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Operational behaviour of major part assembly system of an automobile incorporating human error in maintenance

Surabhi Sengar and S. B. Singh

Department of Mathematics, Statistics and Computer Science, G. B. Pant University of Agriculture and Technology, Pantnagar, India.

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ABSTRACT

Present paper studies the operational behavior of major part assembly system of an automobile incorporating human error. The system consists of eight main units namely: Engine, transmission, fuel supply system, fuel ignition system, exhaust system, cooling system, brake system and build. The whole system can fail due to failure of any of the units. The system satisfies the usual conditions like perfect repair, joint distributions etc. Each operative unit has a constant failure rate but a general repair time distribution. We transform the basic equation into integro differential equation and outline the solution procedure for a repair time distribution with Laplace transform. The transitional state probabilities, asymptotic behaviour and some characteristics such as reliability, availability, MTTF and the cost effectiveness of the system have been obtained using a time dependent version of the supplementary variable method and Gumble-Hougaard copula methodology. At last, some particular cases and numerical examples have been taken to describe the model.

Introduction

During the last 45 years reliability concepts have been applied in various manufacturing and technological fields. The reliability of a system and its maintenance employs an increasing important issue in modern day electronic, manufacturing and industrial systems. In real life, one comes across many complexities of modern day engineering systems. Earlier researchers [1, 2] have described various reliability aspects and its principles in the modern day life. Many authors [3, 4, 5, 6 and 7] discussed reliability and steady state analysis of some realistic engineering systems by using different approaches like probabilistic rational model technique, matlab tool, matrix method etc. Reliability techniques have also been applied to a number of industrial and transportation problems including automobile industry. Here the study is focused on the major part assembly process of an automobile.

The auto industry is often thought of as one of the most global of all industries. Its products have spread around the world, and it is dominated by a small number of companies with worldwide recognition. However, in certain respect the industry is more regional than global, in spite of the globalizing trends evident in the 1990s. One feature of the auto industry in the last 25 years was the way in which leading vehicle manufacturers extended their operations in developing countries. For the global producers, rapidly growing markets in developing countries were spreading vehicle development costs; for establishing cheap production sites for the production of selected vehicles and components; and for access to new markets for higher-end vehicles, which would still be produced in the triad economies. As the complexity and automation of equipments increased, it resulted in severe problems of maintenance and repair. This put forward the tasks of developing a systematic approach to the study of any phenomena and process that can lead to failure free operation or render service for a good or at least reasonable period of time. When production-operations are concern, a maintenance person plays an important role in the reliability of equipment. It is also a well-known fact that a significantly large proportion of total human errors occur during the maintenance phase. Human error in maintenance is a subject, which has not been given the amount of attention that it deserves.

Humans play an important role during the design, installation, production, and maintenance phases of a product. Human error may be defined as the failure to perform a specified task (or the performance of a forbidden action) that could lead to disruption of scheduled operations or result in damage to property and equipment. While human error has existed since the beginning of mankind, only in the last 50 years it has been the subject of scientific inquiry. Various reasons due to which human errors can occur include inadequate lighting in the work area, inadequate training or skill of the manpower involved, poor equipment design, high noise levels, an inadequate work layout, improper tools, and poorly written equipment maintenance and operating procedures. Human error may be classified into six categories: 1.Operating errors, 2. Assembly errors, 3. Design errors, 4. Inspection errors, 5. Installation errors and 6. Maintenance errors. Maintenance error occurs due to incorrect repair or preventive actions. Two typical examples are the incorrect calibration of equipment and application of the wrong grease at appropriate points of the equipment. The occurrence of maintenance errors increases due to the increase in maintenance frequency as the equipment becomes older.

Keeping above facts in view the present paper proposes a methodology to develop a decision-making aid tool whose objective is to assess the dependability and performances of a manufacturing system incorporating human error. In practical situation data collected or available for the complex repairable industrial systems are vague, ambiguous, qualitative and imprecise in nature due to various practical constraints. So it is

Tele:
E-mail addresses: sursengar@gmail.com

not easy to calculate reliability indices of such systems up to a desired accuracy. If reliability indices of these systems have been calculated, then they have high range of uncertainty. The objective of the present study is to compute the operational behaviour of major part assembly system of an automobile incorporating human error in maintenance. Considered system consists of eight sub-units working in series as: engine, transmission, fuel supply system, fuel ignition system, exhaust system, cooling system, brake system and build. The study is focused on the system failures due to human errors in maintenance. It is assumed that the engine can fail due to operating error (EO). Transmission consists of five units namely: Power train, gear, clutch, differential device and drive shaft. Here, (1) the system can fail due to improper installation of power train and gear in transmission (TI). (2) It is in reduced efficiency state due to improper working of differential device (D). (3) The system is in failed state due to failure of drive shaft in transmission (DS). Fuel supply system (FSS) consists of five units like: fuel pump, FIP (fuel ignition pump), fuel injector, indirect injection, and valve. It is assumed that the system can fail due to error in inspection of FIP timing and fuel pump in fuel supply system. Further, it can also fail by misfiring (M) and low pressure (LP) of fuel ignition system having maintenance error and inspection error. Exhaust system consists of four units they are: manifold, TIP, catalytic converter and silencer. It is assumed that the system can fail due to bad assembly exhaust system (EA) and is in reduced efficiency state due to improper maintenance of cooling system having overheating problem (OVH). Braking system (BR) consists of brake pedal having drum brake, disk brake and hydraulic brake. Any design error in the brake system can cause system breakdown. Finally, Building system (BU) consists of chasis, axle, drive axle, suspension and steering. It is assumed that the system can fail due to installation error in axle and design error in chasis and steering in build system. Considered system can completely fail due to failure of any of the subsystems. Once the system fails due to improper working of differential device and drive shaft in transmission two types of repairs are involved to repair the system, similarly when the fuel ignition system can fail due to misfiring and low pressure, again joint repairs are involved so, joint probability distribution is applied to repair the system with the help of copula [9, 10, 11]. All failures follow exponential time distribution whereas all repairs follow general time distribution. The transition state diagram and state specification of the considered system is shown in Figure-1 and Table-1 respectively. With the help of Supplementary variable technique, Laplace transformation and copula methodology, following reliability measures of the system have been evaluated:

