



The influence of heat transfer on MHD Peristaltic flow through a vertical asymmetric porous channel

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ABSTRACT

MHD peristaltic flow of a conducting fluid with heat transfer in a vertical asymmetric channel through porous medium is investigated under long wave length and low Reynolds number assumptions. The flow is examined in a wave frame of reference moving with the velocity of the wave. The channel asymmetry is produced by choosing the peristaltic wave on the walls to have different amplitudes and phase. The analytical solutions have been computed for the axial velocity and temperature. The pressure rise and the frictional force are obtained. The effects of different parameters on the pressure rise, velocity, temperature and shear stress are discussed numerically. The phenomenon of trapping is further investigated.

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1. Introduction

Peristaltic transport is a form of material transport induced by a progressive wave of area contraction or expansion along the wave length of a distensible tube mixing and transporting the fluid in the direction of the propagation. This phenomenon is known as peristalsis. It plays an important role in transporting many physiological fluids in the body. It may be involved in movement of ovum in the female fallopian tubes, in the transport of lymph in the lymphatic vessels and in the vasomotion of small blood vessels. Many modern mechanical devices have been designed on the principle of peristaltic pumping for transporting industrial and biofluids, The blood pump in the heart-lung machine and the peristaltic pump which transport noxious fluid in the nuclear industry are some of the devices working on the principle of peristalsis.

The study of fluid flows and heat transfer through porous medium has attracted much attention recently. This is primarily because of numerous applications of flow through porous media, such as storage of radioactive nuclear waste material transfer, separation process in chemical industries, filtration, transpiration cooling, transport process in aquifers and ground water pollution. Examples of natural porous media are beach sand, sandstone, limestone, ryebread, wood, the human lung etc.

Many investigators studied peristaltic flow through porous media with heat transfer. Hayat [1] have analyzed peristaltic flow of a Maxwell fluid including the Hall effect through porous medium. Vajravelu et al. [7] analyzed the peristaltic flow and heat transfer in a vertical porous medium. Mekheimer and Abd Elmaboud [2] studied the influence of heat transfer and magnetic field on peristaltic transport of a Newtonian fluid in a vertical annulus. Subba Reddy, Ramachandra Rao and Sreenadh [6] analyzed the peristaltic motion of a power-law fluid in an asymmetric channel. Srinivas and Kothandapani [5] investigated

the influence of heat and mass transfer on MHD peristaltic flow through porous space with compliant walls. Srinivas and Gayatri [3] studied the peristaltic transport of a Newtonian fluid in a vertical asymmetric channel with heat transfer and porous medium. Vajravelu et al. [8] analyzed the peristaltic transport of a Casson fluid in contact with a Newtonian fluid in a circular tube with permeable wall. Vajravelu et al. [9] analyzed the influence of heat transfer on peristaltic transport of a Jeffrey fluid in a vertical porous stratum.

The study of MHD plays an important role in agriculture, engineering and petroleum industries. For instance, it may be used to deal with problems such as cooling of nuclear reactors by liquid sodium, magnetotherapy and so on.

In view of the above observations, we study the MHD peristaltic flow of a conducting fluid with heat transfer in a vertical asymmetric channel through porous medium, under long wave length and low Reynolds number assumptions. The flow is examined in a wave frame of reference moving with the velocity of the wave. The analytical solutions have been obtained for the axial velocity, temperature and the pressure gradient. The effects of different parameters on the velocity and pressure rise are discussed through graphs.

2. Mathematical formulation

We consider the motion of an incompressible viscous fluid in a two dimensional vertical asymmetric channel induced by sinusoidal wave train propagating with constant speed C along the channel walls (see Fig.1).

