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Power flow analysis of radial and weakly meshed distribution systems with uncertain load and line parameters

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ABSTRACT

This paper presents power flow analysis of radial and meshed distribution systems taking into consideration of uncertain variations in load and line parameters. Interval arithmetic technique is used to model the power flow equations of distribution system to obtain the bus voltages and line active and reactive power flows. INTLAB Toolbox [14] is used to obtain the Hull of voltage profiles, line active and reactive power flows and line losses. Here we have studied the case of voltage profile, which is the key factor in deciding the active and reactive power flows in each line. The developed algorithm has been studied by considering Radial Distribution test systems of 15-Bus, 30-Bus, 33-Bus and 69-Bus available in the literature. The authors have verified the results obtained by the respective test systems with the developed algorithm for base case load and line data. The developed algorithm is easily modifiable to study the weakly meshed distribution system and this has been demonstrated for 33-Bus and 69-Bus test systems for which the Hull voltage profiles have been presented.

Introduction

The analysis of distribution system is an important area of activity as distribution systems provide the final link between the bulk power system and consumers. Many load flow algorithms specially suited for distribution system analysis have been proposed in literature, but the dominant method used for load flow analysis in distribution systems is Forward-Backward Sweep algorithm [1, 2]. When planning electricity distribution networks, a part of the data used in the calculations is more or less uncertain. The loads vary with time and it is not possible to predict an exact value for the peak load of a certain year. It is also possible that errors exist in our data for the existing network (e.g. geographical spread, size, complexity of network and incorrect line length or conductor type). For future expansions planning of electric power distribution systems there is a need for algorithms which takes care of uncertain variations in loads and system parameters for future load forecasting. For several years, deterministic methods based on engineering judgment and experiences, and also on worst case conditions, have been used successively on distribution system expansion planning. Over the past decade there have been several attempts to systematically take account of uncertainties in evaluation of power system analysis;

Teng [3], proposed a direct approach load flow for distribution system that inputs are crisp in nature and we will use that form of representation in this paper for interval inputs of network. In [4], Wang and Alvarado first proposed the application of interval arithmetic method for power flow analysis of transmission networks. In their study, they have taken a small 5-bus system for illustration.

In transmission system, Kenarangui and Seifi [5] considered fuzziness not only in power generation and loads but also in the availability of generating units. Kauhaniemi [6], uses approximate formula for the voltage drop in fuzzy load flow for distribution system. Das [7], applied interval arithmetic on radial distribution power flow. P K Satpathy, D Das, P B Dutta Gupta [8] modeled the uncertain variation in loads at each bus as fuzzy load distribution. Bijwe and Viswanadha Raju [9, 10], applied fuzzy set thoery for modeling uncertainty of input parameters in weakly meshed distribution system. Kim et al. [11], Considered multiple uncertainties for evaluation of available transfer capability using fuzzy power flow (Newton–Raphson method) and uses fuzzy liguistic description for modeling of demads of load points and Hao et al. [12], uses fuzzy set formulation in Newton–Raphson power flow method to form a fuzzy power flow.

In any particular period, the load demand varies over a certain range or an 'interval'. Hence, instead of having the 'snapshot' of the system at a particular instant, it would be more appropriate to find out the system conditions, when the load demand is varying over an interval. Power flow analysis of a distribution system, when the load demand is varying over an interval, can be performed by repeated simulations or by the use of interval arithmetic [12]. The interval arithmetic method for power flow solution of balanced radial and Meshed distribution systems is studied using INTLAB commands [13]; In this study, the load demands in the system at different buses are assumed to vary over intervals. The power flow algorithm chosen is essentially a backward sweep/forward sweep algorithm. However the real arithmetic calculations in the algorithm have been replaced by complex interval arithmetic calculations.

