



Solving Fully Fuzzy Linear systems with trapezoidal Fuzzy number Matrices using Moore-Penrose generalised inverse

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ABSTRACT

In this article we apply Moore- Penrose generalised inverse method to solve a fully fuzzy linear system. By this method we obtain the solution of fully fuzzy linear system of equations in the case of Trapezoidal fuzzy number matrices. A numerical example is also given.

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1. Introduction

Several authors have given various methods to solve a fully fuzzy linear system (FFLS). These methods hold for non singular matrices. The advantage of Moore-Penrose generalised inverse is it does not depend on nonsingular matrices. Moore Penrose inverse can be found even in the case of singular matrices. It also coincides with A^{-1} in the case of non singular matrices. Therefore all types of FFLS can be solved by using generalised inverse technique. The fact that every matrix is having a generalised inverse has been fully exploited.

The structure of this paper is organized as follows. In section 2 preliminary concepts of trapezoidal fuzzy number matrices has been discussed. In section 3 preliminary concepts of Moore-Penrose generalised inverse have been discussed. In section 4 an algorithm to solve FFLS in the form of Trapezoidal fuzzy number matrices by using generalised inverse has been introduced. In section 5 a numerical example is given. In section 6 conclusion is given.

2. Preliminary concepts of trapezoidal fuzzy number matrices

(a) The characteristic function of a crisp set assigns a value of either 1 or 0 to each individual in the universal set, thereby discriminating between members and non members of the crisp set under consideration. This function can be generalised such that the values assigned to the elements of the universal set fall within a specified range and indicate the membership grade of these elements in the set in question. Larger values denote higher degrees of set membership. Such a function is called a membership function and the set defined by it is a fuzzy set. Thus μ_A the membership function by which a fuzzy set A is defined has the form $\mu_A : X \rightarrow [0,1]$.

(b) A trapezoidal fuzzy number denoted by $A = (m, n, \alpha, \beta)$ has the membership function

$$\mu_A(x) = \begin{cases} 0 & x \leq \alpha \\ \frac{x - \alpha}{m - \alpha} & \alpha \leq x \leq m \\ 1 & m \leq x \leq n \\ \frac{\beta - x}{\beta - n} & n \leq x \leq \beta \\ 0 & x \geq \beta \end{cases}$$

(c) Addition of 2 fuzzy numbers

$$M = (m, n, \alpha, \beta) \text{ and } N = (x, y, \gamma, \delta).$$

$$M \oplus N = (m, n, \alpha, \beta) \oplus (x, y, \gamma, \delta) = (m + x, n + y, \alpha + \gamma, \beta + \delta)$$

(d) Multiplication of two fuzzy numbers.

$$M \otimes N = (m, n, \alpha, \beta) \otimes (x, y, \gamma, \delta) = (mx, ny, m\gamma + x\alpha, n\delta + y\beta)$$

3. Preliminaries in Moore- Penrose inverse

(a) The Moore Penrose generalised inverse of a matrix A , not necessarily square is a matrix A^+ that satisfies the conditions

$$(i) AA^+A = A$$

$$(ii) A^+AA^+ = A^+$$

(b) If A has order $n \times m$ then A^+ has order $m \times n$.

(c) A^+ is Unique.

(d) $A^+ = A^{-1}$ for non singular A .

$$(e) (A^+)^+ = A$$

(f) Any matrix has a Moore Penrose inverse.

4. An Algorithm for solving FFLS by generalised inverse method.

For solving FFLS $A \otimes X = B$ where

$$\mathcal{X} = (A, B, M, N)$$

$$X = (x, y, z, w)$$

$$\mathcal{B} = (b, g, h, k)$$

$$Ax = b \Rightarrow x = A^+b$$

$$By = g \Rightarrow y = B^+g$$

$$Az + Mx = h \Rightarrow z = A^+(h - Mx)$$

$$Bw + Ny = k \Rightarrow w = B^+(k - Ny)$$

As A^{-1} and B^{-1} are not found A and B need not be singular. A^+ and B^+ exist for all A and B .

Three cases arise

Case (i) : Square non singular matrices

$A^+ = A^{-1}$ is unique solution.

Case (ii): System which admit infinite number of solutions

For $AX = B$,

if $R(A) = R(A, B) <$ number of unknowns the equations are consistent and have infinite no of solutions.

In this case A^+ gives the solution with minimum Euclidean norm.

(i.e) if X is a n -dimensional column vector

$$\|X\|_2 = \sqrt{\langle X, X \rangle}$$

where $\langle X, X \rangle = X \cdot \bar{X} = X \cdot X$ (If X is real) is called the Euclidean inner product.

$\therefore A^+$ picks the solution in the least squares sense.

Case(iii): Inconsistent systems

In this case A^+ gives the solution that is best in the least square sense.

