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Optimization of fuzzy inventory model with imperfect production lotsizing and marketing planning problem

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ABSTRACT

Traditional economic production / order quantity (EPQ/EOQ) models make various simplifying assumptions in order to arrive at a closed form solution for the optimal lotsize in a production facility. The assumption of fixed unit cost in EOQ / EPQ models has also been tackled by many researchers. For example, Cheng (1991) formulated a problem where production cost is affected by both the product's demand and the process reliability. Lee (1994), by formulating the unit cost on a function of the order quantity, took the economies of scale into account. Quality level of the product and specifications of the adopted manufacturing process also affect the unit product's cost. Therefore, in this paper we consider a profit maximizing firm who wants to jointly determine the optimal lotsizing, pricing and marketing decisions along with manufacturing requirements in terms of flexibility and reliability of the process. The objective is to determine the optimal order lotsize to maximize the total profit by employing the type of fuzzy numbers which are triangular. We propose two fuzzy inventory model in which first model with fuzzy reliability level and second model with fuzzy reliability level and second model with fuzzy reliability and fuzzy economic production quantity (EPQ) is presented. For ech case we employ the signed distance, a ranking method for fuzzy numbers, to find the estimate of total profit per unit time in the fuzzy sense and then derive the corresponding optimal lotsize. Numerical examples are provided to illustrate the results of proposed models.

1. Introduction

EPQ is assumed that factors such as unit production cost, product's demand and setup cost are fixed and know in advance and that the items produced are all of a perfect quality (Chen, 2000). However, in reality these assumptions are rarely satisfied. Therefore to take into account the real life situations and to respond more appropriately to increasing competitiveness in the global business environment, recently many researchers have studied the production lotsizing models under more realistic conditions. (Bay.S, 2009 & Jaber, 2000) extended the traditional EPQ model considering that produced or received items are not of perfect quality. In their model, each lot contains a random fraction of imperfect quality items, where these poor quality items can later be sold as a single batch at the end of the screening process. Maddah and Jaber (2008) resolved a flaw in the Salameh & Jaber's work and obtained a simple expression for the optimal order quantity. Moreover, they let several batches of poor quality items to be consolidated and shipped. Jaber (2006) also extended the classical EOQ model by considering an imperfect production process in which set-up cost is not constant and due to the learning effect, decreases after each set-up, Khan, Jaber and Bonney (2011) investigated the effect of both imperfect quality items and inspection errors in the EOQ models. (Cheng, T.C., 2006)

In this paper, Duffin (1967) we investigate an integrated pricing, lotsizing and marketing planning model in which optimal levels for product's quality along with flexibility and reliability of the production process also need to be determined.

This paper makes several contribution, to the current literature. First of all, we have comprehensively included all the relevant and meaningful factors and decision variables that different researchers considered in formulating demand, unit production cost and interest and depreciation cost functions in our proposed power functions. Secondly to the best of our knowledge, for the first time we have accounted for the relationship between process reliability and maintenance costs in the area of integrated lotsizing and marketing planning. Third in order to better model the situation as it in the real world, we have incorporated constraints on production capacity, storage space and marketing budget in our mathematical model. And finally, unlike other works in this domain which find optimal solutions for geometric programs with zero or one degrees of difficulty, by using recent advances in optimization software. We have been able to determine optimal levels for decision variables with a high degree of difficulty. (Esmaeili, 2010)

In section 2, a brief review of the work done by Seyed Jafar, 2012 model is given. In section 3, some definitions and properties about fuzzy sets related to this study are introduced. Section 4 presents two fuzzy models as described earlier. Section 5 provides numerical examples to illustrate the results of the proposed models. Fathian, M. (2009)

2. Mathematical Model

Parameters Glickman, T.S. (1976)

D $\rightarrow$ Total market demand covered by manufacturer

C $\rightarrow$ Unit Production Cost

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I $\rightarrow$ Percent inventory holding cost (per unit per unit time)
P $\rightarrow$ Total Market Demand
P $\rightarrow$ Percent manufacturer's share of total market demand
W $\rightarrow$ Space requirement for each item
W $\rightarrow$ Total space available for holding produced items
B $\rightarrow$ Total budget available to the marketing department
A $\rightarrow$ Resource requirements for each item
R $\rightarrow$ Total resources available
N(r) $\rightarrow$ Maintenance costs per production cycle
Y(S, r) $\rightarrow$ Total cost of interest and depreciation for the production process in each cycle.

