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Building up connections in the mind of the learner; a method to improve mathematics' understanding

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ABSTRACT

Understanding in mathematics is mainly concerned with building up connections between mathematical concepts. Literature reveals if we are succeeded in embedding one mathematical concept into another then this lead to better understanding in mathematics. In pursuance of this idea twenty pre-lectures were prepared from grade 10^{th} level mathematics textbook keeping in view the predictions of information processing model. In each pre-lecture efforts were made to build up connections between the previous, existing and coming mathematical concepts. To find out the effectiveness of pre-lectures and to in-depth the students concepts twenty post-tests were prepared keeping in view Blooms' taxonomy. The subjects of this study consisted of two secondary schools in Cantt/ Garrison setup Peshawar Pakistan. The whole sample (n=114) consisted of boys (n=78) and girls (n=36). The boys' sample (n=78) further divided into with pre-lecture group (n=39) and without pre-lecture group (n=39). Similarly the girls' sample further divided into with pre-lecture group (n= 18) and without pre-lecture group (n=18). The data collection was continued through out the whole academic session 2011-12. The data analysis was made by using t-test at 5% level of significance. The results revealed as whole the students with pre-lecture performed significantly better than the students without pre-lecture. The findings outlined that the use of pre-lecture strengthened mathematical connections, which further enhanced their understanding in mathematics.

1. Introduction

In a traditional setup to reach at the right answer is widely rewarded and most of the time understanding in mathematics is confused with this phenomenon. This is a matter of fact that irrespective of knowing about why and where of a mathematical problem many students achieve the right end merely confining to remembering formulae or following a procedure. But in such situations still they have lacking the ability to cope with the mathematical problems in a modified or generalized form. This type of learning describes by Ausuble (1963) as rote learning. This learning goal is achieved by many students just putting information into memory system by repetition in disconnected form from other learned material. Orton (2004) views because of this majority of the students in schools and even at colleges operate at algorithmic level which is not a real essence for understanding in mathematics. The other extreme to this type of learning is meaningful learning. This type of learning is accomplished when a proper connection is built up between existing and coming ideas in the mind of the learner. The matter of connection and disconnection is very important in the context to understanding in mathematics. Pointing towards the same Skemp (1976) describes two types of understanding in mathematics .i.e. instrumental and relational. He views instrumental understanding as simply knowing "rules without reasons". On the other hand relational understanding in mathematics is concerned with knowing both "what to do" and "why". More deeply he describes this type of understanding in mathematics as "building up a conceptual structure". It means to assimilate a concept or group of concepts into a suitable schema .i.e. to form a connection between ideas, facts or procedure. According to Nickerson (1985), "the richer the conceptual context in which one can embed a new fact, the more one can be said to understand the concept". Similarly Hiebart and Carpenter (1992) elaborate, "a mathematical idea, procedure or fact is thoroughly understood if it is linked to existing networks with stronger or numerous connections". To Orton (2004) "if there were no relevant concepts already in the mind to which new knowledge could be linked, the new knowledge would have to be learned by rote and stored in an arbitrary and disconnected manner". Citing the above literature clearly indicate that the objective to build up a connection between different mathematical ideas lead the learner towards understanding in mathematics. Superficially the above ideas are very encouraging to guide us towards the real essence of understanding in mathematics. But more deeply unless and unstill, if we are ignorant about what is actually done at the time to attend, retain, store and retrieve information in the mind of the learner, then the measures to connect one mathematical idea to the other would be futile. Therefore there is need to unfold this phenomenon and take some concrete steps in the light to interconnect mathematical ideas in the mind of the learner, enrich existing one and further pave the way that it can be accessed in several cross references. In this connection Johnstone (1997) predictive model of mathematics and science education stress that there is need to stimulate the background knowledge in the long term memory of the learner,

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which further guarantee proper interaction of the ideas and minimize the chances of fragmentation.

The Predictive Model of Mathematics and Science Education

This model signifies that the teaching process should be accomplished in a scientific way. If we are ignorant about the existing knowledge in the long term memory of the learner then to embed one idea into other would be impossible. The preparation of long term memory of the learner is absolutely essential to internalize a new idea or information. If concentration is not diverted to this phenomenon then the incoming idea cannot be properly interacted with the existing one, then learning would be done in disconnected and fragmented fashion.