5. Numerical Example.

In this section we apply the algorithm for solving FFLS of fuzzy trapezoidal numbers.

$$\begin{pmatrix} (1,4,4,4) & (2,6,6,6) & (3,6,1,2) & (2,4,3,6) \\ (2,6,6,6) & (8,18,4,3) & (8,12,4,3) & (6,9,4,6) \\ (3,6,1,2) & (8,12,4,3) & (14,11,4,2) & (11,9,4,8) \\ (2,4,3,6) & (6,9,4,6) & (11,9,4,8) & (10,13,2,3) \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} (10,36,172,160) \\ (20,42,216,186) \\ (30,40,56,100) \\ (40,36,82,82) \end{pmatrix}$$

Solution

$$A = \begin{pmatrix} 1 & 2 & 3 & 2 \\ 2 & 8 & 8 & 6 \\ 3 & 8 & 14 & 11 \\ 2 & 6 & 11 & 10 \end{pmatrix}$$

$$B = \begin{pmatrix} 4 & 6 & 6 & 4 \\ 6 & 18 & 12 & 9 \\ 6 & 12 & 11 & 9 \\ 4 & 9 & 9 & 13 \end{pmatrix}$$

$$A^+ = \begin{pmatrix} 4 & -\frac{1}{4} & -\frac{3}{2} & 1 \\ -\frac{1}{4} & \frac{15}{16} & -\frac{1}{8} & 0 \\ -\frac{3}{2} & -\frac{1}{8} & \frac{5}{4} & -1 \\ 1 & 0 & -1 & 1 \end{pmatrix}$$

$$B^+ = \begin{pmatrix} \frac{33}{16} & \frac{7}{24} & -\frac{7}{4} & \frac{3}{8} \\ \frac{7}{24} & \frac{1}{4} & -\frac{1}{2} & \frac{1}{12} \\ -\frac{7}{4} & -\frac{1}{2} & 2 & -\frac{1}{2} \\ \frac{3}{8} & \frac{1}{12} & -\frac{1}{2} & \frac{1}{4} \end{pmatrix}$$

$$x = A^+b = \begin{pmatrix} 4 & -\frac{1}{4} & -\frac{3}{2} & 1 \\ -\frac{1}{4} & \frac{5}{16} & -\frac{1}{8} & 0 \\ -\frac{3}{2} & -\frac{1}{8} & \frac{5}{4} & -1 \\ 1 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 10 \\ 20 \\ 30 \\ 40 \end{pmatrix} = \begin{pmatrix} 30 \\ 0 \\ -20 \\ 20 \end{pmatrix}$$

$$y = B^+g = \begin{pmatrix} \frac{33}{16} & \frac{7}{24} & -\frac{7}{4} & \frac{3}{8} \\ \frac{7}{24} & \frac{1}{4} & -\frac{1}{2} & \frac{1}{12} \\ -\frac{7}{4} & -\frac{1}{2} & 2 & -\frac{1}{2} \\ \frac{3}{8} & \frac{1}{12} & -\frac{1}{2} & \frac{1}{4} \end{pmatrix} \begin{pmatrix} 36 \\ 42 \\ 40 \\ 36 \end{pmatrix} = \begin{pmatrix} 30 \\ 4 \\ -22 \\ 6 \end{pmatrix}$$

$$z = A^+(h - Mx) = \begin{pmatrix} 4 & -\frac{1}{4} & -\frac{3}{2} & 1 \\ -\frac{1}{4} & \frac{5}{16} & -\frac{1}{8} & 0 \\ -\frac{3}{2} & -\frac{1}{8} & \frac{5}{4} & -1 \\ 1 & 0 & -1 & 1 \end{pmatrix} \left[\begin{pmatrix} 172 \\ 216 \\ 56 \\ 82 \end{pmatrix} - \begin{pmatrix} 4 & 6 & 1 & 3 \\ 6 & 4 & 4 & 4 \\ 1 & 4 & 4 & 4 \\ 3 & 4 & 4 & 2 \end{pmatrix} \begin{pmatrix} 30 \\ 0 \\ -20 \\ 20 \end{pmatrix} \right] = \begin{pmatrix} 32 \\ 5 \\ -22 \\ 18 \end{pmatrix}$$

$$w = B^+(k - Ny) = \begin{pmatrix} \frac{33}{16} & \frac{7}{24} & -\frac{7}{4} & \frac{3}{8} \\ \frac{7}{24} & \frac{1}{4} & -\frac{1}{2} & \frac{1}{12} \\ -\frac{7}{4} & -\frac{1}{2} & 2 & -\frac{1}{2} \\ \frac{3}{8} & \frac{1}{12} & -\frac{1}{2} & \frac{1}{4} \end{pmatrix} \left[\begin{pmatrix} 160 \\ 186 \\ 100 \\ 82 \end{pmatrix} - \begin{pmatrix} 4 & 6 & 2 & 6 \\ 6 & 3 & 3 & 6 \\ 2 & 3 & 2 & 8 \\ 6 & 6 & 8 & 3 \end{pmatrix} \begin{pmatrix} 30 \\ 4 \\ -22 \\ 6 \end{pmatrix} \right] = \begin{pmatrix} 28 \\ 4 \\ -24 \\ 8 \end{pmatrix}$$

$$x = \begin{pmatrix} 30 \\ 0 \\ -20 \\ 20 \end{pmatrix}, y = \begin{pmatrix} 30 \\ 4 \\ -22 \\ 6 \end{pmatrix}, z = \begin{pmatrix} 32 \\ 05 \\ -22 \\ 18 \end{pmatrix}, w = \begin{pmatrix} 28 \\ 4 \\ -24 \\ 8 \end{pmatrix}$$

6. Conclusion

In this paper FFLS with trapezoidal fuzzy number matrices is solved by Moore Penrose method. This method does not have any condition and can be used for any type of matrix.

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