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# Effect of variable suction and radiative heat transfer on magnetohydrodynamic couette flow through a porous medium in the slip flow regime

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**ABSTRACT**

The objective of this paper is to analyze the effect of variable suction and radiative heat transfer on unsteady magnetohydrodynamic free convective couette flow of a viscous incompressible electrically conducting fluid in the slip flow regime in presence of variable suction and radiative heat source. The governing equations of the flow field are solved employing perturbation technique and the expressions for the velocity, temperature, skin friction and the rate of heat transfer i.e. the heat flux in terms of Nusselts number $N_u$ are obtained. The effects of the pertinent parameters such as magnetic parameter $M$, permeability parameter $K_p$, Grashof number for heat transfer $G_r$, radiation parameter $F$, suction parameters $\alpha_1,\alpha_2$; slip flow parameters $h_1$, $h_2$; Prandtl number $P_r$ etc. on the flow field have been studied and the results are presented graphically and discussed quantitatively. The problem has some relevance in geophysical, astrophysical and cosmical studies.



## 1.Introduction

The phenomenon of magnetohydrodynamic couette flow with heat transfer has been a subject of interest of many researchers because of its possible applications in many branches of science and technology. Channel flows through porous media have several engineering and geophysical applications such as in the field of chemical engineering for filtration and purification process, in the field of agricultural engineering to study the underground resources, in the petroleum industry to study the movement of natural gas, oil and water through the oil channels and reservoirs.

In recent years, flow through porous media has been a subject of general interest of many researchers. A series of investigations have been made by different scholars where the porous medium is either bounded by horizontal or vertical surfaces. Tao [1] studied the magnetohydrodynamic effects on the formation of couette flow. Sattar [2] reported the free and forced convection boundary layer flow through a porous medium with large suction. Sattar and Alam [3] analyzed the effect of thermal diffusion and transpiration on MHD free convective and mss transfer flow past an accelerated vertical porous plate. Attia and Kotb [4] have investigated the magnetohydrodynamic flow between two parallel plates with heat transfer. Das *et al*. [5] investigated the hydromagnetic flow and heat transfer between two stretched/squeezed horizontal porous plates. Nagraju *et al*. [6] estimated the effect of simultaneous radiative and convective heat transfer in a variable porosity medium. Taneja and Jain [7] explained the hydromagnetic flow in the slip flow regime with time dependent suction. Das and his associates [8] discussed the hydromagnetic flow and heat transfer of an elastico-viscous fluid between two horizontal parallel porous plates employing finite difference scheme.

The problem of oscillatory MHD slip flow along a porous vertical wall in a medium with variable suction in the presence of radiation was analyzed numerically by Ogulu and Prakash [9]. Makinde [10] investigated the free convection flow with thermal radiation and mass transfer past a moving vertical porous plate. Das and his co-workers [11] discussed the laminar flow of an elastico-viscous Rivlin-Ericksen fluid through porous parallel plates with suction and injection, the lower plate being stretched. Ogulu and Motsa [12] investigated the problem of radiative heat transfer to magnetohydrodynamic couette flow with variable wall temperature. Cortell [13] studied the flow and heat transfer of a fluid through a porous medium over a stretching surface with internal heat generation/absorption and suction/ blowing. Das and his co-workers [14] analyzed the effect of heat source and variable magnetic field on unsteady hydromagnetic flow of a viscous stratified field past a porous flat moving plate in the slip flow regime. In a separate paper Das *et al*. [15] studied the hydromagnetic three dimensional couette flow and heat transfer. Recently, Das and his associates [16] estimated the effect of mass transfer on free convective MHD flow of a viscous fluid bounded by an oscillating porous plate in the slip flow regime in presence of heat source. Sharma and Singh [17] investigated the unsteady MHD free convective flow and heat transfer along a vertical porous plate with variable suction and internal heat generation

The study reported herein theoretically analyzes the effect of variable suction and radiative heat transfer on unsteady hydromagnetic free convective couette flow of a viscous incompressible electrically conducting fluid in the slip flow regime. The governing equations of the flow fluid are solved for velocity, temperature, skin friction and rate of heat transfer and the effects of the pertinent parameters on the flow fluid have

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been analyzed and the results are presented graphically and discussed quantitatively.

## 2. Formulation of the problem

Consider a two dimensional unsteady free convective magnetohydrodynamic flow of a viscous incompressible electrically conducting fluid between two vertical parallel porous plates placed at a distance $d$ apart in the slip flow regime in presence of variable suction and radiative heat source. Let the medium between the plates be filled with a porous material of permeability

$$K'(t') = K_0\left(1+\varepsilon B e^{-\omega' t'}\right)$$

and a time dependent suction

$$v'(t') = -v'_0\left(1+\varepsilon A e^{-\omega' t'}\right)$$

be applied at the plate $y$=0 and the same injection velocity be applied at the pate $y$=1. We choose $x$-axis along the plate and $y$-axis normal to it. Under the above conditions the equations governing the flow are:

Momentum equation:

$$\frac{\partial u'}{\partial t'} - v'_0\left(1+\varepsilon A e^{-\omega' t'}\right)\frac{\partial u'}{\partial y'} = g\beta(T'-T'_h) + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho}u' - \frac{\nu}{K'(t')}u' \quad (1)$$

Energy equation:

$$\frac{\partial T'}{\partial t'} - v'_0\left(1+\varepsilon A e^{-\omega' t'}\right)\frac{\partial T'}{\partial y'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y'} \quad (2)$$

The boundary conditions of the problem are:

$$u' - U_1 = L_1\frac{\partial u'}{\partial y'},\ \frac{\partial T'}{\partial y'} = -\frac{q}{k},\quad at\ y'=0,$$

$$u' - U_2 = L_2\frac{\partial u'}{\partial y'},\ T' = T'_h,\quad at\ y'=d, \quad (3)$$

where $L_1 = \frac{(2-\mu_1)}{\mu_1}L$, $L$ being the mean free path and $\mu_1$, the Maxwell's reflection coefficient.