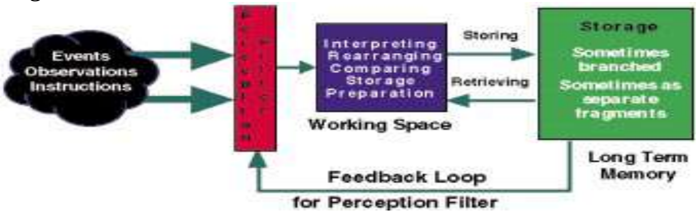

Fig 1: Johnstone (1997) predictive model of mathematics and science education

This model predicts that:

- If working memory is overloaded, learning more or less cease.
- If perception filter works in efficient way then working memory overload is less likely.
- The filtration process is controlled by what is already held in the long term memory.
- If the knowledge is stored in linked fashion, it will be easily recalled.

According to Johnstone (1997) we have a filtration system on the onset for incoming information. According to Zaman (1998) it is a selective process as an expert only process a limited part of incoming information and choose the important or of interest or of greater impact. For a novice it is difficult to differentiate between what is important and unimportant. This is because the first one bears the context such as his previous knowledge, interest and misconceptions. Whereas the second one lacking these qualities. A similar situation is occurred while we are going to install a new program in the computer. If the supporting file for new program is present in the computer memory then it is accepted otherwise rejected. The presence of context in the mind of the learner is an important step which let the perception filter works in an efficient way. Thus there are better chances of interaction of new and existing ideas in the mind of the learner.

After admitting information through perception filter then it enters into an important part of the mind known as working memory space. Working memory is the main work place where reasoning, understanding and problem solving is accomplished. According to Baddeley (1990) working memory is that part of the mind where information is temporarily held and manipulated. Working memory not only receives information from the external world but also receives information from long term memory. According to Miller (1956) this part of the mind is of limited capacity. It is not only of limited capacity but to Johnstone (1997) "it is a shared space in which there is a tradeoff between what has to be held in conscious memory and the processing activities to handle it, transform it, manipulate it and get it ready for storage in the long term memory store. If there is too much to hold, there is not enough space for processing; if a lot of processing is required, we cannot store much". In most of the classrooms situations the limited capacity of this part of mind is ignored and students are exposed to a bulk of material at a time, which results in confusion rather than understanding. According to Johnstone & Letton (1988) in problem solving it is easily overloaded due to holding a lot of data and at the same time having no space for processing necessary to find a solution. This shared space can be used in an efficient way, when a file which is needed, readily available in the long term memory at the time to process new knowledge. Thus in this way the new knowledge will easily interacted with the existing knowledge in the long term memory.

As said above if a new mathematical idea is embed into existing one then it is understood thoroughly and at the time of need it can be used productively. Similarly Johnstone (1994) says, "Storage is most efficient, from retrieval point of view, if the new knowledge is linked to existing material in the long term memory to form a branched network which can be accessed in several ways". To Zaman (1998) "long term memory has links with perception filter and working memory. It is a large store where facts are kept, concepts developed and attitudes formed". To use perception filter and working memory in efficient way there must be some related files in the long term memory; this leads us towards the idea of pre-learning. Following this idea concentration is given to the mental preparation of the student before a lecture. It is a careful process concentrating on important and ignoring irrelevant ideas to the coming topic.

2. Statement of the problem

According to Hiebert & Carpenter (1992) understanding in mathematics is dependent on using procedure and understanding concepts. But in traditional setups merely confining to follow procedure and attaining the right solution is considered to be the main goal for understanding in mathematics. Students usually seem competent to follow rules and procedures while solving mathematical problems. But to many mathematics' educationists this is a blind attempt and still many students don't know what their answer mean? According to Miller (1992), many students solve mathematical problems only relying on algorithmic approach and don't know the real meaning behind the procedure. Due to such practices in schools students lack the capability to cope with mathematical problems in more complex or abstract form. Such practices are accomplished by the students with a mindset lacking to build up connections between the different mathematical ideas. To achieve the goal of interlinking different mathematical ideas, literature reveals proper concentration is not given to the mind of the learner that; How vigorously a learner can perceive the incoming idea? How properly he/she can retain it and then allow it to interact with the existing one? And how successfully will this idea be available to retrieve it at the time of need? According to Johnstone (1997), to ensure the proper accomplishment of these distinct processes a teacher has to prepare the mind of the learner to successfully embed one idea into the other. According to Hayes (1988), "teachers are more expert learners whose understanding about how to learn the subject matter is what students need at least as much as they do the factual information". This phenomenon leads us towards the idea of pre-learning.