The wall deformations are given by

$$H_1 = d_1 + a_1 \cos\left[\frac{2\pi}{\lambda}(X - ct)\right] \quad (1)$$

(Right wall)

$$H_2 = -d_2 - b_1 \cos\left[\frac{2\pi}{\lambda}(X - ct) + \phi\right] \quad (\text{Left wall}) \quad (2)$$

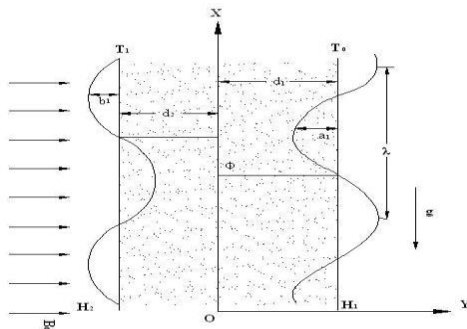


Fig.1: Physical Model

where a_1, b_1 are the amplitudes of the waves, λ is the wave length, $d_1 + d_2$ is the width of the channel, the phase difference ϕ varies in the range $0 \leq \phi \leq \pi$, $\phi = 0$ corresponds to symmetric channel with waves out of phase and $\phi = \pi$ with waves are in phase and further a_1, b_1, d_1, d_2 and ϕ satisfy the condition (Mishra and Ramachandra Rao, [3]).

$$a_1^2 + b_1^2 + 2a_1b_1 \cos\phi \leq (d_1 + d_2)^2 \quad (3)$$

The right hand side wall is maintained at temperature T_0 and left hand side wall at temperature T_1 . Let k_0 be the permeability of the porous medium between the flexible rigid walls. X-axis is chosen between the two flexible walls and Y-axis is taken perpendicular to X-axis. Cartesian coordinate system is used.

Under the assumptions that the channel length is an integral multiple of the wave length λ and the pressure difference across the ends of the channel is a constant, the flow is inherently unsteady in the laboratory frame (X,Y) and become steady in the wave frame (x,y) which is moving with velocity 'c' along the wave. The transformation between these two frames is given by $x = X - ct$, $y = Y$, $u = U - c$, $v = V$, $p(x) = P(x,t)$ (4)

where u, v are velocity components in the wave frame (x,y), p and P are pressures in wave and fixed frames of references respectively.

The non-dimensional quantities are

$$\bar{x} = \frac{x}{\lambda}, \bar{y} = \frac{y}{d_1}, \bar{u} = \frac{u}{c}, \bar{v} = \frac{v}{c\delta}, \delta = \frac{d_1}{\lambda}, \bar{p} = \frac{d_1^2 p}{\mu c \lambda}, \bar{t} = \frac{ct}{\lambda}, h_1 = \frac{H_1}{d_1}, h_2 = \frac{H_2}{d_1}, d = \frac{d_2}{d_1}, a = \frac{a_1}{d_1}, b = \frac{b_1}{d_1}, R = \frac{\rho c d_1}{\mu}, \theta = \frac{T - T_0}{T_1 - T_0}, Pr = \frac{\mu c}{k}, \beta = \frac{Q_0 d_1^2}{k(T_1 - T_0)}, K = \frac{k_0}{d_1^2}, Gr = \frac{\alpha g (T_1 - T_0) d_1^3}{\nu^2}, M^2 = \frac{\sigma B_0^2 d_1^2}{\mu} \quad (5)$$

where R is the Reynolds number, δ is the dimensionless wave number, K is the permeability parameter, Gr is the Grashof number, Pr is the Prandtl number, ν is the kinematic viscosity of the fluid, β is the non-dimensional heat source/sink parameter and M is the magnetic parameter.

The basic equations can be written in non-dimensional form as below

$$\delta R \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \left(\delta^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \left(\frac{1}{K} + M^2 \right) (u+1) + Gr \theta \quad (6)$$

$$\delta^3 R \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \delta^2 \left(\delta^2 \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\delta^2}{K} v \quad (7)$$

$$\delta Pr R \left(u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} \right) = \delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \beta \quad (8)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (9)$$

Using long wave length approximation and dropping terms of order δ and higher, the governing equations (2.6) - (2.8) reduce to