The radiative heat flux $q_r$ is given by

$$\frac{\partial q_r}{\partial y'} = 4(T'-T'_h)I, \quad (4)$$

where $I = \int k_{\lambda w}\frac{\partial e_{b\lambda}}{\partial T}d\lambda$, $k_{\lambda w}$ is the absorption coefficient at the wall, $e_{b\lambda}$ is Planck's function and $\lambda$ is the frequency, $u'$ is the velocity, $T'$ is the temperature, $B_0$ is the uniform transverse magnetic field, β is the volumetric coefficient of expansion for heat transfer, β* is the volumetric coefficient of expansion for mass transfer, $k$ is the thermal conductivity, ν is the kinematic viscosity, $C_p$ is the specific heat at constant pressure, σ is the electrical conductivity, $g$ is the acceleration due to gravity, $A$ and $B$ are the real positive constants, $t$ is the time and ε is a small positive number such that ε$A$<<1 and ε$B$<<1.

Introducing the following non-dimensional variables and parameters,

$$y = \frac{y' v'_0}{\nu},\ t = \frac{t' v_0'^2}{\nu},\ \omega = \frac{\nu\omega'}{v_0'^2},\ u = \frac{u'}{v'_0},\ \nu = \frac{\eta_0}{\rho},\ M = \left(\frac{\sigma B_0^2}{\rho}\right)\frac{\nu}{v_0'^2},\ K_p = \frac{v_0^2 K'_0}{\nu^2},$$

$$\theta = \frac{k v'_0(T'-T'_h)}{\nu q},\ P_r = \frac{\rho\nu C_p}{k},\ G_r = \frac{\nu^2 g\beta q}{k v_0'^4},\ F = \frac{4\nu I}{\rho C_p v_0'^2},$$

$$\alpha_1 = \frac{U_1}{v_0},\ \alpha_2 = \frac{U_2}{v_0},\ R = \frac{v'_0 d}{\nu},\ h_1 = \frac{L_1 v'_0}{\nu},\ h_2 = \frac{L_2 v'_0}{\nu}. \quad (5)$$

in equations (1)-(2), we get the following non-dimensional equations

$$\frac{\partial u}{\partial t} - \left(1+\varepsilon A e^{-\omega t}\right)\frac{\partial u}{\partial y} = G_r\theta + \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K_p\left(1+\varepsilon B e^{-\omega t}\right)}\right)u, \quad (6)$$

$$\frac{\partial\theta}{\partial t} - \left(1+\varepsilon A e^{-\omega t}\right)\frac{\partial\theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2\theta}{\partial y} - F\theta. \quad (7)$$

The corresponding boundary conditions are:

$$u = \alpha_1 + h_1\frac{\partial u}{\partial y},\quad \frac{\partial\theta}{\partial y} = -1,\quad at\ y=0,$$

$$u = \alpha_2 + \frac{\partial u}{\partial y},\quad \theta = 0,\quad at\ y=d. \quad (8)$$

## 3. Method of Solution

We now seek the solutions for equations (6)-(7) under boundary conditions (8) for a particular case $R$=1, which is valid for an incompressible fluid. In order to solve equations (6) - (7), we assume

$$u(y,t) = u_0(y) + \varepsilon u_1(y)\,e^{-\omega t} + O(\varepsilon^2), \quad (9)$$

$$\theta(y,t) = \theta_0(y) + \varepsilon\theta_1(y)\,e^{-\omega t} + O(\varepsilon^2). \quad (10)$$

Using equations (9)-(10) in equations (6)-(7), we get the following zeroth order and first order equations:

Zeroth order:

$$-\frac{\partial u_0}{\partial y} = G_r\theta_0 + \frac{\partial^2 u_0}{\partial y^2} - \left(M + \frac{1}{K_p}\right)u_0, \quad (11)$$

$$-\frac{\partial\theta_0}{\partial y} = \frac{1}{P_r}\frac{\partial^2\theta_0}{\partial y} - F\theta_0. \quad (12)$$

First order:

$$-\omega u_1 - A\frac{\partial u_0}{\partial y} - \frac{\partial u_1}{\partial y} = G_r\theta_1 + \frac{\partial^2 u_1}{\partial y^2} - \left(M + \frac{1}{K_p}\right)u_1 + \frac{B u_0}{K_p}, \quad (13)$$

$$-\omega\theta_1 - A\frac{\partial\theta_0}{\partial y} - \frac{\partial\theta_1}{\partial y} = \frac{1}{P_r}\frac{\partial^2\theta_1}{\partial y} - F\theta_1. \quad (14)$$