Decision Variables

P $\rightarrow$ Unit price of the product
M_i $\rightarrow$ Volume of investments in marketing method i = 1, . . . , I per unit time
Q $\rightarrow$ Economic Production Quantity (EPQ)
Q $\rightarrow$ Quality level of the product from the customers point of view.
R $\rightarrow$ Reliability level of the production process (percent of non-defective items in a batch)
S $\rightarrow$ Set up cost

Profit = $PrQ - S - cQ - \frac{(ic)Q^2r^2}{2D} - Y(S, r) - N(r) - Tx\sum_{i=1}^{I} M_i$

Inventory Holding Cost in each cycle Jung.H (2001)

$$= ic\int_{t=0}^{T} I(t)dt$$

$$= ic\int_{t=0}^{T} (rQ - Dt)\, dt$$

$$= ic\left(\frac{r^2Q^2}{2D}\right)$$

The annual profit of the manufacturer as follows :

$$\pi(Q, r) = \left[P(rQ) - S - cQ - \frac{(ic)Q^2r^2}{2D} - Y(s, r) - N(r) - TX\sum_{i=1}^{I} M_i\right] \times \frac{D}{rQ}$$

$$= PD - \frac{SD}{rQ} - \frac{CD}{r} - \frac{(ic)Qr}{2} - Y(s, r)\frac{D}{rQ} - \frac{N(r)D}{rQ} - T\sum_{i=1}^{I} M_i \frac{D}{rQ}$$

$$= PD - \frac{CD}{r} - \frac{(ic)Qr}{2} - \frac{D}{rQ}\left[S + Y(s, r) + N(r) + T\Sigma M_i\right]$$

$$Q = \sqrt{\frac{2D\left[S + Y(s, r) + N(r) + T\Sigma M_i\right]}{icr^2}}$$

3. Preliminaries

Before presenting the fuzzy inventory models, we introduce some definitions and properties about fuzzy numbers with relevant operations.

Definition 1 :

For $0 \le \alpha \le 1$, the fuzzy set $\tilde{a}_z$ defined on R = (-α, α) is called an α-level fuzzy point if the membership function of $\tilde{a}_z$ is given by

$$\mu_{\tilde{a}_z}(x) = \begin{cases} \alpha, & x = a \\ 0, & x \ne a \end{cases}$$

Definition 2 :

The fuzzy set $\tilde{A}$ = (a, b, c) where a < b < c and defined on R, is called the triangular fuzzy number, if the membership function of $\tilde{A}$ is given by (Khan, M. (2011))

$$\mu_{\tilde{A}}(x) = \begin{cases} (x - a)/(b - a), & a \le x \le b \\ (c - x)/(c - b), & b \le x \le c \\ 0, & \text{otherwise} \end{cases}$$

Definition 3 :

For $0 \le \alpha \le 1$, the fuzzy set $[a_\alpha, b_\alpha]$ defined on R is called an α-level fuzzy interval if the membership function of $[a_\alpha, b_\alpha]$ given by

$$\mu_{\alpha B(\alpha)}(x) = \begin{cases} \alpha, & a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

Definition 4 :