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \left(\frac{1}{K} + M^2 \right) (u+1) + Gr \theta \quad (10)$$

$$0 = -\frac{\partial p}{\partial y} \quad (11)$$

$$\frac{\partial^2 \theta}{\partial y^2} + \beta = 0 \quad (12)$$

The dimensional volume flow rate in the laboratory and wave frames are

$$Q = \int_{H_2(X,t)}^{H_1(X,t)} U(X,Y,t) dY, \quad q = \int_{h_2(x)}^{h_1(x)} u(x,y) dy \quad (13)$$

where h_1 and h_2 are function of x alone. From equations (2.4) and (2.13) we can write

$$Q = q + ch_1(x) - ch_2(x) \quad (14)$$

The time-averaged flow rate over a period T at a fixed position X is

$$\bar{Q} = \frac{1}{T} \int_0^T Q dt = q + c d_1 + c d_2 \quad (15)$$

The dimensionless mean flow Θ in the laboratory frame and F in the wave frame are related as

$$\Theta = F + 1 + d \quad \text{Where} \quad \Theta = \frac{\bar{Q}}{cd_1}, \quad F = \frac{q}{cd_1} = \int_{h_2}^{h_1} u dy \quad (16)$$

which in

$$h_1(x) = 1 + a \cos(2\pi x), \quad h_2(x) = -d - b \cos(2\pi x + \phi)$$

represents the dimensionless form of the peristaltic walls, and a, b, d and ϕ satisfy the relation

$$a^2 + b^2 + 2abc \cos\phi \leq (1+d)^2 \quad (17)$$

The corresponding boundary conditions in the wave frame are

$$u = -1, \theta = 0 \quad \text{at} \quad y = h_1; \quad u = -1, \theta = 1 \quad \text{at} \quad y = h_2 \quad (18)$$

3. Solution of the problem

Solving equations (2.10) and (2.12) by using the boundary conditions (2.18), we get the solution as follows

$$\theta = \frac{1}{2(h_2 - h_1)} \left[\beta h_1 h_2 (h_1 - h_2) - 2h_1 + [2 + \beta(h_2^2 - h_1^2)]y - \beta(h_2 - h_1)y^2 \right] \tag{19}$$

$$u = \frac{dp}{dx} \left[L_1 \cosh \sqrt{\frac{1}{K} + M^2} y - L_2 \sinh \sqrt{\frac{1}{K} + M^2} y - s_1 \right] + \beta Gr s_1 \left[L_1 \cosh \sqrt{\frac{1}{K} + M^2} y - L_2 \sinh \sqrt{\frac{1}{K} + M^2} y - s_1 \right] + Gr s_1 \left[\frac{\sinh \sqrt{\frac{1}{K} + M^2} (y - h_1)}{\sinh \sqrt{\frac{1}{K} + M^2} (h_1 - h_2)} + \frac{\beta}{2} (h_1^2 - y^2) + L_7 (y - h_1) \right] - 1 \tag{20}$$

Using equation (2.16), we can find the dimensionless mean flow F and the pressure gradient in wave frame as

$$F = \frac{dp}{dx} L_6 + Gr s_1 \left(\beta L_6 + L_5 + \frac{\beta}{6} L_8 + L_7 L_9 \right) - (h_1 - h_2) \tag{21}$$

$$\frac{dp}{dx} = \frac{1}{L_6} \left(F - Gr s_1 \left[L_5 + \frac{\beta}{6} L_8 + L_7 L_9 \right] + (h_1 - h_2) \right) - \beta Gr s_1 = \frac{1}{L_6} \left(\Theta - 1 - d - Gr s_1 \left[L_5 + \frac{\beta}{6} L_8 + L_7 L_9 \right] + (h_1 - h_2) \right) - \beta Gr s_1 \tag{22}$$

where

$$s_1 = \frac{1}{\left(\frac{1}{K} + M^2 \right)}, \quad s_2 = \frac{\sinh \sqrt{\frac{1}{K} + M^2} h_1}{\sinh \sqrt{\frac{1}{K} + M^2} (h_1 - h_2)}$$

$$s_3 = \cosh \sqrt{\frac{1}{K} + M^2} h_1 - \cosh \sqrt{\frac{1}{K} + M^2} h_2$$

$$L_1 = \frac{s_1}{\cosh \sqrt{\frac{1}{K} + M^2} h_1} (1 + s_2 s_3)$$

$$L_2 = \frac{s_1 s_3}{\sinh \sqrt{\frac{1}{K} + M^2} (h_1 - h_2)}$$

$$L_3 = L_1 \frac{\left(\sinh \sqrt{\frac{1}{K} + M^2} h_1 - \sinh \sqrt{\frac{1}{K} + M^2} h_2 \right)}{\sqrt{\frac{1}{K} + M^2}}$$