The corresponding boundary conditions are

$$u_0 = \alpha_1 + h_1\frac{\partial u_0}{\partial y},\ u_1 = h_1\frac{\partial u_1}{\partial y},\ \frac{\partial\theta_0}{\partial y} = -1,\ \frac{\partial\theta_1}{\partial y} = 0,\quad at\ y=0,$$

$$u_0 = \alpha_2 + h_2\frac{\partial u_0}{\partial y},\ u_1 = h_2\frac{\partial u_1}{\partial y},\ \theta_0 = 0,\ \theta_1 = 0,\quad at\ y=1. \quad (15)$$

The solutions of equations (11) - (14) under boundary condition (15) are given by

$$u(y,t)=\left(A_5e^{n_1y}+A_6e^{n_2y}-B_3e^{m_1y}-B_4e^{m_2y}\right)+\varepsilon e^{-\omega t}\left(A_7e^{n_3y}+A_8e^{n_4y}\right.$$
$$\left.+B_5e^{m_1y}+B_6e^{m_2y}-B_7e^{m_3y}-B_8e^{m_4y}-B_9e^{n_1y}-B_{10}e^{n_2y}\right), \qquad (16)$$

$$\theta(y,t)=\left(A_1e^{m_1y}+A_2e^{m_2y}\right)+\varepsilon e^{-\omega t}\left(A_3e^{m_3y}+A_4e^{m_4y}-B_1e^{m_1y}-B_2e^{m_2y}\right). \qquad (17)$$

The wall shear stress i.e. the skin friction at the wall is given by

$$\tau=\left(\frac{\partial u}{\partial y}\right)_{y=0}=\left(\frac{\partial u_0}{\partial y}\right)_{y=0}+\varepsilon e^{-\omega t}\left(\frac{\partial u_1}{\partial y}\right)_{y=0} \qquad (18)$$

Using equation (16) in equation (18), it is given by

$$\tau=\left(n_1A_5+n_2A_6-m_1B_3-m_2B_4\right)+\varepsilon e^{-\omega t}\left(n_3A_7+n_4A_8\right.$$
$$\left.+m_1B_5+m_2B_6-m_3B_7-m_4B_8-n_1B_9-n_2B_{10}\right). \qquad (19)$$

The rate of heat transfer i.e. the heat flux at the wall in terms of Nusselts number is given by

$$N_u=\left(\frac{\partial\theta}{\partial y}\right)_{y=0}=\left(\frac{\partial\theta_0}{\partial y}\right)_{y=0}+\varepsilon e^{-\omega t}\left(\frac{\partial\theta_1}{\partial y}\right)_{y=0} \qquad (20)$$

Using equation (17) in equation (20), it is given by

$$N_u=\left(m_1A_1+m_2A_2\right)+\varepsilon e^{-\omega t}\left(m_3A_3+m_4A_4-m_1B_1-m_2B_2\right) \qquad (21)$$

where, $m_1=-\frac{P_r}{2}+\frac{1}{2}\sqrt{P_r^2+4P_rF}$, $m_2=-\frac{P_r}{2}-\frac{1}{2}\sqrt{P_r^2+4P_rF}$,

$m_3=-\frac{P_r}{2}+\frac{1}{2}\sqrt{P_r^2-4P_r(\omega-F)}$, $m_4=-\frac{P_r}{2}-\frac{1}{2}\sqrt{P_r^2-4P_r(\omega-F)}$,

$n_1=-\frac{1}{2}+\frac{1}{2}\sqrt{1+4\left(M+\frac{1}{K_p}\right)}$, $n_2=-\frac{1}{2}-\frac{1}{2}\sqrt{1+4\left(M+\frac{1}{K_p}\right)}$,

$n_3=-\frac{1}{2}+\frac{1}{2}\sqrt{1+4\left(M+\frac{1}{K_p}-\omega\right)}$,

$n_4=-\frac{1}{2}-\frac{1}{2}\sqrt{1+4\left(M+\frac{1}{K_p}-\omega\right)}$, $A_1=\frac{e^{m_2}}{\left(m_2e^{m_1}-m_1e^{m_2}\right)}$,

$A_2=-\frac{A_1e^{m_1}}{e^{m_2}}$, $A_3=\frac{e^{m_4}\left(B_1m_1+B_2m_2\right)-m_4\left(B_1e^{m_1}+B_2e^{m_2}\right)}{\left(m_3e^{m_4}-m_4e^{m_3}\right)}$,

$A_4=\frac{B_1m_1+B_2m_2-A_3m_3}{m_4}$,

$A_5=\frac{E_3E_5-E_2E_6}{E_1E_5-E_2E_4}$, $A_6=\frac{E_3-A_5E_1}{E_2}$, $A_7=\frac{E_9E_{11}-E_8E_{12}}{E_7E_{11}-E_8E_{10}}$,

$A_8=\frac{E_9-A_7E_7}{E_8}$,

$B_1=\frac{AP_rA_1m_1}{m_1^2+m_1P_r+P_r(\omega-F)}$, $B_2=\frac{AP_rA_2m_2}{m_2^2+m_2P_r+P_r(\omega-F)}$,

$B_3=\frac{G_rA_1}{m_1^2+m_1-\left(M+\frac{1}{K_p}\right)}$, $B_4=\frac{G_rA_2}{m_2^2+m_2-\left(M+\frac{1}{K_p}\right)}$,

