# Discrete Mathematics 

# The P-Value models with its mathematical formulas and solution-boxes 

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## Introduction

## The $\mathbf{P}$-Value Models

The One's (1's) Place Value Concept
NB:

- The word 'and' = Addition (+)
- The movement on each place value is from both 'right' to 'left' and vice versa.
- A straight line is divided into nine (9) equal parts
- The One's (1's) Place Value starts from zero (0) and ends at nine (9)

Fig (1): Movement One


Movement One (Jump one step from zero to the right and or vice versa)
0 to $1=1$
1 to $2=1$
2 to $3=1$
3 to $4=1$
4 to $5=1$
5 to $6=1$
6 to $7=1$
7 to $8=1$
8 to $9=1$
Fig (2): Movement Two



#### Abstract

The P-Value models were designed by the researcher to teach Place Values in mathematics. The P-Value models are tools which is methodology in nature. It depicts yet another / alternative practical way of teaching Place Values in Mathematics to the usual / known method (Appendix A; Appendix B) of teaching Place Value (Kumassah, 2012; Price 2002; Price, 2001). The mathematical formulas of the P-Value models shows the step (s) / movement (s) involved in arriving at a specific number (s) / figure (s) and also the total / sum / overall number (s) / figure (s) in each P-Value models. The mathematical formulas of the P-Value models can be employed in teaching and used at the industry level. The researcher is of the view that the use of the P-Value models may help teachers teach Place Value effectively and students learn meaningfully of Place Values in mathematics. The PValue model solution-boxes serve as a calculator to the actual calculators. It gives the same answers as that of the actual calculators and also shows and reveals the hidden mathematical operations of the actual calculator. Instead of a student manipulating the hidden operations of an actual calculator, with the P-Value model solution-box he/she practically sees and manipulates that hidden operations of a calculator.


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$\begin{aligned} & \text { Movement Two (Jump two steps from zero to the right and } \\ & \text { or vice versa) } \\ & 0 \text { to } 2=0 \text { to } 1 \text { and }(+) 1 \text { to } 2=1+1=2 \times 1=2 \\ & 1 \text { to } 3=1 \text { to } 2 \text { and (+) } 2 \text { to } 3=1+1=2 \times 1=2 \\ & 2 \text { to } 4=2 \text { to } 3 \text { and }(+) 3 \text { to } 4=1+1=2 \times 1=2 \\ & 3 \text { to } 5=3 \text { to } 4 \text { and (+) } 4 \text { to } 5=1+1=2 \times 1=2 \\ & 4 \text { to } 6=4 \text { to } 5 \text { and (+) } 5 \text { to } 6=1+1=2 \times 1=2 \\ & 5 \text { to } 7=5 \text { to } 6 \text { and }(+) 6 \text { to } 7=1+1=2 \times 1=2 \\ & 6 \text { to } 8=6 \text { to } 7 \text { and }(+) 7 \text { to } 8=1+1=2 \times 1=2 \\ & 7 \text { to } 9=7 \text { to } 8 \text { and (+) } 8 \text { to } 9=1+1=2 \times 1=2 \\ & \text { NB: } 2 \times 1=1+1 \text {, means one }(1) \text { has been repeated two times. }\end{aligned}$
Fig (3): Movement Three


Movement Three (Jump three steps from zero to the right and or vice versa)
0 to $3=0$ to 1 and 1 to 2 and 2 to $3=1+1+1=3 \times 1=3$
1 to $4=1$ to 2 and 2 to 3 and 3 to $4=1+1+1=3 \times 1=3$
2 to $5=2$ to 3 and 3 to 4 and 4 to $5=1+1+1=3 \times 1=3$
3 to $6=3$ to 4 and 4 to 5 and 5 to $6=1+1+1=3 \times 1=3$
4 to $7=4$ to 5 and 5 to 6 and 6 to $7=1+1+1=3 \times 1=3$
5 to $8=5$ to 6 and 6 to 7 and 7 to $8=1+1+1=3 \times 1=3$
6 to $9=6$ to 7 and 7 to 8 and 8 to $9=1+1+1=3 \times 1=3$
NB: $3 \times 1=1+1+1$, means one (1) has been repeated three times.

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Fig (4): Movement Four


Movement Four (Jump four steps from zero to the right and or vice versa)
0 to $4=0$ to 1 and 1 to 2 and 2 to 3 and 3 to $4=1+1+1+1=$ $4 \times 1=4$
1 to $5=1$ to 2 and 2 to 3 and 3 to 4 and 4 to $5=1+1+1+1=$ $4 \times 1=4$
2 to $6=2$ to 3 and 3 to 4 and 4 to 5 and 5 to $6=1+1+1+1=$ $4 \times 1=4$
3 to $7=3$ to 4 and 4 to 5 and 5 to 6 and 6 to $7=1+1+1+1=$ $4 \times 1=4$
4 to $8=4$ to 5 and 5 to 6 and 6 to 7 and 7 to $8=1+1+1+1=$ $4 \times 1=4$
5 to $9=5$ to 6 and 6 to 7 and 7 to 8 and 8 to $9=1+1+1+1=$ $4 \times 1=4$
NB: $4 \times 1=1+1+1+1$, means one (1) has been repeated four times.