$$L_4 = \frac{L_2 s_3}{\sqrt{\frac{1}{K} + M^2}}$$

$$L_5 = \frac{1 - \cosh \sqrt{\frac{1}{K} + M^2} (h_2 - h_1)}{\sqrt{\frac{1}{K} + M^2} \sinh \sqrt{\frac{1}{K} + M^2} (h_1 - h_2)}$$

$$L_6 = L_3 - L_4 - s_1 (h_1 - h_2)$$

$$L_7 = \frac{-2 + \beta (h_1^2 - h_2^2)}{4(h_1 - h_2)}, \quad L_8 = 2h_1^3 + h_2 (h_2^2 - 3h_1^2)$$

$$L_9 = -h_1^2 + h_2 (2h_1 - h_2)$$

The non-dimensional forms for pressure rise Δp and the frictional forces at the left and the right walls $F_1(t)$ and $F_2(t)$ respectively, are given as follows

$$\Delta p = \int_0^1 \frac{\partial p}{\partial x} dx \tag{23}$$

$$F_1(t) = \int_0^1 \frac{\partial p}{\partial x} (-h_1^2) dx \tag{24}$$

$$F_2(t) = \int_0^1 \frac{\partial p}{\partial x} (-h_2^2) dx \tag{25}$$

The non-dimensional shear stress can be obtained as

$$\tau = \frac{\bar{\tau}}{\left(\frac{\mu c}{d_1} \right)} = \frac{\partial^2 \psi}{\partial y^2} \tag{26}$$

and it reduces to

$$\tau = \frac{dp}{dx} \left[\frac{L_1 \sinh \sqrt{\frac{1}{K} + M^2} y - L_2 \cosh \sqrt{\frac{1}{K} + M^2} y}{\sqrt{\frac{1}{K} + M^2}} + \beta Gr s_1 \frac{\left[L_1 \sinh \sqrt{\frac{1}{K} + M^2} y - L_2 \cosh \sqrt{\frac{1}{K} + M^2} y \right]}{\sqrt{\frac{1}{K} + M^2}} \right] + Gr s_1 \left[\frac{\cosh \sqrt{\frac{1}{K} + M^2} (y - h_1)}{\sqrt{\frac{1}{K} + M^2} \sinh \sqrt{\frac{1}{K} + M^2} (h_1 - h_2)} - \beta y + L_7 \right] \tag{27}$$

4. Results and discussion

The equation (2.20) gives the expression for velocity in terms of y. Velocity profiles are plotted in fig. 2 to study the effects of different parameters such as permeability parameter K, magnetic parameter M, Grashof number Gr on the velocity distribution. It reveals that the velocity profiles are parabolic. From Fig. 2a and Fig. 2c we observe that the velocity increases with increasing K and Gr. Fig. 2b and Fig. 2d shows that velocity decreases with increasing M and ϕ .

The equation (2.19) gives the expression for temperature in terms of y. Temperature profiles are plotted in Fig. 3 to show the effects of heat source/sink parameter β and phase difference ϕ . We observe that the temperature increases with increasing β and decreases with the increase in ϕ .

We have calculated the pressure rise Δp in terms of the mean flow Θ from equation (2.23). The variation of pressure rise with the mean flow for different Gr is shown in Fig. 4a. It is noticed that the pressure rise decreases with the increase in Θ . We observe that for a given Θ , pressure rise increases with increasing Gr. Also for fixed Δp , the increase in Gr increases the mean flow. The variation of pressure rise with the mean flow for different β is shown in Fig. 4b. We notice that for a given

Θ , pressure rise increases with increasing β . Also for fixed Δp , the increase in β increases the mean flow. Fig. 4c shows the variation of pressure rise with the mean flow for different values of permeability parameter K. We find that for fixed Θ , pressure rise decreases with increasing K. Also for a given Δp , the increase in K decreases the mean flow. The variation of pressure rise with the mean flow for different values of magnetic parameter M is shown in Fig. 4d. We notice that for a given Θ , pressure rise increases with increasing M. Also for fixed Δp , the increase in M increases the mean flow.