(1) Transition state probabilities of the system.
(2) Asymptotic behaviour of the system.
(3) Various measures such as availability, reliability, MTTF and cost effectiveness of the system.

We also perform a parametric investigation which provides numerical results to show the effects of different system parameters to the reliability, availability, MTTF and cost which may be helpful to managerial staff of the industry in the decision making.

2. Brief Introduction of Gumbel-Hougaard Family Copula

A number of authors including [8] have studied the family of copulas extensively. The Gumbel-Hougaard family copula is defined as:

$$C_\theta(u_1,u_2)=\exp(-((-\log u_1)^\theta+(-\log u_2)^\theta)^{1/\theta}),\quad 1\le\theta\le\infty\pm\pm$$

For $\theta = 1$ the Gumbel-Hougaard copula models independence, for $\theta\to\infty$ it converges to comonotonicity.

State specification

Tables 1 and 2 show the state specification and notations of the considered system:

The following assumption has been taken into the considerations in this study.

- Initially at t=0, all subsystems are operating well.
- At $t = 0$ all the components are perfect and $t > 0$ they start operating.
- Failures are statistically independent.
- The repair time of the subsystems are assumed to be arbitrarily distributed.
- Repaired subsystem/ plant(s) works like new.
- All failures follow exponential time distribution.
- Considered system can completely fail due to failure of any of the subsystems.
- Joint probability distribution has been obtained with the help of copula for repair when the system failed due to failure of transmission and fuel supply system.

State transition diagram of model

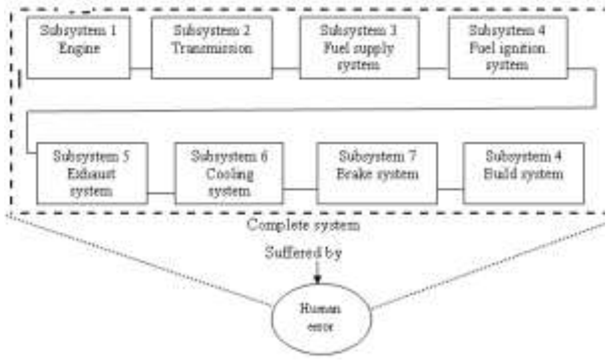

Figure 1: Diagram of investigated system

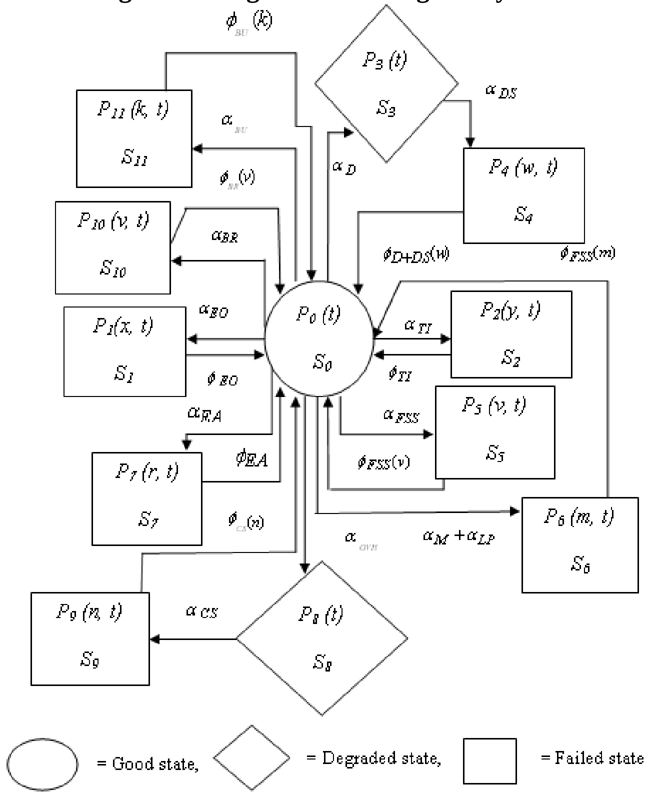

Figure 2: State transition diagram

Mathematical formulation of the model

Using elementary probability considerations and limiting procedure, we obtain the following set of difference-differential equations governing the behaviour of considered system, continuous in time and discrete in space:

$$\left[\frac{d}{dt}+\alpha_{EO}+\alpha_{TI}+\alpha_{D}+\alpha_{FSS}+(\alpha_{M}+\alpha_{LP})+\alpha_{EA}+\alpha_{OVH}+\alpha_{BR}+\alpha_{BU}\right]P_0(t)=$$