3. Objectives of the study

The purpose of this study was to investigate the effect of using pre-lecture on the students' understanding in mathematics.

4. Methodology

4.1 Research Design

In this study pre-lecture post-test randomized experimental group design was used. This design involved two groups .i.e.

with pre-lecture and without pre-lecture. With pre-lecture group of students got instruction through information processing model whereas without pre-lecture group of students through traditional teaching methods. The post-test mean scores of the both groups then analyzed and compared to judge the effectiveness of the treatment. This design is the same as pre-test post-test equivalent group design but except no pre-test is assigned to the sample.

4.2 Duration of Experimental Survey:

The experimentation process was continued for the whole academic session 2011-12.

4.3 Sample of the Study:

The project of this study was employed in two schools in Federal Government Educational Institutions Canntt/Garrison setup Peshawar Pakistan. A total of 114 grade 10^{th} level students (boys, girls) were randomly selected from the sampled schools. Majority of the students were belonging to lower middle class civilian and low ranked armed forces families. The average ages of the students were between 14-16 years.

4.4 Content of the Study:

For the present study twenty topics were selected from grade10^{th} mathematics text book. The topics selection was made keeping in view the topic selection criteria for Board examinations in the subject of mathematics by Federal Board of Intermediate & Secondary Education Islamabad. For this purpose five years previous papers were thoroughly screened out and finally the topics were selected. One of the benefits of this selection was that the focus concentrated on the main topics of various chapters. Secondly the selection of twenty topics didn't confine to one or two chapters but to a greater extent covered the main topics of the entire syllabus.

4.5 Procedure & Time Table of the Study

This study was continued in the above mentioned sampled schools for the whole academic session 2011-12. In one of the two sampled schools .i.e. in boys section the researcher himself implemented the research plan. For this purpose the researcher used to conduct two lectures on daily basis with grade 10^{th} boys with and without pre-lecture. In the girl section there was only one female math teacher responsible to conduct the routine classes at grade 10^{th} level. She was given two days training prior to implementation of research plan. The regional authority of the school was taken into confidence in this regard.

The whole sample was divided into with and without pre-lecture groups. The students in with pre-lecture group got instruction through pre-lecture (information processing model), whereas the students in without pre-lecture group got instruction through traditional teaching method.

One week before the scheduled lectures each student in with pre-lecture group handed over pre-lectures and they were instructed to complete the preparatory work before coming to the lecture. On the delivery day of the scheduled lectures during the first five minutes of the lecture questions were raised about the preparatory work. After finding the satisfactory answers then progress was made to start the topic. The topic was detailed in accordance with the main headings of the pre-lecture.

On the alternate day of scheduled lecture each student in both groups got through a twenty minutes post-test. If any student remained absent on the test day then on the alternate day special arrangements were made such as to conduct lecture and post-test in library or any other free period.

According to Farooq and Shah (2008) mathematics' achievement is highly affected by the students' attitude towards this subject. Therefore before going to employ the research plan it was necessary to assess this factor in detail and find out any significant difference between the attitude of the sample with and without pre-lecture.

4.6 Instruments

In this study three instruments were used for data collection. The first instrument used in this study before implementing the experiment was the attitude questionnaire, for the purpose to come to know about the attitude of the sample with and without pre-lecture. The other instrument was pre-lectures handed over to the "with pre-lecture" groups time to time in accordance with the scheduled syllabus to be taught. Similarly post-tests were conducted to both the groups in order to find out the effectiveness of the new methodology. The detail of each one is given below;

4.7 Attitude Scale

Keeping in view the importance of attitude as it affects the students' performance in mathematics; an attitude questionnaire was developed on the pattern Likert (1932) five points scale. The questionnaire was composed of 20 items. According to Sommer *et, al.* (1997), "an attitude scale should yield consistent results". Therefore to ensure about the reliability of the attitude questionnaire the first ten items were designed in the manner to point towards the positive aspects, whereas the other ten items pointing the same in negative form. The second part was constructed to check the consistency of the responses between the positive and negative statements of the attitude questionnaire.

4.8 Pre-lecture

Educationists and psychologists view the importance of previous knowledge as an important factor in building up connection between existing and incoming ideas. This is because to Jhonstone (1997) the humans are patterns seekers as new things are related into an existing system to make sense of it. The new idea is rejected when it doesn't make any sense. In other words the new idea is rejected when there is no relevant file in the mind of the learner. On the basis of relatedness and disconnection of the existing and new knowledge Ausuble (1978) presents the idea of meaningful and rote learning. Meaningful learning is done when a connection is established between existing and new knowledge. Rote learning is done when the coming knowledge finds no connection with the existing one.