The variations of frictional forces at the left and right walls with mean flow are calculated from the equations (2.24) and (2.25) for different values of M and are shown in Fig. 5a and 5b. We observe that the frictional forces at the left and right walls have the opposite behavior compared to the pressure rise.

The stress distribution on the left and right walls of the channel for different values of M is presented in Fig. 6a and 6b. Fig. 6a is plotted to study the effect of M on the shear stress at the right side wall. It is clear that the shear stress is symmetric about $x = 0$. We observe that it increases with increasing M and it takes the maximum value on the up stream side. Fig. 6b is plotted to study the effect of M on the shear stress at the left side wall. It is clear that the shear stress is symmetric about $x = 0$. We observe that it increases with increasing M and it takes the maximum value on the up stream side.

The formation of an internally circulating bolus of fluid by closed streamlines is called trapping and this trapped bolus is pushed ahead along with the peristaltic wave. The stream lines for different values of mean flow $\square\square$ are plotted in Fig. 7. It is observed that the size of bolus decreases as \square increases. The effect of M on trapping is illustrated in Fig. 8. It can be seen that volume of the bolus decreases with the increase of M. The stream lines for different values of K \square are plotted in Fig. 9. We notice that the size of bolus increases with increasing K.

5. Conclusions

A study is made in order to explain the effect of heat transfer on MHD peristaltic flow through a vertical asymmetric porous channel. The effects of various emerging parameters on the axial velocity, pressure rise, shear stress and stream line flow pattern are seen with the help of graphs. From the present study the following conclusions can be drawn.

- (1) The velocity increases with the increase of K and Gr and decreases by increase M and ϕ in the middle part of the channel.
- (2) The temperature increases with the increase in β and decreases with the increase in ϕ .
- (3) The pressure rise increases with the increase each of Gr, β and M where as it decreases as K increases.
- (4) The pressure rise for different values of Gr, β , M and K becomes grater with decreasing the mean flow Θ and reaches maximum at zero flow rate. There is an inversely linear relation between Δp and Θ .
- (5) The friction force has the opposite behavior compared to pressure rise.
- (6) The size of trapped bolus increases with the increase in K

and decreases with increase in Θ and M.

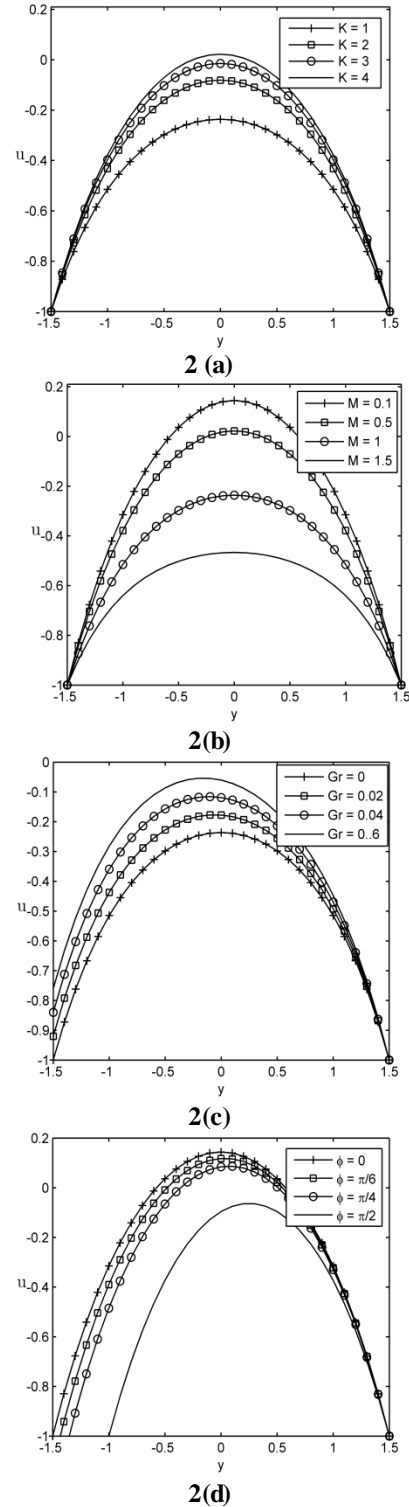
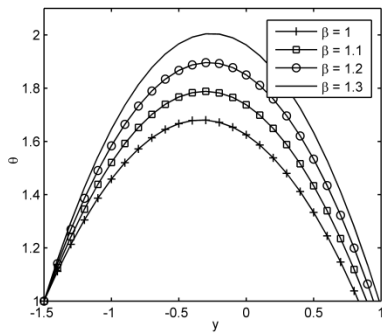
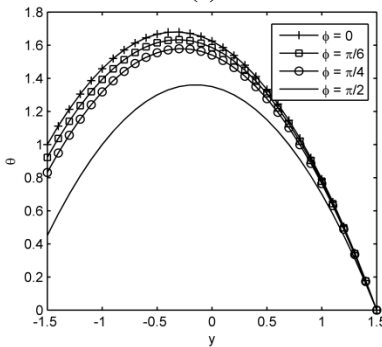