$$\int_0^\infty \phi_{EO}(x)P_1(x,t)dx+\int_0^\infty \phi_{TI}(y)P_2(y,t)dy+$$

$$\int_0^\infty \phi_{D+DS}(w)P_4(w,t)dw+$$

$$\int_0^\infty \phi_{FSS}(v)P_5(v,t)dv+\int_0^\infty \phi_{M+LP}(m)P_6(m,t)dm+\int_0^\infty \phi_{EA}(r)P_7(r,t)dr+$$

$$\int_0^\infty \phi_{CS}(n)P_9(n,t)dn+\int_0^\infty \phi_{BR}(l)P_{10}(l,t)dl+$$

$$\int_0^\infty \phi_{BU}(k)P_{11}(k,t)dk \quad \dots (1)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\phi_{EO}(x)\right]P_1(x,t)=0 \quad \dots (2)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\phi_{TI}(y)\right]P_2(y,t)=0 \quad \dots (3)$$

$$\left[\frac{d}{dt}+\alpha_{DS}\right]P_3(t)=\alpha_D P_0(t) \quad \dots (4)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial w}+\phi_{D+DS}(w)\right]P_4(w,t)=0 \quad \dots (5)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial v}+\phi_{FSS}(v)\right]P_5(v,t)=0 \quad \dots (6)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial m}+\phi_{LP+M}(m)\right]P_6(m,t)=0 \quad \dots (7)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial r}+\phi_{EA}(r)\right]P_7(r,t)=0 \quad \dots (8)$$

$$\left[\frac{d}{dt}+\alpha_{CS}\right]P_8(t)=\alpha_{OVH}P_0(t) \quad \dots (9)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial n}+\phi_{CS}(n)\right]P_9(n,t)=0 \quad \dots (10)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial v}+\phi_{BR}(v)\right]P_{10}(v,t)=0 \quad \dots (11)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial k}+\phi_{BU}(k)\right]P_{11}(k,t)=0 \quad \dots (12)$$

The boundary conditions of designed system are defined as:
The state transition probability of the system in a state = failure rate × probability of the previous state. Therefore

$$P_1(0,t)=\alpha_{EO}P_0(t) \quad \dots (13)$$

$$P_2(0,t)=\alpha_{TI}P_0(t) \quad \dots (14)$$

$$P_4(0,t)=\alpha_{DS}P_3(t) \quad \dots (15)$$

$$P_5(0,t)=\alpha_{FSS}P_0(t) \quad \dots (16)$$

$$P_6(0,t)=\alpha_{LP+M}P_0(t) \quad \dots (17)$$

$$P_7(0,t)=\alpha_{EA}P_0(t) \quad \dots (18)$$

$$P_9(0,t)=\alpha_{CS}P_8(t) \quad \dots (19)$$

$$P_{10}(0,t)=\alpha_{BR}P_0(t) \quad \dots (20)$$

$$P_{11}(0,t)=\alpha_{BU}P_0(t) \quad \dots (21)$$

Initial Condition

$P_0(0)=1$, and other state probabilities are zero at t = 0 … (22)

In the above equations *t* and *x* both represent times. The supplementary variable *x*, which represents the elapsed repair time of the system, varies only when the system is in degraded or failed state, and its rate of variation is exactly equal to that of the schedule time, represented by *t*.

Solution of the model

Taking Laplace transforms of equation (1) through (21) subject to initial condition (22) and then on solving them one by one; we obtain the following transition state probabilities of the system:

$$\overline{P_0}(s)=\frac{1}{B(s)} \quad \dots (23)$$

$$\overline{P_1}(s)=\frac{\alpha_{EO}\times D_{\phi_{EO}}(s)}{B(s)} \quad \dots (24)$$

$$\overline{P_2}(s)=\frac{\alpha_{TI}\times D_{\phi_{TI}}(s)}{B(s)} \quad \dots (25)$$

$$\overline{P_3}(s)=\frac{\alpha_D}{[s+\alpha_{DS}]}\times\frac{1}{B(s)} \quad \dots (26)$$

$$\overline{P_4}(s)=\frac{\alpha_D\alpha_{DS}}{[s+\alpha_{DS}]}\times\frac{D\phi_{D+DS}(s)}{B(s)} \quad \dots (27)$$

$$\overline{P_5}(s)=\frac{\alpha_{FSS}\times D_{\phi_{FSS}}(s)}{B(s)} \quad \dots (28)$$

$$\overline{P_6}(s)=\frac{\alpha_{LP+M}\times D\phi_{LP+M}(s)}{B(s)} \quad \dots (29)$$

$$\overline{P_7}(s)=\frac{\alpha_{EA}\times D_{\phi_{EA}}(s)}{B(s)} \quad \dots (30)$$

$$\overline{P_8}(s)=\frac{\alpha_{OVH}}{[s+\alpha_{CS}]}\times\frac{1}{B(s)} \quad \dots (31)$$

$$\overline{P_9}(s)=\frac{\alpha_{CS}\times\alpha_{OVH}\times D_{\phi_{CS}}(s)}{(s+\alpha_{CS})\times B(s)} \quad \dots (32)$$

$$\overline{P_{10}}(s)=\frac{\alpha_{BR}\times D_{\phi_{BR}}(s)}{B(s)} \quad \dots (33)$$

$$\overline{P_{11}}(s) = \frac{\alpha_{BU} \times D_{\phi_{BU}}(s)}{B(s)} \quad \ldots (34)$$