The proposed Algorithm has been developed using INTLAB Toolbox in MATLAB. For any n-Bus distribution system the proposed algorithm is used to obtain the Hull of voltage profiles, line active and reactive power flows and line losses. Here we have studied the case of voltage profile, which is the key factor in deciding the active and reactive power flows in (2)

each line by varying the line minimum and maximum limits of R/X ratio of line parameters and the load power factors. The effectiveness of developed algorithm has been studied by considering Radial Distribution test systems of 15-Bus, 30-Bus, 33-Bus and 69-Bus available in the literature. The authors have verified the results obtained for the respective test systems with the developed algorithm for base case load and line data with the results available and they are found to be in tolerable limits. The developed algorithm is easily modifiable to study the weakly meshed distribution system.

Interval Power Flow Analysis

A.Radial distribution system

The proposed method is developed based on a derived matrix load-current to branch current matrix (LCBC) and equivalent current injections [6]. In this section, the developed procedure will be described in detail. For bus-i, the complex load S_i is expressed as

$$S_i = P_i + jQ_i$$
, $i=1,2,...,n$ (1)
and the corresponding equivalent Load currents at the kth iteration of solution is

 $I_i^k = ((P_i + jQ_i)/V_i^k)^*$

Where \dot{V}_i^k and I_i^k are the bus voltage and equivalent current injection of bus-i at the kth iteration, respectively. Where I_i^k is the node currents and P_i^k, Q_i^k represents the active and reactive

$$\begin{bmatrix} B1\\ B2\\ B3\\ B4\\ B5\\ \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1\\ 0 & 1 & 1 & 1 & 1\\ 0 & 0 & 1 & 1 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I2\\ I3\\ I4\\ I5 + B6\\ I6 - B6 \end{bmatrix} - ----(3)$$

power load at ith node.

[<i>B</i> 1]		Γ1	1	1	1	1]	[I2]]
B 2		0	1	1	1	1	<i>I</i> 3	
<i>B</i> 3	-	0	0	1	1	0	<i>I</i> 4	(3)
B 4		0	0	0	1	0	I5 + B6	
B 5		0	0	0	0	1	I6 - B6	

The current injection is obtained from equation (2), the relationship between the bus current injections and branch currents can be obtained by applying Kirchhoff's current law (KCL) to the distribution network. An algorithm of finding nodes beyond all branches is used to find the branch currents as functions of equivalent current injections.

B.Meshed Distribution System



Fig-1: Simple weakly meshed distribution system

Some distribution feeders serving high-density load areas contain loops created by closing normally switches. The proposed method for radial is extended for weakly meshed distribution feeders. Existence of loops in the system does not affect the bus current injections, but new branches need to be added to the system. Fig.3 shows a simple case with a one loop. Taking the new branch current into account, the current injections of bus 5 and bus 6 will be

 $I_{5}^{1} = I_{5} + B_{6}, I_{6}^{1} = I_{6} - B_{6}$

Modification of equation (2.14) we have The above equation can be rewritten as

$$\begin{bmatrix} B1\\ B2\\ B3\\ B4\\ B5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1\\ 0 & 1 & 1 & 1 & 1 & 1\\ 0 & 0 & 1 & 1 & 0 & I4\\ 0 & 0 & 0 & 1 & 0 & I5\\ 0 & 0 & 0 & 0 & 1 & I6 \end{bmatrix} \begin{bmatrix} I2\\ I3\\ I4\\ I5\\ I6 \end{bmatrix} + \begin{bmatrix} 1 & 1\\ 1 & 1\\ 1 & 0\\ 1 & 0\\ 0 & 1 \end{bmatrix} \begin{bmatrix} B6\\ -B6 \end{bmatrix} - - - (4)$$