Let $\tilde{B}$ be a fuzzy set on R, and $0 \le \alpha \le 1$. The α-cut B(a) of $\tilde{B}$ consists of points x such that $\mu_{\tilde{B}}(x) \ge \alpha$, that is B(α) = $[B_L(\alpha),\ B_U(\alpha)]$. Then we have

$$\tilde{B} = \bigcup_{0 \le \alpha \le 1} \alpha B(\alpha)$$

(or)

$$\mu_{\tilde{B}}(x) = \bigcup_{0 \le \alpha \le 1} \alpha C_{B(\alpha)}(x)$$

where

(i) αB(α) is a fuzzy set with membership function

$$\mu_{\alpha B(\alpha)}(x) = \begin{cases} \alpha, & a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

(ii) $C_{B(\alpha)}(x)$ is a characteristic function B(α)

(ie) $C_{B(\alpha)}(x) = \begin{cases} 1, & x \in B(\alpha) \\ 0, & x \notin B(\alpha) \end{cases}$

Definition 5 :

For any a and 0 ∈ R, define the signed distance from a to 0 as $d_0(a, 0) = 0$. If 0 > 0, the distance from a to 0 is a = $d_0(a, 0)$;

If a < 0, the distance from a to 0 is –a = $-d_0(a, o)$.

Hence $d_0(a, 0) = a$ is called the signed distance from a to 0.

Let Ω be the family of all fuzzy sets $\tilde{B}$ defined on R with which the α-cut B(α) = $[B_L(\alpha), B_U(\alpha)]$ exists for every α ∈ [0, 1], and both $B_L(\alpha)$ and $B_U(\alpha)$ are continuous functions on α ∈ [0, 1]. Then for any $\tilde{B}$ ∈ Ω, we have

$$\tilde{B} = \bigcup_{0 \le \alpha \le 1} [B_L(\alpha)_\alpha,\ B_U(\alpha)_\alpha]$$

Definition 6 :

For $\tilde{B}$ ∈ Ω, define the signed distance of $\tilde{B}$ to $\tilde{0}$, (ie y axis) as

$$d\left(\tilde{B}, \tilde{0}_1\right) = \int_0^1 d\left([B_L(\alpha)_\alpha,\ B_U(\alpha)_\alpha],\ \tilde{0}_1\right) dx$$

$$= \frac{1}{2}\int_0^1 \left(B_L(\alpha)_\alpha + B_U(\alpha)_\alpha\right) dx$$

According to this, we obtain the following property.

Property 1 :

For the triangular fuzzy number $\tilde{A} = (a, b, c)$ the α-cut of $\tilde{A}$ is $[A_L(\alpha),\ A_U(\alpha)]$, α ∈ [0, 1] where $A_L(\alpha) = a + (b – a)\alpha$ and $A_U(\alpha) = c - (c – b)\alpha$.

The signed distance of $\tilde{A}$ to $\tilde{0}_1$ is

$$d(\tilde{A}, \tilde{0}_1) = \frac{1}{4}(a + 2b + c)$$

Furthermore, for two fuzzy sets $\tilde{B}, \tilde{G} \in \Omega$ where $\tilde{B} = \bigcup_{0 \le \alpha \le 1} [B_L(\alpha)_\alpha, B_U(\alpha)_\alpha]$ and $\tilde{G} = \bigcup_{0 \le \alpha \le 1} [G_L(\alpha)_\alpha, G_U(\alpha)_\alpha]$ and a $\in$R,

(i) $\tilde{B} + \tilde{G} = \bigcup_{0 \le \alpha \le 1} [B_L(\alpha)_\alpha + G_L(\alpha)_\alpha, B_U(\alpha)_\alpha + G_U(\alpha)_\alpha]$

(ii) $\tilde{B} - \tilde{G} = \bigcup_{0 \le \alpha \le 1} [B_L(\alpha)_\alpha - G_L(\alpha)_\alpha, B_U(\alpha)_\alpha - G_U(\alpha)_\alpha]$

(iii) $\tilde{K}_1 \times \tilde{B} = \begin{cases} \bigcup_{0 \le \alpha \le 1} [(KB_L(\alpha))_\alpha, (KB_U(\alpha))_\alpha] \\ \bigcup_{0 \le \alpha \le 1} [(KB_U(\alpha))_\alpha, (KB(\alpha))_\alpha] \\ \tilde{0} \qquad K = 0 \end{cases}$