Fig (5): Movement Five


Movement Five (Jump five steps from zero to the right and or vice versa)
0 to $5=0$ to 1 and 1 to 2 and 2 to 3 and 3 to 4 and 4 to $5=1+1$ $+1+1+1=5 \times 1=5$
1 to $6=1$ to 2 and 2 to 3 and 3 to 4 and 4 to 5 and 5 to $6=1+1$ $+1+1+1=5 \times 1=5$
2 to $7=2$ to 3 and 3 to 4 and 4 to 5 and 5 to 6 and 6 to $7=1+1$ $+1+1+1=5 \times 1=5$
3 to $8=3$ to 4 and 4 to 5 and 5 to 6 and 6 to 7 and 7 to $8=1+1$ $+1+1+1=5 \times 1=5$
4 to $9=4$ to 5 and 5 to 6 and 6 to 7 and 7 to 8 and 8 to $9=1+1$ $+1+1+1=5 \times 1=5$
NB: $5 \times 1=1+1+1+1+1$, means one (1) has been repeated five times

Fig (6): Movement Six


## Movement Six (Jump six steps from zero to the right and or

 vice versa)0 to $6=0$ to 1 and 1 to 2 and 2 to 3 and 3 to 4 and 4 to 5 and 5 to $6=1+1+1+1+1+1=6 \times 1=6$
1 to $7=1$ to 2 and 2 to 3 and 3 to 4 and 4 to 5 and 5 to 6 and 6 to $7=1+1+1+1+1+1=6 \times 1=6$
2 to $8=2$ to 3 and 3 to 4 and 4 to 5 and 5 to 6 and 6 to 7 and 7 to $8=1+1+1+1+1+1=6 \times 1=6$
3 to $9=3$ to 4 and 4 to 5 and 5 to 6 and 6 to 7 and 7 to 8 and 8 to $9=1+1+1+1+1+1=6 \times 1=6$
NB: $6 \times 1=1+1+1+1+1+1$, means one (1) has been repeated six times

Fig (7): Movement Seven


Movement Seven (Jump seven steps from zero to the right and or vice versa)
0 to $7=0$ to 1 and 1 to 2 and 2 to 3 and 3 to 4 and 4 to 5 and 5 to 6 and 6 to $7=1+1+1+1+1+1+1=7 \times 1=7$
1 to $8=1$ to 2 and 2 to 3 and 3 to 4 and 4 to 5 and 5 to 6 and 6 to 7 and 7 to $8=1+1+1+1+1+1+1=7 \times 1=7$
2 to $9=2$ to 3 and 3 to 4 and 4 to 5 and 5 to 6 and 6 to 7 and 7 to 8 and 8 to $9=1+1+1+1+1+1+1=7 \times 1=7$
NB: $7 \times 1=1+1+1+1+1+1+1$, means one (1) has been repeated seven times

Fig (8): Movement Eight


Movement Eight (Jump eight steps from zero to the right and or vice versa)
0 to $8=0$ to 1 and 1 to 2 and 2 to 3 and 3 to 4 and 4 to 5 and 5 to 6 and 6 to 7 and 7 to $8=1+1+1+1+1+1+1+1=8 \times 1=8$ 1 to $9=1$ to 2 and 2 to 3 and 3 to 4 and 4 to 5 and 5 to 6 and 6 to 7 and 7 to 8 and 8 to $9=1+1+1+1+1+1+1+1=8 \times 1=8$ NB: $8 \times 1=1+1+1+1+1+1+1+1$, means one ( 1 ) has been repeated eight times

Fig (9): Movement Nine


Movement Nine (Jump nine steps from zero to the right and or vice versa)
0 to $9=0$ to 1 and 1 to 2 and 2 to 3 and 3 to 4 and 4 to 5 and 5 to 6 and 6 to 7 and 7 to 8 and 8 to $9=1+1+1+1+1+1+1+1+$ $1=9 \times 1=9$
NB: $9 \times 1=1+1+1+1+1+1+1+1+1$, means one (1) has been repeated nine times