To establish proper files in the mind of the learner is an important step towards what is meant by building up connections between mathematical ideas which further leads to understanding in mathematics. Various researches in the field of science education used the idea of pre-learning especially in laboratory work and showed considerable improvements in students' understanding. Zaman (1996) highlights the same as the use of pre-lab activity significantly improved the students' attitude positively towards the laboratory work. Zaman (1998) reports a significant improvement in the performance of the students used pre-lab before the start of the experiment. Similarly Safdar (2010) reports a significant improvement in the students' performance in the physics theory and practical work when they adopted pre-learning activities.

Keeping in view the importance of building up connections between mathematical ideas in the mind of the learner twenty pre-lectures were prepared from grade 10^{th} mathematics text books. To build up connections between mathematical ideas in the minds of the students, pre-lectures were detailed in the manner as; where did they use the concept or related sub concepts, what target they needed to achieve, how to achieve the target, why to achieve the target and where to use the learned concept. According to Allen (1992) doing mathematics leads to knowing and understanding mathematics. Therefore some problems related to the coming topic were also given at the end of each pre-lecture. The students were strictly instructed to solve the problems before coming to the scheduled lecture. Moreover the students were also instructed to read each step carefully given in the pre-lecture, consult the relevant portion in the text book and write their questions which might have raised during the preparatory phase.

4.9 Post-test

To judge the effectiveness of pre-lectures and provide an opportunity to the students to think critically about their performance, the idea of post-test was used in this study. Highlighting its importance Zaman (2003) elaborates that post-test provide an opportunity to the students to think independently and help in developing skills for problem solving. Further he views post-test help the students to link the new learning to existing one in an enriched manner. Safdar (2002) asserts post-test help the students to draw conclusions and plan their own activities for future problem solving. Moreover the results obtained through post-test provide information about the students seriousness towards pre-lecture preparation, their weaknesses in the topic area, background knowledge and the teacher own weaknesses regarding the lecture.

Keeping in view the above ideas twenty post-tests were prepared on the basis of pre-lecture and the approved text material taught to the sample. Each post-test used in this study comprised four objective type questions and two or three short questions related to approved topics. These questions were prepared in accordance with cognitive domain of Bloom's (1956). In each post-test efforts were made to set the questions in forty sixty ratio between lower and higher order cognitive levels of the cognitive domain. If possible efforts were made to relate the questions to daily life situations in order to check the students' understanding in the core concept and their generalizing ability.

To check the validity and reliability of the instruments three months pilot study was conducted in the said schools for the session 2010-11. For pilot study one class each at grade 10^{th} level was selected and a sample of ten students was used. Moreover the mathematics teachers were also consulted in preparation of research instruments.

5. Results and Discussion

5.1 Attitude

Before going to analyze the sample' attitude towards mathematics it was necessary to determine the reliability of the attitude questionnaire. This was done by employing internal consistency method .i.e. each pair of items measuring the same dimension. To check the consistency of the responses made by the sample chi-square test was used. The results depicted strong internal consistency between the responses of the sample towards positive and negative statements of the attitude questionnaire. After confirming about the internal consistency of the attitude questionnaire, then it was safe to be used to draw reliable results regarding the sample' attitude towards mathematics. Originally the cells in strongly disagree column of the table appeared to have expected frequencies below 5. See table 1;

Table 1: The frequency distribution of the responses of the sample with & without pre-lecture

Respondents Items	Frequencies				
	SA	A	N	D	SD
With	20	24	2	9	2
Without	18	21	9	5	4
With	11	25	10	8	3
Without	16	12	14	12	3
With	11	21	16	6	3
Without	18	17	12	8	2
With	17	19	10	8	3
Without	17	17	12	7	4
With	15	22	10	7	3
Without	15	22	10	6	4
With	12	12	17	10	6
Without	15	17	7	15	3
With	15	19	16	5	2
Without	20	20	8	4	5
With	27	19	4	5	2
Without	31	16	2	3	5
With	8	21	15	9	4
Without	14	19	14	8	2
With	23	18	5	8	4
Without	25	14	8	6	4

Table 2: Chi-square test results to the responses of the sample with & without pre-lecture