LILC Matrix

Current of tie branch '6' can be calculated as

$B_6 = V_5 - V_6 / Z_{66}$	(5)
$V(i+1) = V(i) - Z_{ki}(m) \times B(m)$	(6

Interval Mathematics in INTLAB

Interval Mathematics considers a set of methods for handling intervals that approximate uncertain data. These methods are based on the definition of both interval arithmetic and optimal scalar product. The maximal accuracy principle guarantees (by means of the outward rounding) the automatic control of errors in numerical computation. A real interval X is a nonempty subset of real numbers R, $X = [x1, x2] = \{x \in \mathbb{R} \mid x1 \le$ $x \le x^2$, where x1 is the *infimum* and x2 is the *supremum*. The set of real intervals is denoted by IR. An interval $X = [x1; x2] \in IR$ may not be representable on a machine if x1 and x2 are not machine numbers. In order to obtain a rounded interval \tilde{X} such that $X \in \tilde{X}$ (i.e., \tilde{X} is an approximation of X), x1 and x2 must be rounded downward and upward, respectively, which is called outward rounding. The midpoint, the diameter and the radius of an interval X are given, respectively, by mid(X) = X = 1/2 (x1 + 1)/2 (x1 +x2), diam(X) = x2 - x1 and $rad(X) = \frac{1}{2} diam(X)$. X can be also denoted by $X = \langle mid(X), rad(X) \rangle$. Interval arithmetic operations are defined such that the interval result encloses all possible real results, which guarantees the reliability of interval methods.

The operations in INTLAB are defined by

 $X * Y = \{x * y \mid x \in X, y \in Y\}$, for $* \in \{-,+,\times,\div\}$ and for X = [x1, x2] and $Y = [y1, y2] \in IR$ they are given by X+Y = [x1+y1, x2+y2]X-Y = [x1-y2, x2-y1] $X \times Y = [min\rho, max\rho]$ with $\rho = \{x1y1, x1y2, x2y1, x2y2\}$ $X \div Y = X \times [y_2^{-1}, y_1^{-1}]$ if $0 \notin Y$

The Matlab toolbox IntLab [15] uses the routine setround to control the rounding mode (see IEEE standard for floating point arithmetic [IEE 85]. Real intervals may be stored by either infimum and supremum or midpoint and radius. IntLab enables basic interval operations to be performed on real and complex interval scalars, vectors and matrices.

Proposed Algorithm

The computational steps involved in solving the power flow problem of a single source network in INTLAB Toolbox [15] are given in the following:

Step-1: Read the system data. Construct the tree and number the branches. Assume the initial voltage at all buses as source bus voltage.

Step-2: Declare the variables and parameters initially as complex interval variables using the command 'cintval' in INTLAB Toolbox.

Step-3: The interval variations in the loads and line parameters are specified using the 'cintval(a,r)' command, where 'a' nominal value of load and 'r' specifies the lower and upper interval variations form nominal value of load.

Step-4: Once the variables and the interval variations of loads and line parameters have been declared it is usual to compute current injections at each bus using eqn. (2)

Step-5: From the computed current injections; compute the branch currents in each branch-using LCBC matrix from eqn. (3).

Step-6: For meshed network, LILC matrix is formed from LCBC matrix as given by eqn.(4). The order of LILC matrix is (number of branches)**X**(twice the number of tie switches)

Step-7: Compute the current injections at tie line nodes using eqn. (5)

Step-8: Using LILC matrix currents are injected in to the tie-switch nodes in opposite directions.

Step-9: Compute the voltage magnitude at the receiving end bus of each branch using eqn. (6).

Step-10: Once currents are injected using LILC matrix, branch currents are updated and again radial load flow is run which completes one mesh iteration.

Step-11: Repeat step-4 to step-11 till the max magnitude of voltage difference between consecutive iterations is less than 0.0001 p.u

Studied Radial and Meshed Systems

Example-1: 33-Bus System

The Fig-2 and Fig-3the interval hull voltage profiles for 33-Bus radial and meshed distribution system are obtained with active and reactive loads varying by 80% of nominal value as lower interval and 120% of nominal value as upper interval, also the line parameters are varied by 1% in Fig-2 and 3% in Fig-3 in per unit. The upper graph in Fig-2 and Fig-3 is the interval hull voltage profile for Radial system and the lower graph is for Meshed system.