## Mathematical proves of the One's ( 1 's $=n_{1}$ ) Place Value

Fig 10: Diagram of Mathematical proves of the One's (1's = $n_{1}$ ) Place Value


Let
$\mathrm{n}_{\mathrm{th}}=$ all Place Values,
$\mathrm{y}_{\mathrm{nth}}=$ answer for all Place Values,
$\mathrm{n}_{1}=1$ for One's (1's) Place value,
$\mathrm{b}=$ movement ( s ) within the One's (1's) Place Value
$>\mathrm{y}_{\mathrm{b}}=\mathrm{an}_{(1)}$
For movement one on the One's $\left(\mathrm{n}_{(1)}\right)$ Place Value, $>\mathrm{b}_{1}=1$ and $\mathrm{n}_{(1)}=1$
$>y_{b}=\operatorname{an}_{(1)}=y_{b}=y_{1}=1 n_{(1)}=1 \times 1=1^{2}=1$
$\therefore \mathrm{y}_{1}=1$
For movement two on the One's $\left(\mathrm{n}_{(1)}\right)$ Place Value, $>\mathrm{b}_{2}=2$ and
$\mathrm{n}_{(1)}=1$
$>\mathrm{y}_{\mathrm{b}}=\mathrm{an}_{(1)}=\mathrm{y}_{\mathrm{b}}=\mathrm{y}_{2}=2 \mathrm{n}_{(1)}=2 \times 1=1+1=2$
:. $y_{2}=2$
For movement three on the One's $\left(\mathrm{n}_{(1)}\right)$ Place Value, $>\mathrm{b}_{3}=3$ and $\mathrm{n}_{(1)}=1$
) $\mathrm{y}_{\mathrm{b}}=\mathrm{an}_{(1)}=\mathrm{y}_{\mathrm{b}}=\mathrm{y}_{3}=3 \mathrm{n}_{(1)}=3 \times 1=1+1+1=3$
$\therefore y_{3}=3$
For movement four on the One's $\left(\mathrm{n}_{(1)}\right)$ Place Value, $>\mathrm{b}_{4}=4$ and $\mathrm{n}_{(1)}=1$
) > $\mathrm{y}_{\mathrm{b}}=\mathrm{an}_{(1)}=\mathrm{y}_{\mathrm{b}}=\mathrm{y}_{4}=4 \mathrm{n}_{(1)}=4 \times 1=1+1+1+1=4$
$\therefore y_{4}=4$
For movement five on the One's $\left(\mathrm{n}_{(1)}\right)$ Place Value, $>\mathrm{b}_{5}=5$ and $\mathrm{n}_{(1)}=1$
) $\mathrm{y}_{\mathrm{b}}=\mathrm{an}_{(1)}=\mathrm{y}_{\mathrm{b}}=\mathrm{y}_{5}=5 \mathrm{n}_{(1)}=5 \times 1=1+1+1+1+1=5$
:. $y_{5}=5$
For movement six on the One's $\left(\mathrm{n}_{(1)}\right)$ Place Value, $>\mathrm{b}_{6}=6$ and n (1) $=1$
) ) $\mathrm{y}_{\mathrm{b}}=\mathrm{an}_{(1)}=\mathrm{y}_{\mathrm{b}}=\mathrm{y}_{6}=6 \mathrm{n}_{(1)}=6 \times 1=1+1+1+1+1+1=6$
$\therefore y_{6}=6$
For movement seven on the One's ( $\mathrm{n}_{(1)}$ ) Place Value, > $\mathrm{b}_{7}=7$ and $\mathrm{n}_{(1)}=1$
) $y_{b}=\mathrm{an}_{(1)}=\mathrm{y}_{\mathrm{b}}=\mathrm{y}_{7}=7 \mathrm{n}_{(1)}=7 \times 1=1+1+1+1+1+1+1$ $=7$
$\therefore y_{7}=7$
For movement eight on the One's ( $\mathrm{n}_{(1)}$ ) Place Value, $>\mathrm{b}_{8}=8$ and $\mathrm{n}_{(1)}=1$
$>y_{b}=\mathrm{an}_{(1)}=y_{b}=y_{8}=8 n_{(1)}=8 \times 1=1+1+1+1+1+1+1+$ $1=8$
:. $\mathrm{y}_{8}=8$
For movement nine on the One's ( $\mathrm{n}_{(1)}$ ) Place Value, $>\mathrm{b}_{9}=9$ and $\mathrm{n}_{(1)}=1$
$>\mathrm{y}_{\mathrm{b}}=\mathrm{an}_{(1)}=\mathrm{y}_{\mathrm{b}}=\mathrm{y}_{8}=9 \mathrm{n}_{(1)}=9 \times 1=1+1+1+1+1+1+1+$ $1+1=9$
$\therefore y_{9}=9$
P-Value model solution-boxes
One Place Value Solution Boxes
Solution Box 1a: One's Place Values

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Solution Box 1b: reverse One's Place Values

| 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

The Ten's (10's) Place Value Concept
NB:

- The word 'and' = Addition (+)
- The movement on each place value is from both 'right' to 'left' and vice versa.
- A straight line is divided into nine (9) equal parts
- The Ten's ( 10 's) Place Value starts from zero ( 00 ) and ends at ninety (90)

Fig (11): Movement One


Movement One (Jump one step from zero to the right and or vice versa)
00 to $10=10$
10 to $20=10$
20 to $30=10$
30 to $40=10$
40 to $50=10$
50 to $60=10$
60 to $70=10$
70 to $80=10$
80 to $90=10$
Fig (12): Movement Two


Movement Two (Jump two steps from zero to the right and or vice versa)
00 to $20=00$ to 10 and $(+) 10$ to $20=10+10=2 \times 10=20$
10 to $30=10$ to 20 and $(+) 20$ to $30=10+10=2 \times 10=20$
20 to $40=20$ to 30 and $(+) 30$ to $40=10+10=2 \times 10=20$
30 to $50=30$ to 40 and $(+) 40$ to $50=10+10=2 \times 10=20$
40 to $60=40$ to 50 and $(+) 50$ to $60=10+10=2 \times 10=20$
50 to $70=50$ to 60 and $(+) 60$ to $70=10+10=2 \times 10=20$
60 to $80=60$ to 70 and $(+) 70$ to $80=10+10=2 \times 10=20$
70 to $90=70$ to 80 and $(+) 80$ to $90=10+10=2 \times 10=20$
NB: $2 \times 10=10+10$, means ten (10) has been repeated two times.

Fig (13): Movement Three


Movement Three (Jump three steps from zero to the right and or vice versa)
00 to $30=00$ to 10 and 10 to 20 and 20 to $30=10+10+10=$ $3 \times 10=30$
10 to $40=10$ to 20 and 20 to 30 and 30 to $40=10+10+10=$ $3 \times 10=30$
20 to $50=20$ to 30 and 30 to 40 and 40 to $50=10+10+10=$ $3 \times 10=30$
30 to $60=30$ to 40 and 40 to 50 and 50 to $60=10+10+10=$ $3 \times 10=30$
40 to $70=40$ to 50 and 50 to 60 and 60 to $70=10+10+10=$ $3 \times 10=30$
50 to $80=50$ to 60 and 60 to 70 and 70 to $80=10+10+10=$ $3 \times 10=30$
60 to $90=60$ to 70 and 70 to 80 and 80 to $90=10+10+10=$ $3 \times 10=30$
NB: $3 \times 10=10+10+10$, means ten (10) has been repeated three times.