$B_5=\frac{G_rB_1+AB_3m_1+\frac{BB_3}{K_p}}{m_1^2+m_1-\left(M+\frac{1}{K_p}-\omega\right)}$, $B_6=\frac{G_rB_2+AB_4m_2+\frac{BB_4}{K_p}}{m_2^2+m_2-\left(M+\frac{1}{K_p}-\omega\right)}$,

$B_7=\frac{G_rA_3}{m_3^2+m_3-\left(M+\frac{1}{K_p}-\omega\right)}$, $B_8=\frac{G_rA_4}{m_4^2+m_4-\left(M+\frac{1}{K_p}-\omega\right)}$,

$B_9=\frac{AA_5n_1+\frac{BA_5}{K_p}}{n_1^2+n_1-\left(M+\frac{1}{K_p}-\omega\right)}$, $B_{10}=\frac{AA_6n_2+\frac{BA_6}{K_p}}{n_2^2+n_2-\left(M+\frac{1}{K_p}-\omega\right)}$,

$E_1=1-h_1n_1$, $E_2=1-h_1n_2$,

$E_3=\alpha_1+B_3\left(1-h_1m_1\right)+B_4\left(1-h_1m_2\right)$, $E_4=e^{n_1}\left(1-h_2n_1\right)$,

$E_5=e^{n_2}\left(1-h_2n_2\right)$,

$E_6=\alpha_2+B_3e^{m_1}\left(1-h_2m_1\right)+B_4e^{m_2}\left(1-h_2m_2\right)$,

$E_7=1-h_1n_3$, $E_8=1-h_1n_4$,

$E_9=-B_5\left(1-h_1m_1\right)-B_6\left(1-h_1m_2\right)+B_7\left(1-h_1m_3\right)+B_8\left(1-h_1m_4\right)+B_9\left(1-h_1n_1\right)+B_{10}\left(1-h_1n_2\right)$,

$E_{10}=e^{n_3}\left(1-h_2n_3\right)$,

$E_{11}=e^{n_4}\left(1-h_2n_4\right)$,

$E_{12}=-B_5e^{m_1}\left(1-h_2m_1\right)-B_6e^{m_2}\left(1-h_2m_2\right)+B_7e^{m_3}\left(1-h_2m_3\right)+B_8e^{m_4}\left(1-h_2m_4\right)+B_9e^{n_1}\left(1-h_2n_1\right)+B_{10}e^{n_2}\left(1-h_2n_2\right)$.

## 4. Results and of discussion

The effect of variable suction and radiative heat transfer on unsteady hydromagnetic free convective couette flow of a viscous incompressible electrically conducting fluid in the slip flow regime has been studied. The governing equations of the flow fluid are solved for velocity, temperature, skin friction and the rate of heat transfer and the effects of the pertinent parameters on the flow fluid have been discussed with the aid of velocity profiles 1-8, temperature profiles 9-10 and skin friction profiles 11-14.

### 4.1. Velocity field

The velocity of the flow field suffers a change in magnitude with the variation of the flow parameters. The important flow parameters affecting the velocity field are magnetic parameter $M$, permeability parameter $K_p$, Grashof number for heat transfer $G_r$, radiation parameter $F$, suction parameters $\alpha_1, \alpha_2$ and velocity slip parameters $h_1$, $h_2$.The effects of these parameters on the velocity field have been discussed and analyzed with the help of Figures 1-8.

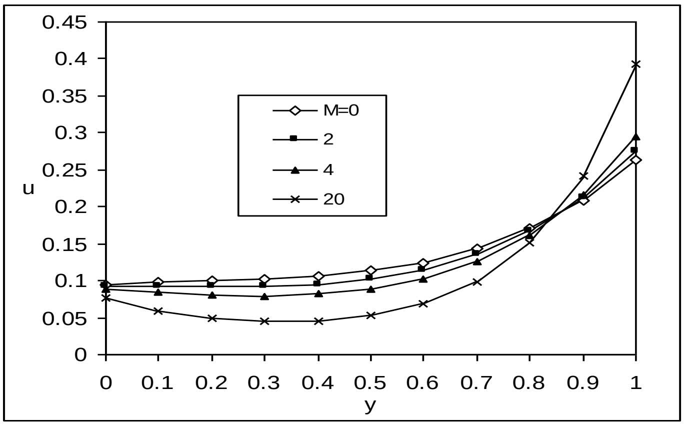


**Figure 1. Velocity profiles against y for different values of M with $K_p$=1, $G_r$=2, F=0.1, $\alpha_1$=0.1, $\alpha_2$=0.2, $h_1$=0.1, $h_2$=0.1, $P_r$=0.71, A=0.5, B=0.5, $\omega$=0.1, t=0.1, $\in$=0.02**

The effect of magnetic parameter $M$ on the velocity field is shown in Figure 1. A growing magnetic parameter is found to retard the velocity of the flow field up to a certain distance and thereafter effect reverses. Figure 2 depicts the effect of permeability parameter $K_p$ on the velocity field. An increase in permeability parameter tends to accelerate the velocity of the flow field up to a certain distance from the plate and thereafter it reverses the effect. Figure 3 elucidates the effect of Grashof number $G_r$ on the velocity field.