where,

$$B(s) = s + \alpha_{EO} + \alpha_{TI} + \alpha_D + \alpha_{FSS} + (\alpha_M + \alpha_{LP}) + \alpha_{EA} + \alpha_{OVH} + \alpha_{BR} + \alpha_{BU}$$
$$-\alpha_{EO} \times \overline{S}_{\phi_{EO}}(s)$$
$$-\alpha_{TI} \times \overline{S}_{\phi_{TI}}(s) - \frac{\alpha_D \alpha_{DS}}{[s+\alpha_{DS}]} \times \overline{S}_{\phi_{D+DS}}(s) -$$
$$\alpha_{FSS} \times \overline{S}_{\phi_{FSS}}(s)$$
$$-\alpha_{LP+M} \times \overline{S}_{\phi_{LP+M}}(s) - \alpha_{EA} \times \overline{S}_{\phi_{EA}}(s)$$
$$-\frac{\alpha_{CS}\alpha_{OVH}}{(s+\alpha_{CS})} \times \overline{S}_{\phi_{CS}}(s) - \alpha_{BR} \times \overline{S}_{\phi_{BR}}(s)$$
$$-\alpha_{BU} \times \overline{S}_{\phi_{BU}}(s) \quad \ldots (35)$$

$$D_i(s) = \frac{1-\overline{S}_i(s)}{s}$$
$$, i = \phi_{EO}, \phi_{TI}, \phi_{D+DS}, \phi_{FSS}, \phi_{LP+M}, \phi_{EA}, \phi_{CS}, \phi_{BR}, \phi_{BU} \quad \ldots (36)$$

$$\phi_{LP+M}(m) = \exp[m^\theta + (\log \phi_{LP+M}(m))^\theta)^{1/\theta}] \quad \ldots (37)$$

$$\phi_{D+DS}(w) = \exp[w^\eta + (\log \phi_{LP+M}(w))^\eta)^{1/\eta}] \quad \ldots (38)$$

$$\overline{S}_i(s) = \int_0^\infty i \exp[-sj - \int_0^j i(j)dj]dj,$$
$$i = \phi_{EO}, \phi_{TI}, \phi_{D+DS}, \phi_{FSS}, \phi_{LP+M}, \phi_{EA}, \phi_{CS}, \phi_{BR}, \phi_{BU}$$
$$j = x, y, w, h, k, r, n, l, q, v. \quad \ldots (39)$$

Also up state and down state probabilities of the system is given by:

$$\overline{P}up(s) = \overline{P}_0(s) + \overline{P}_3(s) + \overline{P}_8(s)$$
$$= \frac{1}{B(s)} \times \left[1 + \frac{\alpha_D}{[s+\alpha_{DS}]} + \frac{\alpha_{OVH}}{[s+\alpha_{CS}]}\right] \quad \ldots (40)$$

$$\overline{P}down(s) = \overline{P}1(s) + \overline{P}2(s) + \overline{P}4(s) + \overline{P}5(s) + \overline{P}6(s) + \overline{P}7(s) + \overline{P}9(s) + \overline{P}10(s) + \overline{P}11(s)$$

$$= \frac{1}{B(s)} \times [$$
$$\alpha_{EO} \times D_{\phi_{EO}}(s) + \alpha_{TI} \times D_{\phi_{TI}}(s) + \frac{\alpha_D \alpha_{DS}}{[s+\alpha_{DS}]} D_{\phi_{D+DS}}(s) + \alpha_{FSS} \times D_{\phi_{FSS}}(s)$$
$$+\alpha_{LP+M} \times D_{\phi_{LP+M}}(s) + \alpha_{EA} \times D_{\phi_{EA}}(s) + \frac{\alpha_{CS} \times \alpha_{OVH} D_{\phi_{CRSG}}(s)}{(s+\alpha_{CS})} +$$
$$\alpha_{BR} \times D_{\phi_{BR}}(s) + \alpha_{BU} \times D_{\phi_{BU}}(s) \quad \ldots (41)$$

Asymptotic behaviour of the system

In long run as t tends to infinity, the state transition probability of system can be obtained using Abel's lemma in Laplace transformation i.e. $\lim_{t\to\infty} F(t) = \lim_{s\to 0} sF(s) = F(say)$, provided that the limit of the right hand side exists. Time independent i.e. steady state probabilities of the system in different states are given by

$$\overline{P}up = \frac{1}{B(0)} \times \left[1 + \frac{\alpha_D}{\alpha_{DS}} + \frac{\alpha_{OVH}}{\alpha_{CS}}\right] \quad \ldots (42)$$

$$\overline{P}down = \frac{1}{B(0)} \times [\ \alpha_{EO} \times \overline{M}_{\phi_{EO}} + \alpha_{TI} \overline{\times M}_{\phi_{TI}}(s) + \frac{\alpha_D}{\alpha_{DS}} +$$
$$\frac{\alpha_D \times \alpha_{DS}}{[\alpha_{DS}]} \times M_{\phi_{D+DS}}$$
$$+\alpha_{FSS} \times M_{\phi_{FSS}} + \alpha_{LP+M} \times M_{\phi_{LP+M}} + \alpha_{EA} \times M_{\phi_{EA}} + \frac{\alpha_{OVH}}{\alpha_{CS}}$$
$$+\frac{\alpha_{CS}\alpha_{OVH}}{\alpha_{CS}} M_{\phi_{CS}} + \alpha_{BR} \times M_{\phi_{BR}} + \alpha_{BU} \times M_{\phi_{BU}}\ ] \quad \ldots (43)$$

where,

$$B(0) = \lim_{s\to 0} B(s) \quad \ldots (44)$$

$$\overline{M}_i = \lim_{s\to 0} \left\{\frac{1-\overline{S}_i(s)}{s}\right\},$$
$$i = \phi_{EO}, \phi_{TI}, \phi_{D+DS}, \phi_{FSS}, \phi_{LP+M}, \phi_{EA}, \phi_{CS}, \phi_{BR}, \phi_{BU} \ldots\ldots.. (45)$$

$$S_{\phi_i}(s) = \frac{\phi_i}{s+\phi_i},$$
$$i = \phi_{EO}, \phi_{TI}, \phi_{D+DS}, \phi_{FSS}, \phi_{LP+M}, \phi_{EA}, \phi_{CS}, \phi_{BR}, \phi_{BU} \quad \ldots (46)$$