Fig.2 Velocity distribution: (a)

- $x = 0, a = 0.5, b = 0.5, d = 1, \beta = 2, Gr = 0, M = 1, \phi = 0$
- (b) $x = 0, a = 0.5, b = 0.5, d = 1, \beta = 2, Gr = 0, K = 1, \phi = 0$
- (c) $x = 0, a = 0.5, b = 0.5, d = 1, \beta = 2, K = 1, M = 1, \phi = 0$
- (d) $x = 0, a = 0.5, b = 0.5, d = 1, \beta = 2, Gr = 0, K = 1, M = 0.1$



3(a)

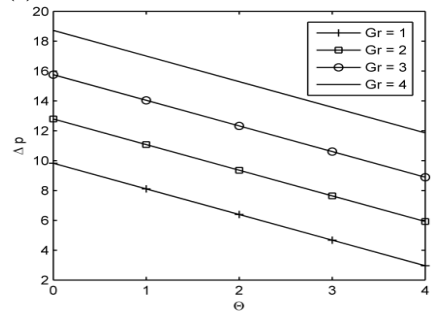


3(b)

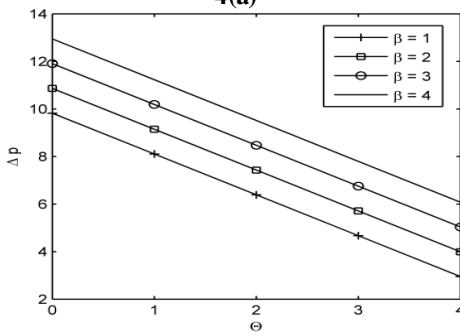
Fig.3 Temperature profiles: (a)

$x=0, a=0.5, b=0.5, d=1, \phi=0$.

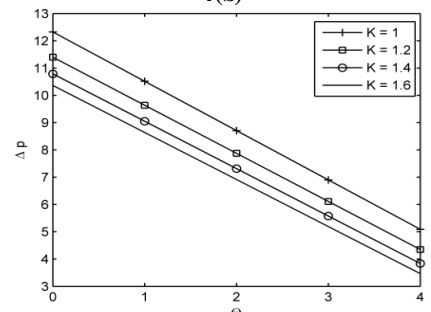
(b) $x=0, a=0.5, b=0.5, d=1, \beta=1$



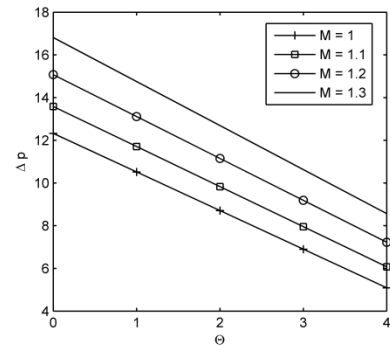
4(a)



4(b)



4(c)



4(d)

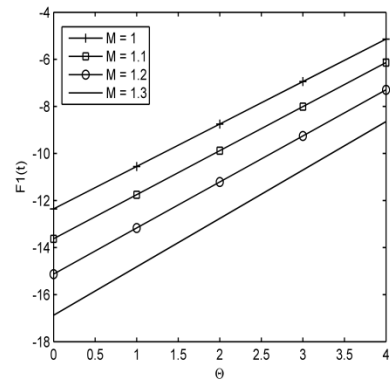
Fig.4 Pressure rise:

(a) $a=0.1, b=0.1, d=1, \beta=1, K=2, M=1, \phi=\pi/2$.