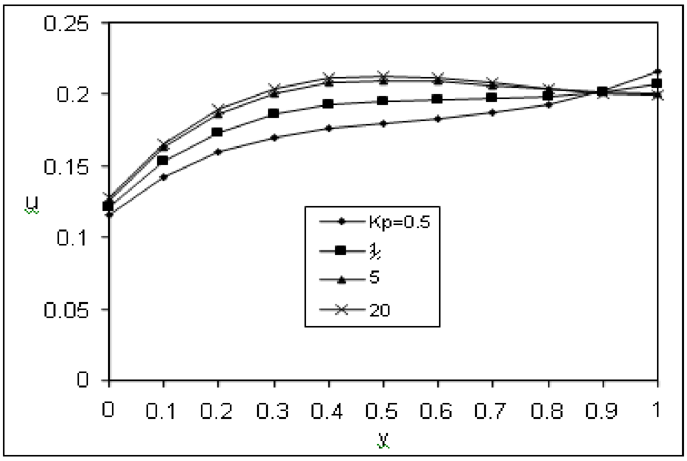


**Figure 2. Velocity profiles against y for different values of $K_p$ with M=1, $G_r$=2, F=0.1, $\alpha_1$=0.1, $\alpha_2$=0.2, $h_1$=0.1, $h_2$=0.1, $P_r$=0.71, A=0.5, B=0.5, $\omega$=0.1, t=0.1, $\in$=0.02**

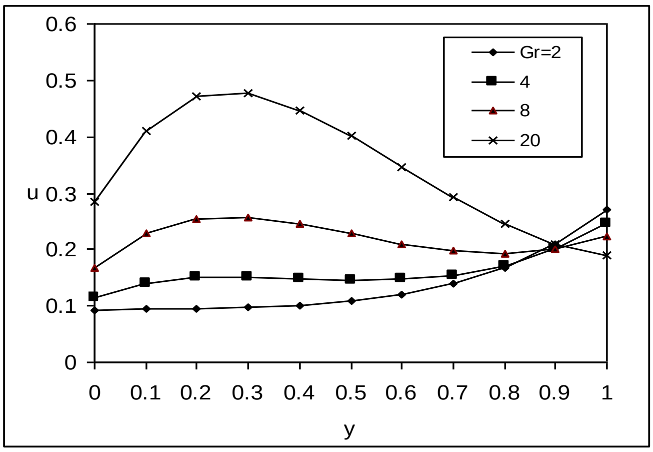


**Figure 3. Velocity profiles against y for different values of $G_r$ with M=1, $K_p$=1, F=0.1, $h_1$=0.1, $h_2$=0.1, $\alpha_1$=0.1, $\alpha_2$=0.2, $P_r$=0.71, A=0.5, B=0.5, $\omega$=0.1, t=0.1, $\in$=0.02**

The velocity of the flow field increases at all points of the flow field due to an increase in Grashof number in the flow field. Figure 4 analyses the effect of radiation parameter $F$ on the velocity field. Comparing the curves of the figure it is observed that a growing radiation parameter retards the velocity of the flow field at all points. The effects of suction parameters $\alpha_1$ and $\alpha_2$ on the velocity field are discussed in Figures 5 and 6. Both the parameters show an increase in velocity of the flow field at all points in the flow field in a different manner. The parameter $\alpha_1$ is tending to converge the velocity profiles to a point while the parameter $\alpha_2$ is tending to diverge the velocity profile from a point. The behaviour of velocity slip parameters $h_1$ and $h_2$ are shown in Figures 7 and 8 respectively. Both the parameters tend to enhance the velocity of the flow field at all points in the similar manner as $\alpha_1$ and $\alpha_2$ but some discrepancy is found in case of $h_2$.

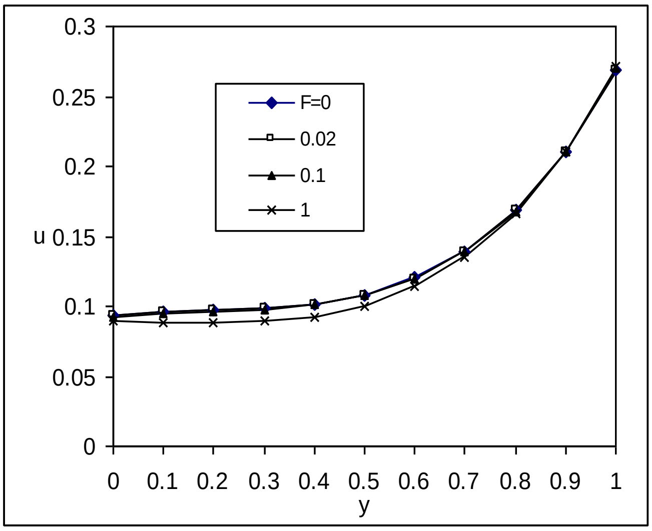


**Figure 4. Velocity profiles against y for different values of F with M=1, $K_p$=1, $G_r$=2, $h_1$=0.1, $h_2$=0.1, $\alpha_1$=0.1, $\alpha_2$=0.2, $P_r$=0.71, A=0.5, B=0.5, $\omega$=0.1, t=0.1, $\in$=0.02**