Particular cases

(1) **Availability analysis**

A particular case is also discussed as given below:

When all repairs follow exponential time distribution:

In this case setting, $\overline{S}_{\phi_i}(s) = \dfrac{\phi_i}{s+\phi_i}, \forall i$,

$$\overline{S}_{\phi_{LP+M}}(s) = \frac{\exp\left[m^\theta + \left[\log \phi_{LP+M}(m)\right]^\theta\right]^{1/\theta}}{s + \exp\left[m^\theta + \left[\log \phi_{LP+M}(m)\right]^\theta\right]^{1/\theta}}$$ and

$$\overline{S}_{\phi_{D+DS}}(s) = \frac{\exp\left[w^\theta + \left[\log \phi_{D+DS}(w)\right]^\eta\right]^{1/\eta}}{s + \exp\left[w^\theta + \left[\log \phi_{D+DS}(w)\right]^\eta\right]^{1/\eta}}$$ in equations (23) through (34) we get,

$$\overline{P}up(s) = \frac{1}{B_1(s)} \times \left[1 + \frac{\alpha_D}{[s+\alpha_{DS}]} + \frac{\alpha_{OVH}}{[s+\alpha_{CS}]}\right] \quad \ldots (47)$$

$$\overline{P}down(s) = \frac{1}{B_1(s)} \times [\frac{\alpha_{EO}}{[s+\phi_{EO}]} + \frac{\alpha_{TI}}{[s+\phi_{TI}]} + \frac{\alpha_D}{[s+\phi_{DS}]} + \frac{\alpha_D \alpha_{DS}}{[s+\phi_{DS}]} \frac{1}{[s+\phi_{D+DS}]} +$$
$$\frac{\alpha_{FSS}}{[s+\phi_{FSS}]} + \frac{\alpha_{LP+M}}{[s+\phi_{LP+M}]} + \frac{\alpha_{EA}}{[s+\phi_{EA}]} + \frac{\alpha_{OVH}}{[s+\alpha_{CS}]} + \frac{\alpha_{CS}\alpha_{OVH}}{[s+\phi_{CS}][s+\alpha_{CS}]}$$
$$+\frac{\alpha_{BR}}{[s+\phi_{BR}]} + \frac{\alpha_{BU}}{[s+\phi_{BU}]}\ ] \quad \ldots (48)$$

where,

$$B_1(s) = s + \alpha_{EO} + \alpha_{TI} + \alpha_D + \alpha_{FSS} + (\alpha_M + \alpha_{LP}) + \alpha_{EA} + \alpha_{OVH} + \alpha_{BR} + \alpha_{BU} - \frac{\alpha_{EO} \times \phi_{EO}}{[s+\phi_{EO}]}$$
$$-\frac{\alpha_{TI} \times \phi_{TI}}{[s+\phi_{TI}]} - \frac{\alpha_D \times \alpha_{DS}}{[s+\alpha_{DS}]} \frac{\phi_{D+DS}}{[s+\phi_{D+DS}]} - \frac{\alpha_{FSS} \times \phi_{FSS}}{[s+\phi_{FSS}]} - \frac{\alpha_{LP+M} \times \phi_{LP+M}}{[s+\phi_{LP+M}]} -$$
$$\frac{\alpha_{EA} \times \phi_{EA}}{[s+\phi_{EA}]}$$
$$-\frac{\alpha_{CS} \times \alpha_{OVH} \times \phi_{CS}}{[s+\phi_{CS}][s+\alpha_{CS}]} - \frac{\alpha_{BR} \times \phi_{BR}}{[s+\phi_{BR}]} - \frac{\alpha_{BU} \times \phi_{BU}}{[s+\phi_{BU}]} \quad \ldots (49)$$

Let us take α_{EO} = 0.005, α_{TI} = 0.008, α_D = 0.009, α_{DS} = 0.01, α_{FSS} = 0.06, α_M = 0.006, α_{LP}=0.005, α_{EA}=0.002,

α_{OVH}=0.004, α_{CS}=0.003, α_{BR}=0.009, α_{BU}=0.004, Φ_i = 1 for i= EO, TI, D, DS, FSS, M, LP, EA, OVH, CS, BR, BU, $\theta = \eta$ = 1, and $x= y= w= v= m= r= n= v=k$ = 1, then putting all these values in equation (47), taking inverse Laplace transformation, we get

P_{up} (t) = -0.002992831292 $e^{(-0.003000000000\ t)}$ +0.09089745085 $e^{(-1.10098171\ t)}$ -0.1045972416 $10^{(-7)}$ $e^{(-.9969905879t)}$ -0.04625367733$e^{(-0.2043987365\ t)}$ -0.009328505569$e^{(-0.00471367337\ t)}$ +0.9676775738 …(50)

Now setting t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, in equation (50), one can obtain Figure 2 which represents the variation of availability with respect to time.