Property 2 :

For two fuzzy sets $\tilde{B}, \tilde{G} \in \Omega$ and $K \in R$

$$d(\tilde{B}, \tilde{0}_1) = \int_0^1 d([B_L(\alpha)_\alpha, B_U(\alpha)_\alpha], \tilde{0}_1)\, dx$$

$$= \frac{1}{2}\int_0^1 (B_L(\alpha)_\alpha + B_U(\alpha)_\alpha)\, dx$$

(i) $d(\tilde{B}(+)\tilde{G}, \tilde{0}_1) = d(\tilde{B}, \tilde{0}_1) + d(\tilde{G}, \tilde{0}_1)$

(ii) $d(\tilde{B}(-)\tilde{G}, \tilde{0}_1) = d(\tilde{B}, \tilde{0}_1) - d(\tilde{G}, \tilde{0}_1)$

(iii) $d(\tilde{K}_1 (\times) \tilde{B}, \tilde{0}_1) = K d(\tilde{B}, \tilde{0}_1)$

4. Fuzzy EOQ Models

Model with Fuzzy Economic Order Quantity

In this subsection we modify the crisp economic order quantity model by incorporating the fuzziness of the EOQQ. For convenience we let Q = 1 – p. Then we fuzzify Q to be a triangular fuzzy number $\tilde{Q} = (Q\text{-}\Delta_1, Q, Q\text{-}\Delta_2)$ where $0 < \Delta_1 < Q$ and $0 < \Delta_2 \le 1 - Q$, Δ_1, Δ_2 are determined by the decision makers. In this case, the total profit unit time is a fuzzy value also, which is expressed as,

$$\tilde{\pi}(Q, r) = PD - \frac{CD}{r} - \frac{(ic)\tilde{Q}r}{2D} + \frac{D}{r\tilde{Q}}[S + Y(s, r) + N(r) + T\Sigma M_i]$$

Now we defuzzify $\tilde{\pi}(Q, r)$ using the signed distance method. The signed distance of $\tilde{\pi}(Q, r)$ is given by

$$d(\tilde{\pi}(Q, r), \tilde{0}_1) = PD - \frac{CD}{r} - \frac{(ic)r}{2D} d(\tilde{Q}, \tilde{0}_1) - \frac{D}{r} d\left(\frac{1}{\tilde{Q}}, \tilde{0}_1\right)[S + Y(s, r) + N(r) + T\Sigma M_i]$$

where $d(\tilde{Q}, \tilde{0}_1)$ and $d\left(\frac{1}{\tilde{Q}}, \tilde{0}_1\right)$ are measured as follows.

The signed distance of fuzzy number $\tilde{Q}$ to $\tilde{0}_1$ is

$$d(\tilde{Q}, \tilde{0}_1) = \frac{1}{4}[(Q\text{-}\Delta_1) + 2Q + (Q\text{-}\Delta_2)]$$

$$= Q + \frac{1}{4}(\Delta_1 - \Delta_2)$$

Also the left and right end points of the α-cut (0 ≤ α ≤ 1) of $\tilde{Q}$ are

$Q_L(\alpha) = (Q - \Delta_2) + \Delta_1\alpha > 0$ and

$Q_U(\alpha) = (Q + \Delta_2) - \Delta_2\alpha > 0$ respectively.

Since $0 < Q_L(\alpha) < Q_U(\alpha)$.

The left and right end points of the α-cut (0 ≤ α ≤ 1) of $\frac{1}{\tilde{Q}}$ are

$\left(\frac{1}{Q}\right)_L(\alpha) = \frac{1}{Q_L(\alpha)} = \frac{1}{(Q - \Delta_1) + \Delta_1\alpha}$ respectively.