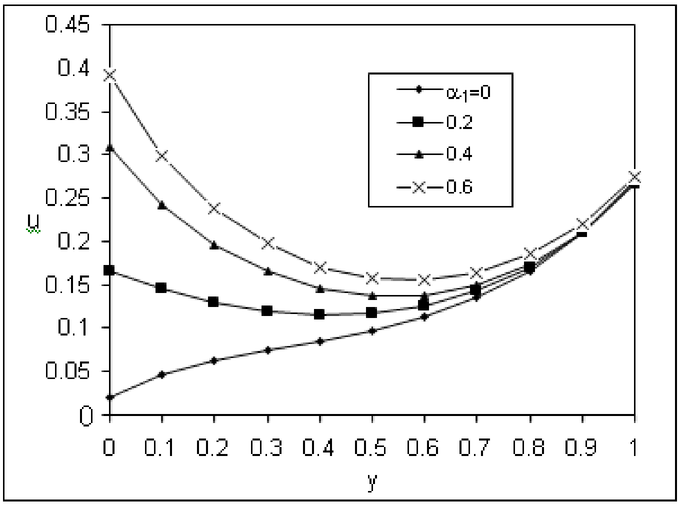


**Figure 5. Velocity profiles against y for different values of $\alpha_1$ with M=1, $K_p$=1, $G_r$=2, F=0.1, $\alpha_2$=0.2, $h_1$=0.1, $h_2$=0.1, $P_r$=0.71, A=0.5, B=0.5, $\omega$=0.1, t=0.1, $\in$=0.02**

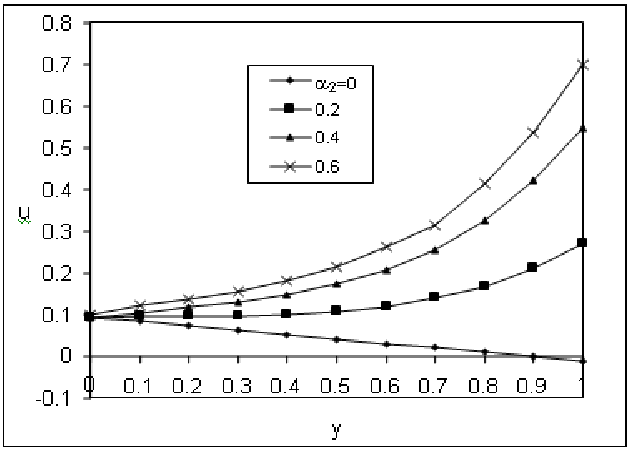


**Figure 6. Velocity profiles against y for different values of $\alpha_2$ with M=1, $K_p$=1, $G_r$=2, F=0.1, $P_r$=0.71, A=0.5, B=0.5, $\alpha_1$=0.1, $h_1$=0.1, $h_2$=0.1, ω=0.1, t=0.1, ∈=0.02**

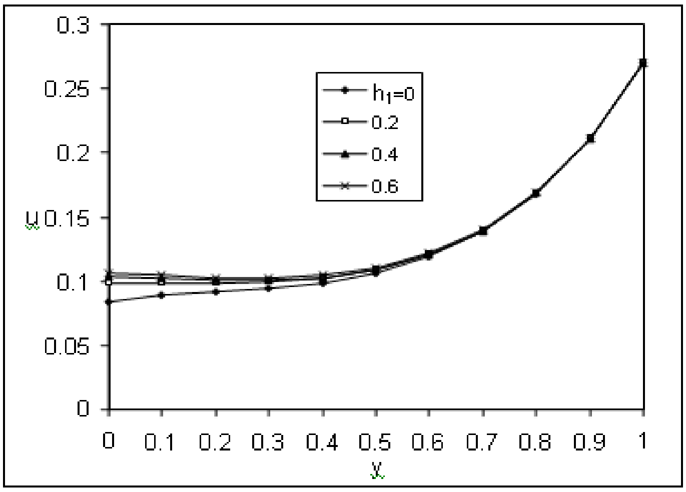


**Figure 7. Velocity profiles against y for different values of $h_1$ with M=1, $K_p$=1, $G_r$=2, F=0.1, $\alpha_1$=0.1, $\alpha_2$=0.2, $h_2$=0.1, $P_r$=0.71, A=0.5, B=0.5,ω=0.1, t=0.1, ∈=0.02**

### 4.2. Temperature field

The temperature of the flow field varies appreciably with the variation of Prandtl number $P_r$ and radiation parameter $F$. Figures 9-10 elucidate the effects of these parameters on the temperature field. Comparing the curves of both the figures it is observed that both the parameters show a decrease in temperature of the flow field at all points, but the effect is more pronounced in case of Prandtl number $P_r$ .

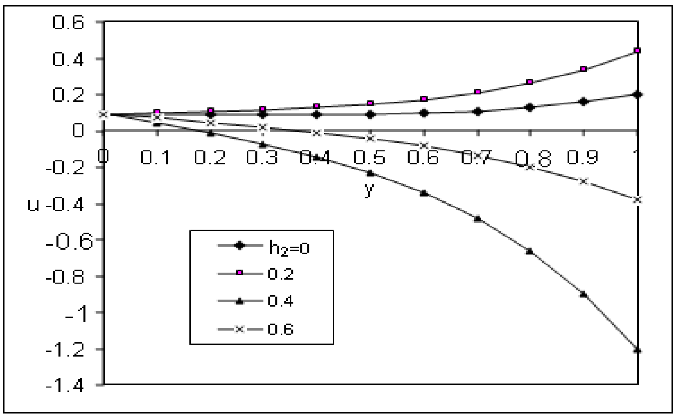


**Figure 8. Velocity profiles against y for different values of $h_2$ with M=1, $K_p$=1, $G_r$=2, F=0.1, $\alpha_1$=0.1, $\alpha_2$=0.2, $h_1$=0.1, $P_r$=0.71, A=0.5, B=0.5, ω=0.1, t=0.1, ∈=0.02**