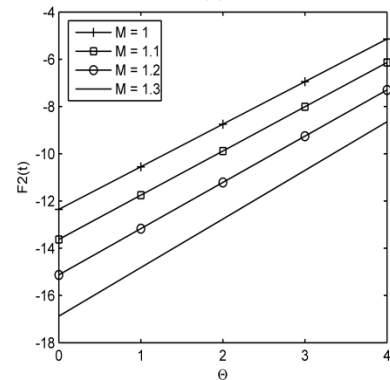
(b) $a=0.1, b=0.1, d=1, Gr=1, M=1, K=2, \phi=\pi/2$.

(c) $Gr=1, M=1, \beta=1, \phi=\pi/2$.

(d) $a=0.1, b=0.1, d=1, Gr=1, \beta=1, K=1, \phi=\pi/2$.



5(a)

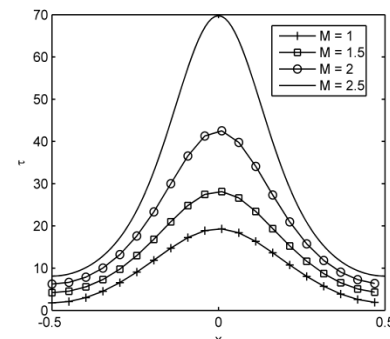


5(b)

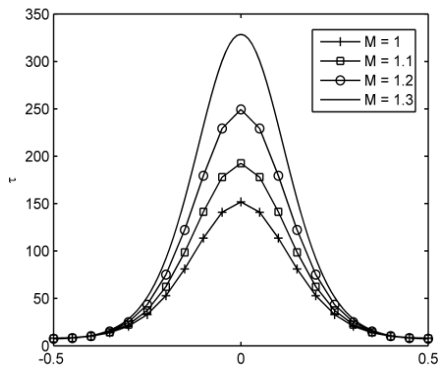
Fig.5 Frictional force:

(a) $y=h_1, a=0.1, b=0.1, d=1, Gr=1, \beta=1, K=1, \phi=\pi/2$.

(b) $y=h_2, a=0.1, b=0.1, d=1, Gr=1, \beta=1, K=1, \phi=\pi/2$.



6(a)

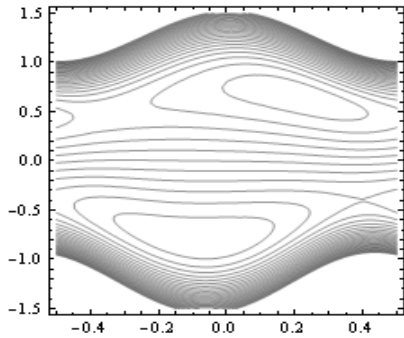


6(b)

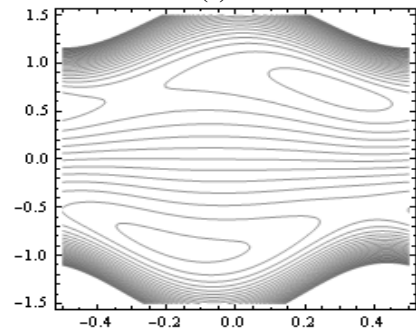
Fig.6 Variation of shear stress:

(a) $y = h_1$, $a = 0.3, b = 0.3, d = -1, Gr = 2, \beta = 2, K = 2, \phi = 0, \theta = 4$.

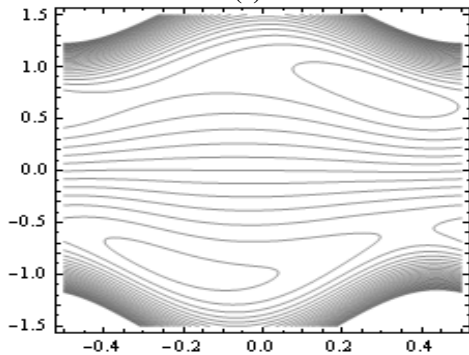
(b) $y = h_2$, $a = 0.3, b = -0.3, d = -1, Gr = 2, \beta = 1, K = 4, \phi = 0, \theta = 4$.