Fig (14): Movement Four


Movement Four (Jump four steps from zero to the right and or vice versa)
00 to $40=00$ to 10 and 10 to 20 and 20 to 30 and 30 to $40=10$ $+10+10+10=4 \times 10=40$
10 to $50=10$ to 20 and 20 to 30 and 30 to 40 and 40 to $50=10$ $+10+10+10=4 \times 10=40$
20 to $60=20$ to 30 and 30 to 40 and 40 to 50 and 50 to $60=10$ $+10+10+10=4 \times 10=40$
30 to $70=30$ to 40 and 40 to 50 and 50 to 60 and 60 to $70=10$ $+10+10+10=4 \times 10=40$
40 to $80=40$ to 50 and 50 to 60 and 60 to 70 and 70 to $80=10$ $+10+10+10=4 \times 10=40$
50 to $90=50$ to 60 and 60 to 70 and 70 to 80 and 80 to $90=10$ $+10+10+10=4 \times 10=40$
NB: $4 \times 10=10+10+10+10$, means ten (10) has been repeated four times.

Fig (15): Movement Five


Movement Five (Jump five steps from zero to the right and or vice versa)
00 to $50=00$ to 10 and 10 to 20 and 20 to 30 and 30 to 40 and 40 to $50=10+10+10+10+10=5 \times 10=50$
10 to $60=10$ to 20 and 20 to 30 and 30 to 40 and 40 to 50 and 50 to $60=10+10+10+10+10=5 \times 10=50$

20 to $70=20$ to 30 and 30 to 40 and 40 to 50 and 50 to 60 and 60 to $70=10+10+10+10+10=5 \times 10=50$
30 to $80=30$ to 40 and 40 to 50 and 50 to 60 and 60 to 70 and 70 to $80=10+10+10+10+10=5 \times 10=50$
40 to $90=40$ to 50 and 50 to 60 and 60 to 70 and 70 to 80 and 80 to $90=10+10+10+10+10=5 \times 10=50$
NB: $5 \times 10=10+10+10+10+10$, means ten (10) has been repeated five times

Fig (16): Movement Six


Movement Six (Jump six steps from zero to the right and or vice versa)
00 to $60=00$ to 10 and 10 to 20 and 20 to 30 and 30 to 40 and 40 to 50 and 50
to $60=10+10+10+10+10+10=6 \times 10=60$
10 to $70=10$ to 20 and 20 to 30 and 30 to 40 and 40 to 50 and 50 to 60 and 60
to $70=10+10+10+10+10+10=6 \times 10=60$
20 to $80=20$ to 30 and 30 to 40 and 40 to 50 and 50 to 60 and 60 to 70 and 70
to $80=10+10+10+10+10+10=6 \times 10=60$
30 to $90=30$ to 40 and 40 to 50 and 50 to 60 and 60 to 70 and 70 to 80 and 80
to $90=10+10+10+10+10+10=6 \times 10=60$
NB: $6 \times 10=10+10+10+10+10+10$, means ten (10) has been repeated six times

Fig (17): Movement Seven


Movement Seven (Jump seven steps from zero to the right and or vice versa)
00 to $70=00$ to 10 and 10 to 20 and 20 to 30 and 30 to 40 and 40 to 50 and 50 to 60 and 60 to $70=10+10+10+10+10+10$ $+10=7 \times 10=70$
10 to $80=10$ to 20 and 20 to 30 and 30 to 40 and 40 to 50 and 50 to 60 and 60 to 70 and 70 to $80=10+10+10+10+10+10$ $+10=7 \times 10=70$
20 to $90=20$ to 30 and 30 to 40 and 40 to 50 and 50 to 60 and 60 to 70 and 70 to 80 and 80 to $90=10+10+10+10+10+10$ $+10=7 \times 10=70$
NB: $7 \times 10=10+10+10+10+10+10+10$, means ten (10) has been repeated seven times.