Then the signed distance of $\frac{1}{\tilde{Q}}$ to $\tilde{0}_1$ is

$$d\left(\frac{1}{\tilde{Q}}, \tilde{0}_1\right) = \frac{1}{2}\int_0^1\left[\left(\frac{1}{Q}\right)_L(\alpha) + \left(\frac{1}{Q}\right)_U(\alpha)\right]$$

$$= \frac{1}{2}\left(\frac{1}{\Delta_1}\text{In}\frac{Q}{Q - \Delta_1} - \frac{1}{\Delta_2}\text{In}\frac{Q}{Q + \Delta_2}\right)$$

which is positive since $\Delta_1 > 0$, $\Delta_2 > 0$.

$\text{In}\left(\frac{Q}{Q - \Delta_1}\right) > 0$ and $\text{In}\left(\frac{Q}{Q + \Delta_2}\right) < 0$

We have

$$\pi^*(Q, r) = d(\pi(Q, r), \tilde{0}_1) =$$

$$PD - \frac{CD}{r} - \frac{(ic)r}{2}\left(Q + \frac{1}{4}(\Delta_2 - \Delta_1)\right) - \frac{D}{r}\frac{1}{2}\left(\frac{1}{\Delta_1}\text{In}\frac{Q}{Q\text{-}\Delta_1} - \frac{1}{\Delta_2}\text{In}\frac{Q}{Q\text{+}\Delta_2}\right)$$

$\pi^*(Q, r)$ is regarded as the estimate of the total profit per unit time in the fuzzy sense. The objective of the problem is to determine the optimal reliability level r^* such that $\pi^*(Q, r)$ has a maximum value. We take the first derivative of $\pi^*(Q, r)$ with respect to r and obtain

$\frac{\partial \pi^*(Q, r)}{\partial r} = 0$ Because $\frac{\partial \pi^*(Q, r)}{\partial r} < 0$

The optimal reliability level is

$$r = \sqrt{\frac{2D\left[C + \frac{1}{2}\left(\frac{1}{\Delta_1}\text{In}\frac{Q}{Q\text{-}\Delta_1} - \frac{1}{\Delta_2}\text{In}\frac{Q}{Q\text{+}\Delta_2}\right)\right]}{IC\left(Q + \frac{1}{4}(\Delta_2 - \Delta_1)\right)}}$$

Model with fuzzy reliability level and fuzzy order quantity

The crisp order quantity Q is fuzzified as the triangular fuzzy number $\tilde{Q} = (Q - \Delta_3, Q, Q + \Delta_4)$, where Δ_3, Δ_4 are determined by the decision makers and should satisfy the conditions $0 < \Delta_3 < Q$ and $0 < \Delta_4$. For this case we express the fuzzy total profit per unit time as

$$\tilde{\pi}(Q, r) = PD - \frac{CD}{r} - \frac{(ic)\tilde{Q}r}{2} - \frac{D}{rQ}[S + Y(s, r) + N(r) + T\Sigma M_i]$$

The signed distance of $\tilde{\pi}(Q, r)$ to $\tilde{0}_1$ is given by

$$d(\tilde{\pi}(Q, r), \tilde{0}_1) = PD - \frac{CD}{r} - \frac{(ic)r}{2} d(\tilde{Q}, \tilde{0}_1) - \frac{D}{r}[S + Y(s, r) + N(r) + T\Sigma M_i]\, d\left(\frac{\tilde{D}}{\tilde{Q}}, \tilde{0}_1\right)$$

where $d(\tilde{Q}, \tilde{0}_1) = Q + \frac{\Delta_2 - \Delta_1}{4}$

$$d(\tilde{D}, \tilde{0}_1) = D + \frac{\Delta_4 - \Delta_3}{4} d$$

We calculate the signed distance $d\left(\frac{\tilde{D}}{\tilde{Q}}, \tilde{0}_1\right)$