(2) Reliability Analysis

Let the failure rates be α_{EO} = 0.005, α_{TI} = 0.008, α_D = 0.009, α_{DS} = 0.01, α_{FSS} = 0.06, α_M = 0.006, α_{LP}=0.005, α_{EA}=0.002, α_{OVH}=0.004, α_{CS}=0.003, α_{BR}=0.009, α_{BU}=0.004, repair rates be Φ_i = 0 for i= EO, TI, D, DS, FSS, M, LP, EA, OVH, CS, BR, BU, $\theta = \eta$ = 1, and $x= y= w= v= m= r= n= v=k$ = 1. Also let the repair follows exponential distribution. Now by putting all these values in equation (40) and taking inverse Laplace transform, using (46) and varying time from t = 0 to t = 10, one can obtain Figure 3 which demonstrate the manner in which reliability varies as time passes.

(3) MTTF Analysis

We know that MTTF = $\lim_{s \to 0} \overline{P}_{up}(s)$

(a) Setting Φ_i = 0 for i= EO, TI, D, DS, FSS, M, LP, EA, OVH, CS, BR, BU = 0, α_{TI} = 0.008, α_D = 0.009, α_{DS} = 0.01, α_{FSS} = 0.06, α_M = 0.006, α_{LP}=0.005, α_{EA}=0.002, α_{OVH}=0.004, α_{CS}=0.003, α_{BR}=0.009, α_{BU}=0.004, $\theta = \eta$ = 1, and $x= y= w= v= m= r= n= v=k$ = 1 and varying α_{EO} as 0.001, 0.002, 0.003, 0.004, 0.005, 0.006, 0.007, 0.008, 0.009,.01 one can obtain Figure 4 which demonstrates variation of MTTF with respect to α_{EO}.

(b) Let us take Φ_i = 0 for i= EO, TI, D, DS, FSS, M, LP, EA, OVH, CS, BR, BU = 0, α_{EO} = 0.005, α_{TI} = 0.008, α_D = 0.009, α_{DS} = 0.01, α_{FSS} = 0.06, α_M = 0.006, α_{EA}=0.002, α_{OVH}=0.004, α_{CS}=0.003, α_{BR}=0.009, α_{BU}=0.004, $\theta = \eta$ = 1, and $x= y= w= v= m= r= n= v=k$ = 1 and varying α_{LP} as 0.001, 0.002, 0.003, 0.004, 0.005, 0.006, 0.007, 0.008, 0.009,.01 Figure 4 can be obtained which shows how MTTF varies as the value of α_{LP} increases.

(c) Fixing Φ_i = 0 for i= EO, TI, D, DS, FSS, M, LP, EA, OVH, CS, BR, BU = 0, α_{EO} = 0.005, α_{TI} = 0.008, α_D = 0.009, α_{DS} = 0.01, α_{FSS} = 0.06, α_{LP}=0.005, α_{EA}=0.002, α_{OVH}=0.004, α_{CS}=0.003, α_{BR}=0.009, α_{BU}=0.004, $\theta = \eta$ = 1, and $x= y= w= v= m= r= n= v=k$ = 1and varying α_M as 0.001, 0.002, 0.003, 0.004, 0.005, 0.006, 0.007, 0.008, 0.009, .01, one can obtain Figure 4 which shows variation of MTTF with respect to α_M.

Putting Φ_i = 0 for i= EO, TI, D, DS, FSS, M, LP, EA, OVH, CS, BR, BU = 0, α_{EO} = 0.005, α_{TI} =

(d) 0.008, α_D = 0.009, α_{DS} = 0.01, α_{FSS} = 0.06, α_M = 0.006, α_{LP}=0.005, α_{EA}=0.002, α_{OVH}=0.004, α_{CS}=0.003, α_{BU}=0.004, $\theta = \eta$ = 1, and $x= y= w= v= m= r= n= v=k$ = 1and varying α_{BR} as 0.001, 0.002, 0.003, 0.004, 0.005, 0.006, 0.007, 0.008, 0.009, .01 one can obtain Figure 4 which represents the manner in which MTTF varies with respect to α_{BR}.

(e) Setting Φ_i = 0 for i = EO, TI, D, DS, FSS, M, LP, EA, OVH, CS, BR, BU = 0, α_{EO} = 0.005, α_{TI} = 0.008, α_D = 0.009, α_{FSS} = 0.06, α_M = 0.006, α_{LP}=0.005, α_{EA}=0.002, α_{OVH}=0.004, α_{CS}=0.003, α_{BR}=0.009, α_{BU}=0.004, $\theta = \eta$ = 1, and $x= y= w= v= m= r= n= v=k$ = 1and varying α_{DS} as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1 one can obtain Figure 5 which demonstrates variation of MTTF with respect to α_{DS}.

(f) Assuming Φ_i = 0 for i= EO, TI, D, DS, FSS, M, LP, EA, OVH, CS, BR, BU = 0, α_{EO} = 0.005, α_{TI} = 0.008, α_D = 0.009, α_{DS} = 0.01, α_M = 0.006, α_{LP}=0.005, α_{EA}=0.002, α_{OVH}=0.004, α_{CS}=0.003, α_{BR}=0.009, α_{BU}=0.004, $\theta = \eta$ = 1, and $x= y= w= v= m= r= n= v=k$ = 1and varying α_{FSS} as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09,0.1 Figure 5 can be obtained which shows how MTTF varies as the value of α_{FSS} increases.