Table 1: Summary of the One's (1's) Place Value

| Movement 1 | Movement 2 | Movement 3 | Movement 4 | Movement 5 | Movement 6 | Movement 7 | Movement 8 | Movement $9$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 to $1=1$ | $\begin{aligned} & 0 \text { to } 2=1 \\ & +1=2 \times 1 \\ & =2 \end{aligned}$ | $\begin{aligned} & 0 \text { to } 3=1 \\ & +1+1= \\ & 3 \times 1=3 \end{aligned}$ | $\begin{aligned} & 0 \text { to } 4=1+1+1 \\ & +1=4 \times 1=4 \end{aligned}$ | $\begin{aligned} & 0 \text { to } 5=1 \\ & +1+1+1 \\ & +1=5 \times 1= \\ & 5 \end{aligned}$ | $\begin{aligned} & 0 \text { to } 6=1+ \\ & 1+1+1+1+ \\ & 1=6 \times 1=6 \end{aligned}$ | $\begin{aligned} & 0 \text { to } 7=1 \\ & +1+1+1 \\ & +1+1+1= \\ & 7 \times 1=7 \end{aligned}$ | $\begin{aligned} & 0 \text { to } 8=1+ \\ & 1+1+1+1+ \\ & 1+1+1=8 \times 1 \\ & =8 \end{aligned}$ | $\begin{aligned} & 0 \text { to } 9=1 \\ & +1+1+1 \\ & +1+1+1+ \\ & 1+1=9 \times 1 \\ & =9 \end{aligned}$ |
| 1 to $2=1$ | $\begin{aligned} & 1 \text { to } 3=1 \\ & +1=2 \times 1 \\ & =2 \end{aligned}$ | $\begin{aligned} & 1 \text { to } 4=1 \\ & +1+1= \\ & 3 \times 1=3 \end{aligned}$ | $\begin{aligned} & 1 \text { to } 5=1+1+1 \\ & +1=4 \times 1=4 \end{aligned}$ | $\begin{aligned} & 1 \text { to } 6= \\ & 1+1+1+ \\ & 1+1=5 \times 1 \\ & =5 \end{aligned}$ | $\begin{aligned} & 1 \text { to } 7=1+ \\ & 1+1+1+1+ \\ & 1=6 \times 1=6 \end{aligned}$ | $\begin{aligned} & 1 \text { to } 8=1 \\ & +1+1+1 \\ & +1+1+1= \\ & 7 \times 1=7 \end{aligned}$ | $\begin{aligned} & 1 \text { to } 9=1+ \\ & 1+1+1+1+ \\ & 1+1+1=8 \times 1 \\ & =8 \end{aligned}$ |  |
| 2 to $3=1$ | $\begin{aligned} & 2 \text { to } 4=1 \\ & +1=2 \times 1 \\ & =2 \end{aligned}$ | $\begin{aligned} & 2 \text { to } 5= \\ & 1+1+1= \\ & 3 \times 1=3 \end{aligned}$ | $\begin{aligned} & 2 \text { to } 6 \quad 1+1+1+ \\ & 1=4 \times 1=4 \end{aligned}$ | $\begin{aligned} & 2 \text { to } 7= \\ & 1+1+1+ \\ & 1+1=5 \times 1 \\ & =5 \end{aligned}$ | $\begin{aligned} & 2 \text { to } 8=1+ \\ & 1+1+1+1+ \\ & 1=6 \times 1=6 \end{aligned}$ | $\begin{aligned} & 2 \text { to } 9=1 \\ & +1+1+1 \\ & +1+1+1= \\ & 7 \times 1=7 \end{aligned}$ |  |  |
| 3 to $4=1$ | $\begin{aligned} & 3 \text { to } 5=1 \\ & +1=2 \times 1 \\ & =2 \end{aligned}$ | $\begin{aligned} & 3 \text { to } 6=1 \\ & +1+1= \\ & 3 \times 1=3 \end{aligned}$ | $\begin{aligned} & 3 \text { to } 7=1+1+1 \\ & +1=4 \times 1=4 \end{aligned}$ | $\begin{aligned} & 3 \text { to } 8=1 \\ & +1+1+1 \\ & +1=5 \times 1= \\ & 5 \end{aligned}$ | $\begin{aligned} & 3 \text { to } 9=1+ \\ & 1+1+1+1+ \\ & 1=6 \times 1=6 \end{aligned}$ |  |  |  |
| 4 to $5=1$ | $\begin{aligned} & 4 \text { to } 6=1 \\ & +1=2 \times 1 \\ & =2 \end{aligned}$ | $\begin{aligned} & 4 \text { to } 7=1 \\ & +1+1= \\ & 3 \times 1=3 \end{aligned}$ | $\begin{aligned} & 4 \text { to } 8=1+1+1 \\ & +1=4 \times 1=4 \end{aligned}$ | $\begin{aligned} & 4 \text { to } 9=1 \\ & +1+1+1 \\ & +1=5 \times 1= \\ & 5 \end{aligned}$ |  |  |  |  |
| 5 to $6=1$ | $\begin{aligned} & 5 \text { to } 7=1 \\ & +1=2 \times 1 \\ & =2 \end{aligned}$ | $\begin{aligned} & 5 \text { to } 8=1 \\ & +1+1= \\ & 3 \times 1=3 \\ & \hline \end{aligned}$ | $\begin{aligned} & 5 \text { to } 9=1+1+ \\ & 1+1=4 \times 1=4 \end{aligned}$ |  |  |  |  |  |
| 6 to $7=1$ | $\begin{aligned} & 6 \text { to } 8=1 \\ & +1=2 \times 1 \\ & =2 \end{aligned}$ | $\begin{aligned} & 6 \text { to } 9=1 \\ & +1+1= \\ & 3 \times 1=3 \end{aligned}$ |  |  |  |  |  |  |
| 7 to $8=1$ | $\begin{aligned} & 7 \text { to } 9=1 \\ & +1=2 \times 1 \\ & =2 \end{aligned}$ |  |  |  |  |  |  |  |
| 8 to $9=1$ |  |  |  |  |  |  |  |  |