The left and right end points of the α-cut $(0 \le \alpha \le 1)$ if $\tilde{Q}$ are

$$\left(\frac{D}{Q}\right)_L(\alpha) = \frac{D_L(\alpha)}{Q_U(\alpha)} = \frac{(D - \Delta_3) + \Delta_3\alpha}{(Q + \Delta_2) - \Delta_2\alpha} \text{ and}$$

$$\left(\frac{D}{Q}\right)_U(\alpha) = \frac{D_U(\alpha)}{Q_L(\alpha)} = \frac{(D + \Delta_4) - \Delta_4\alpha}{(Q - \Delta_1) + \Delta_1\alpha}$$

The signed distance of $\frac{\tilde{D}}{\tilde{Q}}$ to $\tilde{0}_1$ can be derived as

$$d\left(\frac{\tilde{D}}{\tilde{Q}}, \tilde{0}_1\right) = \frac{1}{2}\int_0^1\left[\left(\frac{D}{Q}\right)_L(\alpha) + \left(\frac{D}{Q}\right)_U(\alpha)\right]d\alpha =$$

$$\frac{1}{2}\left[\frac{Q\Delta_4 + D\Delta_1}{\Delta_1^2}\text{In}\frac{Q}{Q - \Delta_1} + \frac{Q\Delta_3 + D\Delta_2}{\Delta_2^2}\text{In}\frac{Q + \Delta_2}{Q} - \frac{\Delta_3}{\Delta_2}\right]$$

which is positive, since the left and right end points of the α-cut of $\frac{\tilde{D}}{\tilde{Q}}$ are positive continuous functions on $0 \le \alpha \le 1$.

$$\tilde{\pi}(Q, r) = d\left(\tilde{\pi}(Q, r), \tilde{0}_1\right) =$$

$$\left(PD\text{-}\frac{CD}{r}\right)\left(r + \frac{\Delta_4 - \Delta_3}{4}\right)\text{-}\frac{(ic)r}{2}\left(Q + \frac{\Delta_2 - \Delta_1}{4}\right)\text{-}\frac{D}{r}\left[S+Y(s, r)+N(r)+T\Sigma M_i\right]$$

$$\frac{1}{2}\left[\frac{Q\Delta_4 + D\Delta_1}{\Delta_1^2}\text{In}\frac{Q}{Q - \Delta_1} + \frac{Q\Delta_3 + D\Delta_2}{\Delta_2^2}\text{In}\frac{Q + \Delta_2}{Q} - \frac{\Delta_3}{\Delta_2}\right]$$

5. Numerical Example

D	=	10000 units/year
S	=	500 units/year
y(S, r)	=	250 units/year
N(r)	=	50 units/year
$T\Sigma M_i$	=	10

i = 2, C = 100, r = 1, Q = 284.6 units/year

Model 1 :

$\Delta_1 = 0.0003$, $\Delta_r = 0.0100$

$d(\tilde{a}, \tilde{0}_1) = 0.8$

$d\left(\frac{1}{\tilde{a}}, \tilde{0}\right) = 0.04$

$r^* = 5.95$

Model 2 :

$d(\tilde{Q}, \tilde{0}_1) = 0.5$

$d(\tilde{p}, \tilde{0}_1) = 0.3$

$d\left(\frac{\tilde{p}}{\tilde{Q}}, \tilde{0}_1\right) = 0.2$

$\Delta_1 = 0.0004$, $\Delta_2 = 0.0200$, $\Delta_3 = 100$, $\Delta_4 = 10$ $r^{**} = 0.46$

6. Conclusion

This paper proposed two fuzzy models for an inventory problem with fuzzy reliability levels. In the first model, the economic quantity level is represented by a fuzzy number, while the annual demand is treated as a fixed constant. In the second model, reliability level and economic quantity are represented by a fuzzy number. For each fuzzy model, a method of defuzzification, namely the signed distance, is employed to find the estimate of total profit per unit time in the fuzzy sense. Numerical examples are carried out to investigate the models.

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