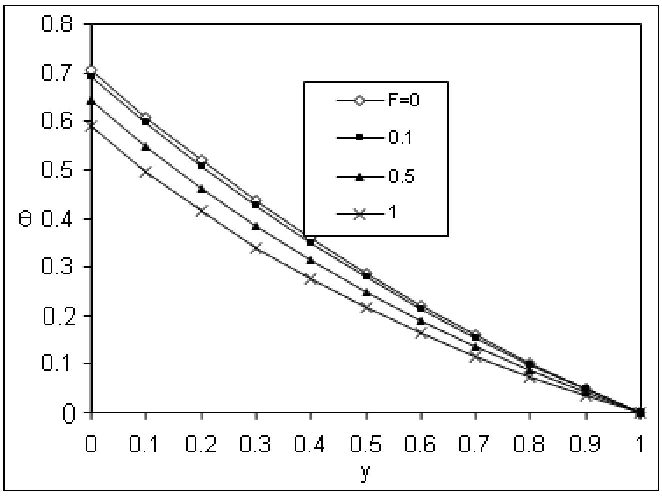


**Figure 9. Temperature profiles against y for different values of F with $P_r$=0.71, A=0.5, ω=0.1, t=0.1, ∈=0.02**

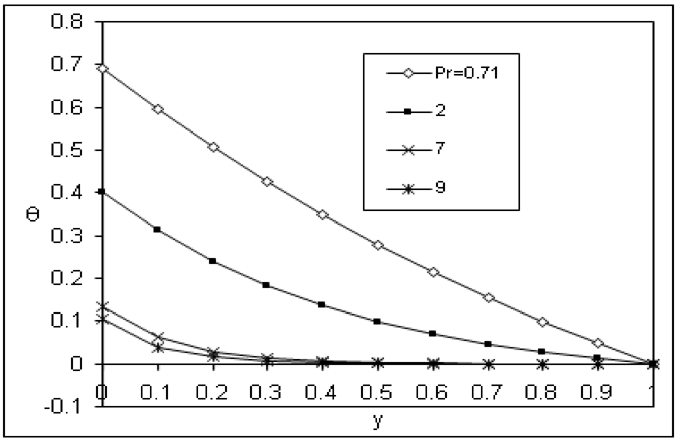


**Figure 10. Temperature profiles against y for different values of $P_r$ with F=0.1, A=0.5, ω=0.1, t=0.1, ∈=0.02**

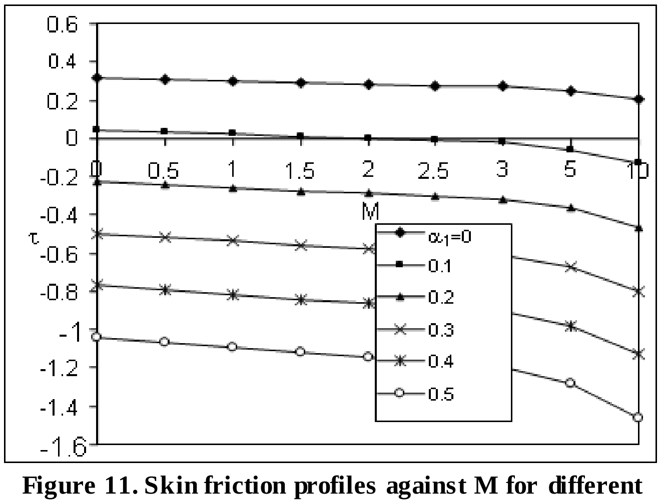


**Figure 11. Skin friction profiles against M for different values of $\alpha_1$ with $K_p$=1, $G_r$=2, F=0.1, $\alpha_2$=0.2, $h_1$=0.1, $h_2$=0.1, $P_r$=0.71, A=0.5, B=0.5, ω=0.1, t=0.1, ∈=0.02**

### 4.3. Skin friction

The variations in the value of skin-friction τ at the wall with the change of flow parameters are shown in Figures 11-14. Figures 11 and 12 depict the effects of suction parameters $\alpha_1$ and $\alpha_2$ on the skin friction at the wall. A growing suction parameter $\alpha_1$ decreases the skin friction at the wall while the other suction parameter $\alpha_2$ increases it at all points. Keeping $\alpha_1$ and $\alpha_2$ constant a growing magnetic parameter is found to decrease the skin friction at the wall. Figure 13 analyzes the effect of Grashof number $G_r$ and magnetic parameter $M$ on the skin friction at the plate. A growing Grashof number enhances the skin friction while the magnetic parameter reduces its value at all points. Figure 14 discusses the effect of radiation parameter $F$ and magnetic parameter $M$ on the skin friction at the wall. A comparison of curves of the figure shows that both the parameters reduce the skin friction at the plate.