Table 2: Summary of the Ten's (10's) Place Value

| Movement 1 | Movement 2 | Movement 3 | Movement 4 | Movement 5 | Movement 6 | Movement 7 | Movement 8 | Movement 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 00 \text { to } 10= \\ & 10 \end{aligned}$ | $\begin{aligned} & 00 \text { to } 20= \\ & 10+ \\ & 10=2 \times 10 \\ & =20 \end{aligned}$ | $\begin{aligned} & 00 \text { to } 30= \\ & 10+ \\ & 10+10= \\ & 3 \times 10=30 \end{aligned}$ | $\begin{aligned} & 00 \text { to } 40= \\ & 10+ \\ & 10+10+ \\ & 10= \\ & 4 \times 10=40 \end{aligned}$ | $\begin{aligned} & 00 \text { to } 50= \\ & 10+ \\ & 10+10+ \\ & 10+ \\ & 10=5 \times 10 \\ & =50 \end{aligned}$ | $\begin{aligned} & 00 \text { to } 60= \\ & 10+ \\ & 10+10+ \\ & 10+ \\ & 10+10= \\ & 6 \times 10=60 \end{aligned}$ | $\begin{aligned} & 00 \text { to } 70 \\ & = \\ & 10+ \\ & 10+10+ \\ & 10+ \\ & 10+10+ \\ & 10=7 \times 10 \\ & =70 \end{aligned}$ | $\begin{aligned} & 00 \text { to } 80= \\ & 10+10+ \\ & 10+10+ \\ & 10+10+ \\ & 10+10= \\ & 8 \times 10=80 \end{aligned}$ | $\begin{aligned} & 00 \text { to } 90= \\ & 10+10+ \\ & 10+10+ \\ & 10+10+ \\ & 10+10+ \\ & 10=9 \times 10 \\ & =90 \end{aligned}$ |
| $\begin{aligned} & 10 \text { to } 20= \\ & 10 \end{aligned}$ | $\begin{aligned} & 10 \text { to } 30= \\ & 10+ \\ & 10=2 \times 10 \\ & =20 \end{aligned}$ | $\begin{aligned} & 10 \text { to } 40= \\ & 10+ \\ & 10+10= \\ & 3 \times 10=30 \end{aligned}$ | $\begin{aligned} & 10 \text { to } 50= \\ & 10+ \\ & 10+10+ \\ & 10= \\ & 4 \times 10=40 \end{aligned}$ | $\begin{aligned} & 10 \text { to } 60= \\ & 10+ \\ & 10+10+ \\ & 10+ \\ & 10=5 \times 10 \\ & =50 \end{aligned}$ | $\begin{aligned} & 10 \text { to } 70= \\ & 10+ \\ & 10+10+ \\ & 10+ \\ & 10+10= \\ & 6 \times 10=60 \end{aligned}$ | $\begin{aligned} & 10 \text { to } 80= \\ & 10+10+ \\ & 10+10+ \\ & 10+10+ \\ & 10=7 \times 10 \\ & =70 \end{aligned}$ | $\begin{aligned} & 10 \text { to } 90= \\ & 10+10+ \\ & 10+10+ \\ & 10+10+ \\ & 10+10= \\ & 8 \times 10=80 \end{aligned}$ |  |
| $\begin{aligned} & 20 \text { to } 30= \\ & 10 \end{aligned}$ | $\begin{aligned} & 20 \text { to } 40= \\ & 10+ \\ & 10=2 \times 10 \\ & =20 \end{aligned}$ | $\begin{aligned} & 20 \text { to } 50 \\ & =\quad 10+ \\ & 10+10= \\ & 3 \times 10=30 \end{aligned}$ | $\begin{aligned} & 20 \text { to } 60= \\ & 10+ \\ & 10+10+ \\ & 10=4 \times 10 \\ & =40 \end{aligned}$ | $\begin{aligned} & 20 \text { to } 70= \\ & 10+ \\ & 10+10+ \\ & 10+ \\ & 10=5 \times 10 \\ & =50 \end{aligned}$ | $\begin{aligned} & 20 \text { to } 80= \\ & 10+ \\ & 10+10+ \\ & 10+ \\ & 10+10= \\ & 6 \times 10=60 \end{aligned}$ | $\begin{aligned} & 20 \text { to } 90= \\ & 10+10+ \\ & 10+10+ \\ & 10+10+ \\ & 10=7 \times 10 \\ & =70 \end{aligned}$ |  |  |
| $\begin{aligned} & 30 \text { to } 40= \\ & 10 \end{aligned}$ | $\begin{aligned} & 30 \text { to } 50= \\ & 10+ \\ & 10=2 \times 10 \\ & =20 \end{aligned}$ | $\begin{aligned} & 30 \text { to } 60 \\ & =10+ \\ & 10+10= \\ & 3 \times 10=30 \end{aligned}$ | $\begin{aligned} & 30 \text { to } 70 \\ & =10+ \\ & 10+10+ \\ & 10= \\ & 4 \times 10=40 \end{aligned}$ | $\begin{aligned} & 30 \text { to } 80= \\ & 10+ \\ & 10+10+ \\ & 10+ \\ & 10=5 \times 10 \\ & =50 \end{aligned}$ | $\begin{aligned} & 30 \text { to } 90= \\ & 10+ \\ & 10+10+ \\ & 10+ \\ & 10+10= \\ & 6 \times 10=60 \end{aligned}$ |  |  |  |
| $\begin{aligned} & 40 \text { to } 50= \\ & 10 \end{aligned}$ | $\begin{aligned} & 40 \text { to } 60= \\ & 10+ \\ & 10=2 \times 10 \\ & =20 \end{aligned}$ | $\begin{aligned} & 40 \text { to } 70= \\ & 10+ \\ & 10+10= \\ & 3 \times 10=30 \end{aligned}$ | $\begin{aligned} & 40 \text { to } 80 \\ & = \\ & 10+ \\ & 10+10+ \\ & 10= \\ & 4 \times 10=40 \\ & \hline \end{aligned}$ | $\begin{aligned} & 40 \text { to } 90= \\ & 10+ \\ & 10+10+ \\ & 10+ \\ & 10=5 \times 10 \\ & =50 \end{aligned}$ |  |  |  |  |

Table 2 continued

| Movement 1 | Movement 2 | Movement 3 | Movement 4 | Movement 5 | Movement 6 | Movement 7 | Movement 8 | Movement 9 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50 to $60=10$ | 50 to $70=10+$ <br> $10=2 \times 10=20$ <br> 50 to $80=10$ <br> + <br> $10+10=$ <br> $3 \times 10=30$ | 50 to $90=$ <br> $10+$ <br> $10+10+10=$ <br> $4 \times 10=40$ |  |  |  |  |  |  |
| 60 to $70=10$ | 60 to $80=10+$ <br> $10=2 \times 10=20$ | 60 to $90=10$ <br> + <br> $10+10=3 \times 10$ <br> $=30$ |  |  |  |  |  |  |
| 70 to $80=10$ | 70 to $90=10+$ <br> $10=2 \times 10=20$ |  |  |  |  |  |  |  |
| 80 to $90=10$ |  |  |  |  |  |  |  |  |