(4) Cost Analysis

Letting α_{EO} = 0.005, α_{TI} = 0.008, α_D = 0.009, α_{DS} = 0.01, α_{FSS} = 0.06, α_M = 0.006, α_{LP}=0.005, α_{EA}=0.002, α_{OVH}=0.004, α_{CS}=0.003, α_{BR}=0.009, α_{BU}=0.004, Φ_i = 1 for i= EO, TI, D, DS, FSS, M, LP, EA, OVH, CS, BR, BU, $\theta = \eta$ = 1, and $x= y= w= v= m= r= n= v=k$ = 1. Furthermore, if the repair follows exponential distribution then using equations (46), we can obtain equation (51). If the service facility is always available, then expected profit during the interval (0, t] is given by

$$E_P(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t$$

where, K_1 and K_2 are the revenue and service cost per unit time respectively, then

E_P (t) = K_1 [0.2602109240 $e^{(-0.5000000000\ t)}$ + 0.003420894509 $e^{(-0.9100000000\ t)}$ +0.3707166241 $e^{(-1.577110005\ t)}$ +0.2282704474 $e^{(-0.8601482323\ t)}$ cos(0.007394121216 t) +0.2901464146 $e^{(-0.8601482323\ t)}$ sin(0.007394121216 t) -0.05867072106 $e^{(-0.7862665523\ t)}$ –0.2283382234 $e^{(-0.7285677930\ t)}$-0.003350319082 $e^{(-0.6042513821\ t)}$-9.234941008 $e^{(-0.1268411365\ t)}$ +8.996326343]-K_2t …..(51)

Keeping K_1 = 1 and varying K_2 at 0.1, 0.2, 0.3 in equation (51), one can obtain Figure 6.

Table 1: State specification of the system

States	Description	System State
S_0	The system is in fully operational condition.	G
S_1	The system is in failed state due to operating error in engine.	F_R
S_2	The system is in failed state due to improper installation of power train and gear in transmission.	F_R
S_3	The system is in reduced efficiency state due to improper working of differential device.	D
S_4	The system is in failed state due to failure of drive shaft in transmission.	F_R
S_5	The system is in failed state due to error in inspection of FIP timing and Fuel pump in Fuel supply system.	F_R
S_6	The system is in failed state due to misfiring and low pressure of fuel ignition system having maintenance error and inspection error.	F_R
S_7	The system is in failed state due to bad assembly exhaust system.	F_R
S_8	The system is in reduced efficiency state due to improper maintenance of cooling system having overheating problem.	D
S_9	The system is in failed state due to failure of cooling system.	F_R
S_{10}	The system is in failed state due to design error in brake system.	F_R
S_{11}	The system is in failed state due to installation error in axle and design error in chasis and steering in build system.	F_R

Note: G= Good state; F_R= Failed state under repair; D = Degraded

Table 2: Notations

Pr	**Probability**
$P_0(t)$	Pr (at time t system is in good state S_0)
$P_i(j,t)$	Pr {the system is in failed state due to the failure of the i^{th} subsystem at time t},where i=1, 2, 4, 5, 6, 7, 9, 10, 11 and j=*x, y, w, v, m, r, n, v, k.*
j	Elapsed repair time, where j= *x, y, w, v, m, r, n, v, k.*
α_{EO}	Engine failure rate due to operating error.
α_{TI} / α_{FSS}	Transmission failure rate due to improper installation of power train and gear. Failure rate of Fuel supply system due to error in inspection of FIP timing and Fuel pump.
α_D / α_{DS}	Transmission failure rate due to improper working of differential device/ drive shaft.
α_M / α_{LP}	Misfiring rate / Rate of low pressure of fuel ignition system due to maintenance error and inspection error.
α_{EA}	Failure rate of exhaust system due to bad assembly.
α_{OVH}	Overheating rate of cooling system due to improper maintenance.
α_{CS}	Failure rate of cooling system.
α_{BR}	Failure rate of brake system due to design error.
α_{BU}	Failure rate of build system due to installation error in axle and design error in chasis and steering.
$\phi i(j)$	General repair rate of i^{th} failure in the time interval (j, $j+\Delta$), where i= EO, TI, D, DS, FSS, M, LP, EA, OVH, CS, BR, BU and j=*x, y, w, v, m, r, n, v, k.*
K_1, K_2	Revenue cost per unit time and service cost per unit time respectively.
$C_\theta(u_1(x),u_2(x))$ $C_\eta(X_1(w),X_2(w))$	The expression for joint probability (failed state to good state) according to Gumbel-Hougaard family is given as: $C_\theta(u_1(m),u_2(m)) = \mu_0(m) = \exp[m^\theta + \{\log\phi_{LP+M}(m)\}^\theta]^{1/\theta}$. $C_\eta(X_1(w),X_2(w)) = X_0(w) = \exp[w^\eta + \{\log\phi_{D+DS}(w)\}^\eta]^{1/\eta}$ where, $u_1 = e^m$, $u_2 = \phi_{LP+M}(m)$ and $X_1 = e^w$, $X_2 = \phi_{D+DS}(w)$,

