machlup (1988)] should also be avoided in physics text as an inappropriate from both scientific and pedagogical point of view.

Finally, let us mention the idea of creating a website listing errors and misconceptions in textbooks. The individual lecturers could add up their knowledge in this respect to a well organized structure listing various topics. We strongly suggest that all Colleges, Polytechnics and Universities should create a website on these identified misconceptions and errors. Our initial Internet search for the keywords: *'errors', 'misprints', 'corrigenda', 'errata',* has, however, revealed no relevant sites. A similar idea was proposed by Hubisz (2000) concerning science textbooks. Interestingly we have located this website due to letter in American Physical Society Newsletter (April, 2001, p.4). Since the URL address was misprinted, we have tracked this site down via the university name (North Carolina State University). Only recently by chance we have learnt of the existing website listing errors in physics textbooks:

(http://www.escape.ca/~dcc/phys/errors.html). It appears that the benefits of such website for teachers, researchers and students in improving general understanding of physics may be substantial.

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