Table 3: Summary of the Hundred's (100's) Place Value

| Movement 1 | Movement 2 | Movement 3 | Movement 4 | Movement 5 | Movement 6 | Movement 7 | Movement 8 | Movement 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 000 \text { to } 100 \\ & =100 \end{aligned}$ | $\begin{aligned} & \hline 000 \text { to } 200 \\ & =100+ \\ & 100= \\ & 2 \times 100= \\ & 200 \end{aligned}$ | $\begin{aligned} & 000 \text { to } 300 \\ & =100+ \\ & 100+100 \\ & =3 \times 100= \\ & 300 \end{aligned}$ | $\begin{aligned} & \hline 000 \text { to } 400 \\ & =100+ \\ & 100+100 \\ & +100= \\ & 4 \times 100= \\ & 400 \end{aligned}$ | $\begin{aligned} & 000 \text { to } 500 \\ & = \\ & 100+ \\ & 100+100 \\ & +100+ \\ & 100= \\ & 5 \times 100= \\ & 500 \end{aligned}$ | $\begin{aligned} & \text { 000 to } 600 \\ & = \\ & 100+ \\ & 100+100 \\ & +100+ \\ & 100+100 \\ & =6 \times 100= \\ & 600 \end{aligned}$ | $\begin{aligned} & \hline 000 \text { to } 700 \\ & = \\ & 100+ \\ & 100+100 \\ & +100+ \\ & 100+100 \\ & + \\ & 100= \\ & 7 \times 100= \\ & 700 \end{aligned}$ | $\begin{aligned} & 000 \text { to } 800 \\ & =100+ \\ & 100+100 \\ & +100+ \\ & 100+100 \\ & + \\ & 100+100 \\ & =8 \times 100= \\ & 800 \end{aligned}$ | $\begin{aligned} & 000 \text { to } 900 \\ & =100+ \\ & 100+100 \\ & +100+ \\ & 100+100 \\ & +100+ \\ & 100+100 \\ & =9 \times 100= \\ & 900 \end{aligned}$ |
| $\begin{aligned} & 100 \text { to } 200 \\ & =100 \end{aligned}$ | $\begin{aligned} & \hline 100 \text { to } 300 \\ & =100+ \\ & 100= \\ & 2 \times 100= \\ & 200 \end{aligned}$ | $\begin{aligned} & \hline 100 \text { to } 400 \\ & =100+ \\ & 100+100 \\ & =3 \times 100= \\ & 300 \end{aligned}$ | $\begin{aligned} & 100 \text { to } 500 \\ & =100+ \\ & 100+100 \\ & +100= \\ & 4 \times 100= \\ & 400 \end{aligned}$ | $\begin{aligned} & 100 \text { to } 600 \\ & =100+ \\ & 100+100 \\ & +100+ \\ & 100= \\ & 5 \times 100= \\ & 500 \end{aligned}$ | $\begin{aligned} & 100 \text { to } 700 \\ & = \\ & 100+ \\ & 100+100 \\ & +100+ \\ & 100+100 \\ & =6 \times 100= \\ & 600 \end{aligned}$ | $\begin{aligned} & 100 \text { to } 800 \\ & = \\ & 100+100 \\ & +100+ \\ & 100+ \\ & 100+100 \\ & + \\ & 100= \\ & 7 \times 100= \\ & 700 \\ & \hline \end{aligned}$ | $\begin{aligned} & 100 \text { to } 900 \\ & =100+ \\ & 100+100 \\ & +100+ \\ & 100+100 \\ & + \\ & 100+100 \\ & =8 \times 100= \\ & 800 \end{aligned}$ |  |
| $\begin{aligned} & 200 \text { to } 300 \\ & =100 \end{aligned}$ | $\begin{aligned} & \hline 200 \text { to } 400 \\ & =100+ \\ & 100= \\ & 2 \times 100= \\ & 200 \end{aligned}$ | $\begin{aligned} & \hline 200 \text { to } 500 \\ & =100+ \\ & 100+100 \\ & =3 \times 100= \\ & 300 \end{aligned}$ | $\begin{aligned} & \hline 200 \text { to } 600 \\ & =100+ \\ & 100+100 \\ & +100= \\ & 4 \times 100= \\ & 400 \end{aligned}$ | $\begin{aligned} & 200 \text { to } 700 \\ & =100+ \\ & 100+100 \\ & +100+ \\ & 10= \\ & 5 \times 100= \\ & 500 \end{aligned}$ | $\begin{aligned} & 200 \text { to } 800 \\ & = \\ & 100+ \\ & 100+100 \\ & +100+ \\ & 100+100 \\ & =6 \times 100= \\ & 600 \end{aligned}$ | $\begin{aligned} & \hline 200 \text { to } 900 \\ & = \\ & 100+100 \\ & +100+ \\ & 100+ \\ & 100+100 \\ & + \\ & 100= \\ & 7 \times 100= \\ & 700 \\ & \hline \end{aligned}$ |  |  |
| $\begin{aligned} & 300 \text { to } 400 \\ & =100 \end{aligned}$ | $\begin{aligned} & 300 \text { to } 500 \\ & =100+ \\ & 100= \\ & 2 \times 100= \\ & 200 \end{aligned}$ | $\begin{aligned} & \hline 300 \text { to } 600 \\ & =100+ \\ & 100+100 \\ & =3 \times 100= \\ & 300 \end{aligned}$ | $\begin{aligned} & \hline 300 \text { to } 700 \\ & =100+ \\ & 100+100 \\ & +100= \\ & 4 \times 100= \\ & 400 \end{aligned}$ | $\begin{aligned} & \hline 300 \text { to } 800 \\ & =100+ \\ & 100+100 \\ & +100+ \\ & 100= \\ & 5 \times 100= \\ & 500 \\ & \hline \end{aligned}$ | $\begin{aligned} & 300 \text { to } 900 \\ & =100+ \\ & 100+100 \\ & +100+ \\ & 100+100 \\ & =6 \times 100= \\ & 600 \\ & \hline \end{aligned}$ |  |  |  |
| $\begin{aligned} & 400 \text { to } 500 \\ & =100 \end{aligned}$ | $\begin{aligned} & \hline 400 \text { to } 600 \\ & =100+ \\ & 100= \\ & 2 \times 100= \\ & 200 \end{aligned}$ | $\begin{aligned} & \hline 400 \text { to } 700 \\ & =100+ \\ & 100+100 \\ & =3 \times 100= \\ & 300 \end{aligned}$ | $\begin{aligned} & \hline 400 \text { to } 800 \\ & = \\ & 100+ \\ & 100+100 \\ & +100= \\ & 4 \times 100= \\ & 400 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 400 \text { to } 900 \\ & =100+ \\ & 100+100 \\ & +100+ \\ & 100= \\ & 5 \times 100= \\ & 500 \\ & \hline \end{aligned}$ |  |  |  |  |
| $\begin{aligned} & 500 \text { to } 600 \\ & =100 \end{aligned}$ | $\begin{aligned} & \hline 500 \text { to } 700 \\ & =100+ \\ & 100= \\ & 2 \times 100= \\ & 200 \end{aligned}$ | $\begin{aligned} & \hline 500 \text { to } 800 \\ & =100+ \\ & 100+100 \\ & =3 \times 100= \\ & 300 \end{aligned}$ | $\begin{aligned} & \hline 500 \text { to } 900 \\ & = \\ & 100+ \\ & 100+100 \\ & +100= \\ & 4 \times 100= \\ & 400 \\ & \hline \end{aligned}$ |  |  |  |  |  |