10. Conclusions

Figure 2 provide information about the changes of availability of the repairable system with respect to time when failure rates are fixed at different values. When failure rates are fixed at lower values α_{EO} = 0.005, α_{TI} = 0.008, α_{D} = 0.009, α_{DS} = 0.01, α_{FSS} = 0.06, α_{M} = 0.006, α_{LP} =0.005, α_{EA} =0.002, α_{OVH} =0.004, α_{CS} =0.003, α_{BR} =0.009, α_{BU} =0.004 availability of the system decreases fast and probability of failure increases, with passage of time and ultimately becomes steady to the value zero after a sufficient long interval of time. From this, one can safely predict the future behaviour of the system at any time for any given set of parametric values, as is evident by the graphical consideration of the model. The reliability of the system initially decreases rapidly with respect to time and later on stabilizes at value 0.4 as shown in Figure 3. By critically examination of Figures 4 and 5 one can see that the MTTF of the syst**em** decreases with the increment in the values of α_{EO}, α_{LP}, α_{M}, α_{BR}, α_{DS} and α_{FSS}. MTTF is found to be highest with respect to α_{FSS}. The value of MTTF varies from 29.9382-27.6353, 30.5031-28.1159, 30.7936-28.3625, 31.6993-29.1291, 29.3939-22.0303 and from 53.8888-21.5555 with respect to α_{EO}, α_{LP}, α_{M}, α_{BR}, α_{DS} and α_{FSS} respectively for considered parameters. When revenue cost per unit time K_1 fixed at one, service cost K_2 = 0.1, 0.2, 0.3 profit has been calculated and results are demonstrated by graphs (Figure 3). The observation outlines that as the service cost decreases profit increases. Here highest and lowest values of expected profit are obtained to be 8.2203 and 0.6647 respectively for considered values.

Thus, in general with this study, behaviour of such complex system can be analyzed and forecast in advance.

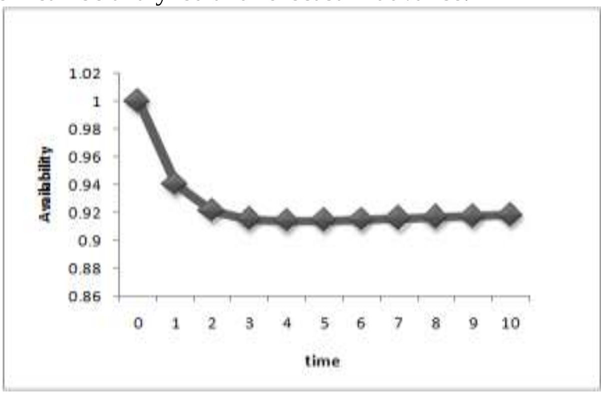

Figure 2: Time vs. Availability

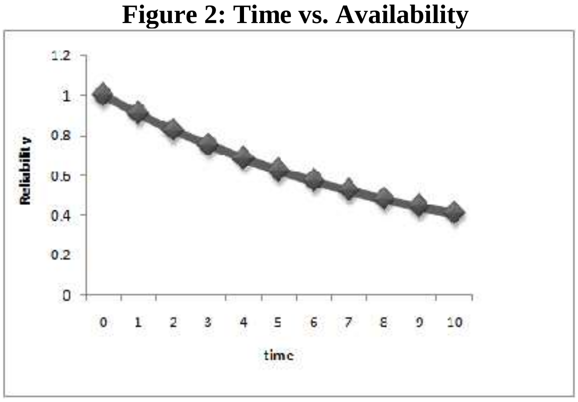

Figure 3: Time vs. Reliability

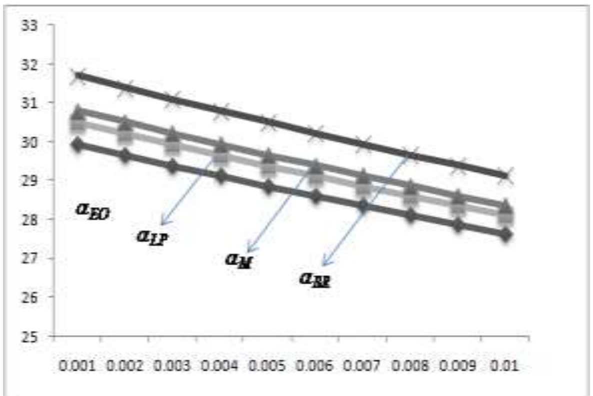

Figure 4: MTTF vs. α_{EO}, α_{LP}, α_{M}, α_{BR}

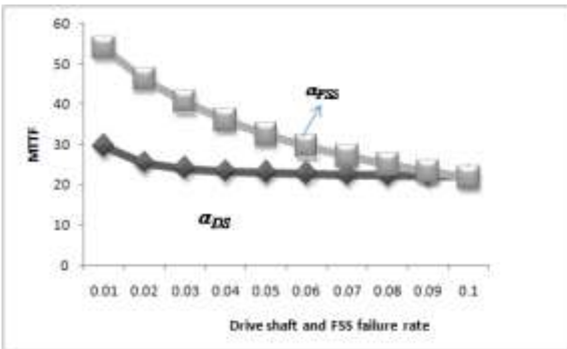

Figure 5 MTTF vs. α_{DS}, α_{FSS}

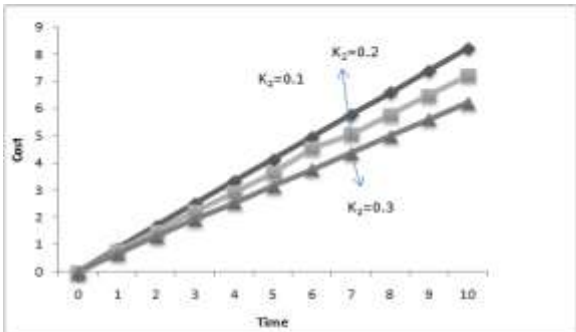

Figure 6: Time vs. cost

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