The lower and upper intervals are taken as symmetrical, i.e 20% variation in load at each bus and 1% or 3% variation in line impedances. The obtained results are very useful for the system planner designing appropriate size and type of the conductors and the insulators on the feeders. [0.30259 3.0259] are the minimum and maximum values of R/X values of impedances. [0.3162 0.9864] are the minimum and maximum value of load power factors at nominal loading conditions.



Fig-2: Represents the obtained Interval Hull Voltage profile for Radial and Meshed Distribution System with impedance variation of 1% of the nominal value.



Fig-3: Represents the obtained Interval Hull Voltage profile for Radial and Meshed Distribution System with impedance variation of 3% of the nominal value.

Example-2: 69-Bus System

The Fig-4 and Fig-5the interval hull voltage profiles for 69-Bus radial and meshed distribution system are obtained with active and reactive loads varying by 80% of nominal value as lower interval and 120% of nominal value as upper interval, also the line parameters are varied by 1% in Fig-4 and 3% in Fig-5 in per unit. The upper graph in Fig-4 and Fig-5 is the interval hull voltage profile for Radial system and the lower graph is for Meshed system.

The lower and upper intervals are taken as symmetrical, i.e 20% variation in load at each bus and 1% or 3% variation in line impedances. The obtained results are very useful for the system planner designing appropriate size and type of the conductors and the insulators on the feeders. [0.2986 3.3571] are the minimum and maximum values of R/X values of impedances. [0.7071 0.8638] are the minimum and maximum value of load power factors at nominal loading conditions.



Fig-4: Represents the obtained Interval Hull Voltage profile for Radial and Meshed Distribution System with impedance variation of 1% of the nominal value.



Fig-5: Represents the obtained Interval Hull Voltage profile for Radial and Meshed Distribution System with impedance variation of 1% of the nominal value.

Results And Discussion 15-bus system

The lower and upper voltage profiles for the studied test systems are obtained by using the Forward/Backward sweep algorithm proposed by [3]. A MATLAB program has been developed using INTLAB ToolBox commands. The uncertain interval variation bars are obtained using the command 'errorbars' in MATLAB. The lower and upper voltage profiles are obtained at $[0.8 \times P_{nominal} \ 1.2 \times P_{nominal}]$ and the feeder impedance variation by $\pm 0.3\%$ of the nominal value in p.u.

The uncertain interval bars of voltages at each bus helps in identifying the limits of variation of voltage profiles which in turn gives the current profiles flowing in branches from which the variations active and reactive power flows in each branch can also computed. For example Fig-7 the lower and upper voltage profiles interval bars gets overlapped at busses 3, 4, 5 and 6. So, for specific loading and line impedance variations the upper and lower interval bars gives the information about which lines are getting overloaded. Also it has been observed that as the bandwidth between the lower and upper interval of load variations are increased/decreased the interval variations in the error bars of voltages are also increased/decreased.



Fig-6 represents the uncertain interval lower and upper bars for a 15-Bus radial distribution system with the lower and upper R/X values of [1.0227 5.1304], Sub-station bus is not included for uncertain variations.



Fig-7 represents the uncertain interval lower and upper bars for a 15-Bus radial distribution system with the lower and upper R/X values of [1.0227 6.413], Sub-station bus is not included for uncertain variations.

Case-I

The R/X ratio is varied over an interval of $[RBX_{min} 1.5 \times RBX_{max}]$, $[RBX_{min} 2 \times RBX_{max}]$, $[RBX_{min} 3 \times RBX_{max}]$ it has been observed that there is a shift in the voltage profile curve, that is, as the maximum value of R/X is increased the voltages at each bus are decreased but there is no much variation in the uncertain interval variation of voltage at each bus when the RBX_{max} is increased form1.5 time the nominal value in p.u. When the RBX_{max} is increased by 2 and 3 time the nominal value in p.u the uncertain interval variation of voltage at each bus is decreased for the 15-Bus system. For the case with interval variation of $[RBX_{min}/3 3 \times RBX_{max}]$, that is as the bandwidth is increased between the interval variation in R/X values of feeder impedances not only there is a shift in the lower and upper voltage profiles but the uncertain variation bars at each bus of lower and upper voltage profiles are also increased and they get overlapped. With this bandwidth if the feeder impedance values are increased by $\pm 1\%$ form $\pm 0.3\%$ it has been observed that there is a shift in the lower and upper voltage profile curves but there is no much variation in the uncertain interval variations in error bars.