Respondents	Frequencies					Results
	SA	A	N	D	SD	
With (57)		44	2	11		4.95< 5.99
W. out (57)		39	9	9		
With (57)		36	10	11		2.28< 5.99
W. out (57)		28	14	15		
With (57)		32	16	9		0.75< 5.99
W. out (57)		35	12	10		
With (57)		36	10	11		0.23< 5.99
W. out (57)		34	12	11		
With (57)		37	10	10		0 < 5.99
W. out (57)		37	10	10		
With (57)		24	17	16		5.42< 5.99
W. out (57)		32	7	18		
With (57)		34	16	7		3.46< 5.99
W. out (57)		40	8	9		
With (57)		46	4	7		0.35< 5.99
W. out (57)		42	5	8		
With (57)		29	15	13		0.68< 5.99
W. out (57)		33	14	10		
With (57)		41	5	12		0.92< 5.99
W. out (57)		39	8	10		

According to chi-square simple rule of thumb the results are doubtful if one or more expected frequencies falls below five. Therefore to follow this rule “disagree” and “strongly disagree” level frequencies are combined as are the “agree” and “strongly agree”. The table 2 details the overall frequency distribution of the sample with and without pre-lecture in original form, whereas the table 3 details the chi-square test results after combining the above said levels frequencies.

In Table 2 all the calculated values are less than the tabulated values at 5% level of significance with degree of freedom 2. This confirms there is no significant difference between the attitude of the students with and without pre-lecture towards mathematics. The frequencies in agree column are by a huge margin greater than the frequencies in disagree column. Therefore as a result it can be said the students in both with and without pre-lecture groups have positive attitude towards mathematics.

5.2 The students’ understanding in mathematics

The students’ understanding in mathematics was analyzed on the basis of post-test the experimental tool developed by researcher. Further to check the reliability of the findings taken by the researcher using experimental tool “post-test”, it was necessary to confirm this from an independent source. The sampled schools were registered with the **Federal Board of Intermediate & Secondary Education Islamabad** therefore the findings of this study were extended to the mean scores obtained in Federal Board Annual Examination (2012) in the subject of mathematics at S.S.C level. To determine the significant difference between the performance of the students with and without pre-lecture t-test was used.

5.2.1 The Post-test performance of the sample

The post-test mean scores made by the whole sample with and without pre-lecture have been analyzed and are presented here in the table 3:

Table 3: t-test result of the post-test mean scores of the sample with & without pre-lecture

Students	N	Means	SD	t_{cal}	t_{tab}	Result
W P L	57	5.54	1.32	4.93	1.981	sig
WT P L	57	4.33	1.29			

LOS = .05, DF = 112

In table 3 the t-test was applied on the post-test mean scores at 5% level of significance and degree of freedom 112. The calculated value found 4.93 is greater than the tabulated value 1.981. The above calculated value indicates that there is significant difference between the mean scores of the students with and without pre-lecture. This further indicates that the students with pre-lecture understanding in mathematics significantly improved as compared to the students without pre-lecture.

5.2.2 The Board Examination performance

The mean scores made by the whole sample with and without pre-lecture in Board Examination have been analyzed and are presented here in the table 4:

Table 4: t-test result of the post Board mean scores of the sample with & without pre-lecture

Students	N	Means	SD	t_{cal}	t_{tab}	Result
W P L	57	56.57	13.7	2.16	1.981	sig
WT P L	57	50.71	15.0			

LOS = .05, DF = 112

In table 4 the t-test was applied on the post-test mean scores at 5% level of significance and degree of freedom 112. The calculated value found 2.16 is greater than the tabulated value 1.981. The above calculated value indicates that there is significant difference between the mean scores of the students with and without pre-lecture. This further indicates that the students with pre-lecture understanding in mathematics significantly improved as compared to the students without pre-lecture.

6. Conclusion

The results that emerged from the study of the students’ responses to the attitude questionnaire tend to confirm that there was no significant difference between the attitude of the sample with and without pre-lecture towards mathematics. Majority of the students in both groups have positive attitude towards mathematics. It further confirms that significant difference between the post-test and post Board mean scores of the sample with and without pre-lecture observed was due to the difference of teaching methods and not because of the attitude of the sample.

The post-test performance of the sample with pre-lecture was significantly better than the performance of the sample without pre-lecture. This was further confirmed by the mean scores difference of the sample with and without pre-lecture in Federal Board Annual Examination (2012). Hence it can be concluded that the pre-lecture strategy employed in this study helped in building up connections in mathematical ideas which led to improvement in students’ understanding in mathematics.

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