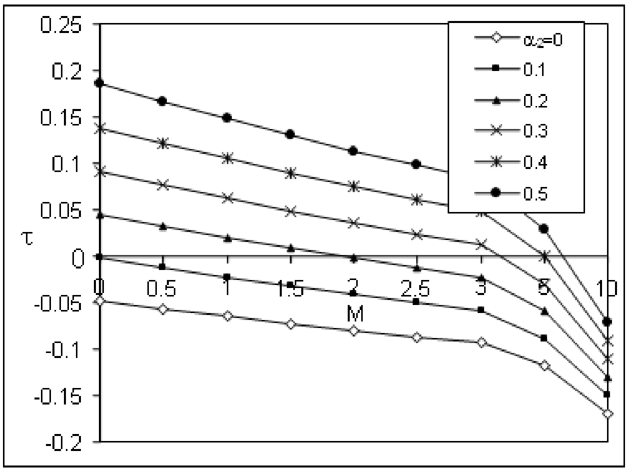


**Figure 12. Skin friction profiles against M for different values of $\alpha_2$ with $K_p$=1, $G_r$=2, F=0.1, $\alpha_1$=0.1, $h_1$=0.1, $h_2$=0.1, $P_r$=0.71, A=0.5, B=0.5, ω=0.1, t=0.1, ∈=0.02**

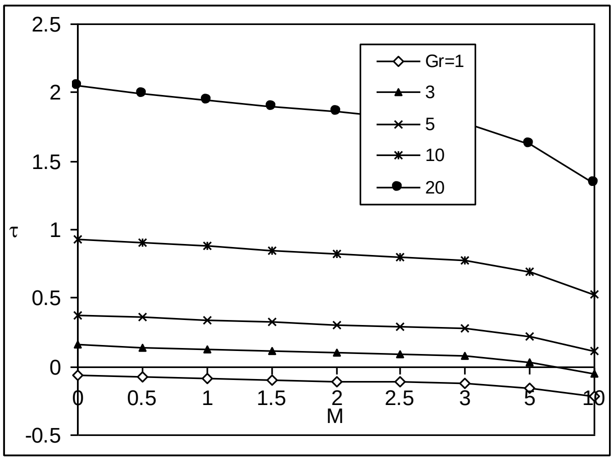


**Figure 13. Skin friction profiles against M for different values of $G_r$ with $K_p$=1, F=0.1, $\alpha_1$=0.1, $\alpha_2$=0.2, $h_1$=0.1, $h_2$=0.1, $P_r$=0.71, A=0.5, B=0.5, ω=0.1, t=0.1, ∈=0.02**

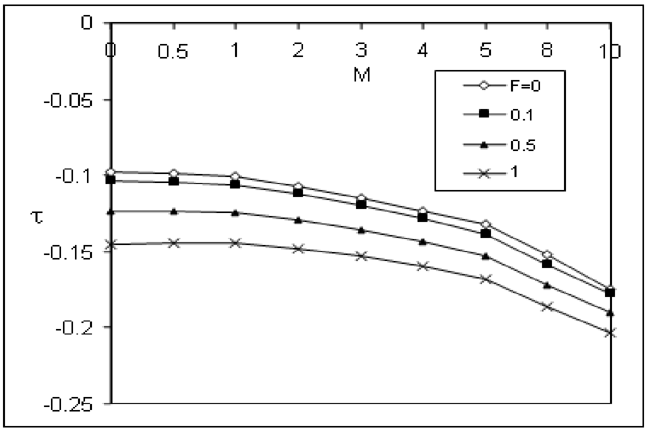


**Figure 14. Skin friction profiles against M for different values of F with $K_p$=1, $G_r$=2, $\alpha_1$=0.1, $\alpha_2$=0.2, $h_1$=0.1, $h_2$=0.1, $P_r$=0.71, A=0.5, B=0.5, ω=0.1, t=0.1, ∈=0.02**

**5. Conclusion**

We summarize below some of the important results of physical interest on the velocity, temperature, skin-friction and the rate of heat transfer at the wall in the flow field.

1. A growing magnetic parameter $M$ is found to retard the velocity of the flow field up to a certain distance and thereafter the effect reverses.

2. An increase in permeability parameter $K_p$ tends to accelerate the velocity of the flow field up to a certain distance from the plate and thereafter it reverses the effect.

3. The velocity of the flow field increases at all points of the flow field due to an increase in Grashof number $G_r$ in the flow field.

4. A growing radiation parameter $F$ retards the velocity of the flow field at all points.

5. Both of the suction parameters $\alpha_1$ and $\alpha_2$ show an increase in velocity of the flow field at all points in the flow field in a different manner. The parameter $\alpha_1$ is tending to converge the velocity profiles to a point while the parameter $\alpha_2$ is tending to diverge the velocity profile from a point.

6. Both of the velocity slip parameters $h_1$ and $h_2$ tend to enhance the velocity of the flow field at all points in the similar manner as $\alpha_1$ and $\alpha_{12}$ but some discrepancy is found in case of $h_2$.

7. Both Prandtl number $P_r$ and radiation parameter $F$ show a decrease in temperature of the flow field at all points, but the effect is more pronounced in case of Prandtl number $P_r$.

8. A growing suction parameter $\alpha_1$ decreases the skin friction at the wall while the other suction parameter $\alpha_2$ increases it at all points. On the other, a growing magnetic parameter is found to decrease the skin friction at the wall.

9. A growing Grashof number $G_r$ enhances the skin friction while the magnetic parameter $M$ reduces its value at all points.

10. Both radiation parameter $F$ and magnetic parameter $M$ reduce the skin friction at the plate.

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