The concluding point is that as the feeder impedance (both R and X together are decreased and increased) there is either voltage profile increase or decrease; there are no much variations in the uncertain interval error bars of voltage at each bus. But as the feeder impedance is increase/decrease there is decrease/increase in the uncertain interval error bars of voltage at each bus. Also the bandwidth of interval variation in R/X helps in planning the distribution system to fix the appropriate ratings of Circuit Breakers and Relay Co-ordinations. **Case-II**

The power factor of the load is varied from 0 to 1. For power factor of 0.0 $(\cos(90^0))$, 0.5 $(\cos(60^0))$, 0.7071 $(\cos(45^0))$, 0.8 $(\cos(36.86^0))$, 0.9063 $(\cos(25^0))$ and 1.0 $(\cos(0^0))$ of Nominal loads at each bus there is a decrease in the lower and upper voltage profiles also the uncertain interval variations at each bus are slightly decreased. But, for the case with power factor of '**zero**' not only there is an increase in the lower and upper voltage profiles, also the uncertain interval variations at each bus are increased and for the case with power factor of '**one**' the lower and upper voltage profiles increases more than the case with power factor '**zero**', apart from this the uncertain interval variation bars are also more than the case with power factor '**zero**'.

As the power factor is close to **'one'** that is for resistive loads uncertain interval variation bars are more for load interval variation over $[0.8 \times P_{nominal} \ 1.2 \times P_{nominal}]$ with $\pm 0.3\%$ in the feeder impedance. Similarly for the case with pure inductive loads that is with power factor of **'zero'** the uncertain interval variation bars are more when compared to the case with power factors of $0.9063 \ (\cos(25^0)), \ 0.8 \ (\cos(36.86^0)), \ 0.7071 \ (\cos(45^0)), \ 0.5 \ (\cos(60^0))$, but less than that for the case with power factor of **'one'**.

30-Bus System

The lower and upper voltage profiles are obtained at $[0.7 \times P_{nominal} 1.3 \times P_{nominal}]$ and the feeder impedance variation by $\pm 0.3\%$ of the nominal value in p.u.

Case-I

For [RBX_{min} $3 \times RBX_{max}$] it has been observed that not only the lower voltage profile is decreasing as there is more drop in voltage across the lines, the uncertain interval variation bars are decreasing. Also as the R/X values of lines from 6 to 29 are increased by three times, then there is more drop in voltage and the uncertain interval variation bars are decreased. This has been absorbed for 15-Bus system also.

Case-II

The power factor of the load is varied from 0 to 1. For power factor of 0.0 $(\cos(90^0))$, 0.5 $(\cos(60^0))$, 0.7071 $(\cos(45^0))$, 0.8 $(\cos(36.86^0))$, 0.9063 $(\cos(25^0))$ and 1.0 $(\cos(0^0))$ of Nominal loads at each bus there is a decrease in the lower and upper voltage profiles also the uncertain interval variations at each bus are slightly decreased. For the case of power factor 'zero' and 'one' the same is observed as with 15-Bus system. Similar observations have been found with the 33-Bus and the 69-Bus

distribution systems at different loading intervals and $\ensuremath{R/X}$ intervals.

Conclusion

Power flow analysis of radial and meshed distribution system with uncertain variations in loads and system parameters is presented using INTLAB Toolbox in MATLAB. The advantage of the method is that the uncertain interval variations in the voltages at each bus and line power flows can be found. This information is useful for the system planner to appropriately fix the ratings of insulators and circuit breakers with large variations in loads that occur in the system. The developed algorithm has been used to study the standard test systems available in the literature and the results have been discussed for variations in R/X ratio and power factors of loads. **References**

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