7(a)



7(b)

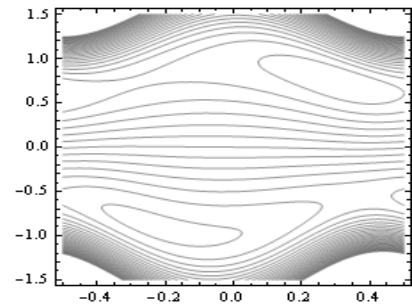


7(c)

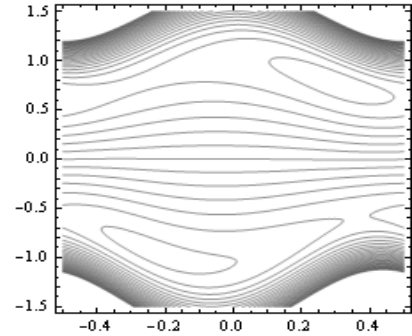
Fig. 7 Streamlines (for different (a) $\Theta = 2$, (b)

$\Theta = 2.5$, (c) $\Theta = 3$), with fixed

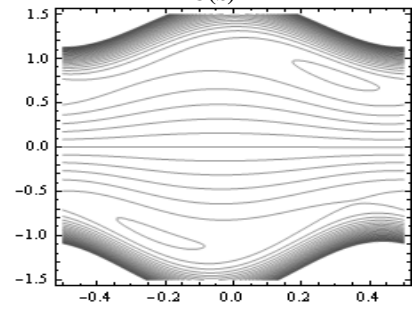
$a = 0.25, b = 0.3, d = 1, \beta = 2, \phi = \pi/8, Gr = 2, K = 0.12, M = 2$



8(a)



8(b)

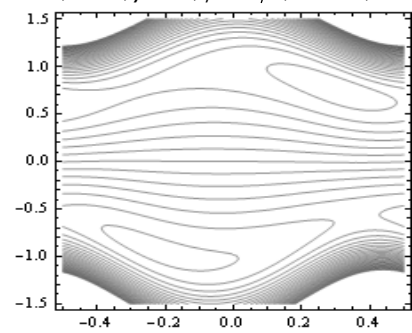


8(c)

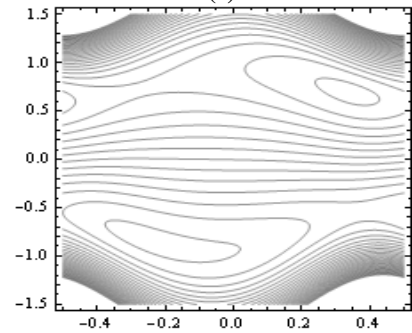
Fig. 8 Streamlines (for different (a) $M = 3$, (b)

$M = 5$, (c) $M = 7$), with fixed

$a = 0.25, b = 0.3, d = 1, \beta = 2, \phi = \pi/8, Gr = 2, K = 0.12, \Theta = 3$



9(a)



9(b)

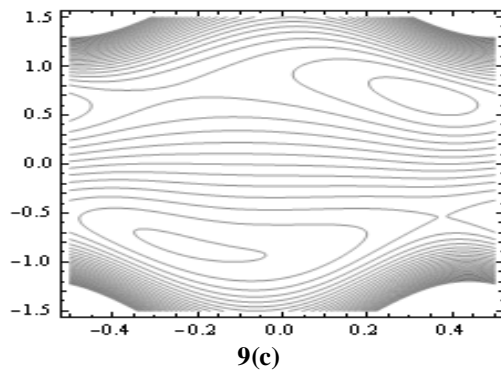


Fig. 9 Streamlines (for different (a) $K = 0.1$, (b) $K = 0.5$, (c) $K = 1$), with fixed

$$a = 0.25, b = 0.3, d = 1, \beta = 2, \phi = \pi/8, Gr = 2, M = 2, \Theta = 3$$

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