Table 3 continued
$\left.\begin{array}{|l|l|l|l|l|l|l|l|l|}\hline \begin{array}{l}\text { Movement } \\ \mathbf{1}\end{array} & \begin{array}{l}\text { Movement } \\ \mathbf{2}\end{array} & \begin{array}{l}\text { Movement } \\ \mathbf{3}\end{array} & \begin{array}{l}\text { Movement } \\ \mathbf{4}\end{array} & \begin{array}{l}\text { Movement } \\ \mathbf{5}\end{array} & \begin{array}{l}\text { Movement } \\ \mathbf{6}\end{array} & \begin{array}{l}\text { Movement } \\ \mathbf{7}\end{array} & \begin{array}{l}\text { Movement } \\ \mathbf{8}\end{array} & \begin{array}{l}\text { Movement } \\ \mathbf{9}\end{array} \\ \hline 600 \text { to } 700 & 600 \text { to } 800 \\ =100 & 600 \text { to } 900 \\ =100+ \\ =100+ \\ 100= \\ 2 \times 100= \\ 200\end{array} \quad \begin{array}{l}100+100 \\ =3 \times 100= \\ 300\end{array}\right)$

Table 4: Summary of the Thousand's (1000's) Place Value

| Movement $1$ | Movement $2$ | Movement 3 | Movement <br> 4 | Movement 5 | Movement 6 | Movement 7 | Movement 8 | Movement 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \hline 0000 \text { to } \\ & 1000= \\ & 1000 \end{aligned}$ | $\begin{aligned} & \hline 0000 \text { to } \\ & 2000= \\ & 1000+ \\ & 1000= \\ & 2 \times 1000= \\ & 2000 \end{aligned}$ | $\begin{aligned} & \hline 0000 \text { to } \\ & 3000= \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 3 \times 1000= \\ & 3000 \end{aligned}$ | $\begin{aligned} & \hline 0000 \text { to } \\ & 4000= \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 4 \times 1000= \\ & 4000 \end{aligned}$ | $\begin{aligned} & \hline 0000 \text { to } \\ & 5000= \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 5 \times 1000= \\ & 5000 \end{aligned}$ | $\begin{aligned} & 0000 \text { to } \\ & 6000= \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 6 \times 1000= \\ & 6000 \end{aligned}$ | $\begin{aligned} & \hline 0000 \text { to } \\ & 7000= \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 7 \times 1000= \\ & 7000 \end{aligned}$ | $\begin{aligned} & \hline 0000 \text { to } \\ & 8000= \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 8 \times 1000= \\ & 8000 \end{aligned}$ | $\begin{aligned} & 0000 \text { to } \\ & 9000= \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 9 \times 1000= \\ & 9000 \\ & \hline \end{aligned}$ |
| $\begin{aligned} & 1000 \text { to } \\ & 2000= \\ & 1000 \end{aligned}$ | $\begin{aligned} & 1000 \text { to } \\ & 3000= \\ & 1000+ \\ & 1000= \\ & 2 \times 1000= \\ & 2000 \end{aligned}$ | $\begin{aligned} & 1000 \text { to } \\ & 4000= \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 3 \times 1000= \\ & 3000 \end{aligned}$ | $\begin{aligned} & \hline 1000 \text { to } \\ & 5000= \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 4 \times 1000= \\ & 4000 \end{aligned}$ | $\begin{aligned} & \hline 1000 \text { to } \\ & 6000= \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 5 \times 1000= \\ & 5000 \end{aligned}$ | $\begin{aligned} & \hline 1000 \text { to } \\ & 7000= \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 6 \times 1000= \\ & 6000 \end{aligned}$ | $\begin{aligned} & \hline 1000 \text { to } \\ & 8000= \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 7 \times 1000= \\ & 7000 \end{aligned}$ | 1000 to $9000=$ $1000+$ $1000+$ $1000+$ $1000+$ $1000+$ $1000+$ $1000+$ $1000=$ $8 \times 1000=$ 8000 |  |
| $\begin{aligned} & \hline 2000 \text { to } \\ & 3000= \\ & 1000 \end{aligned}$ | $\begin{aligned} & 2000 \text { to } \\ & 4000= \\ & 1000+ \\ & 1000= \\ & 2 \times 1000= \\ & 2000 \end{aligned}$ | $\begin{aligned} & \hline 2000 \text { to } \\ & 5000= \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 3 \times 1000= \\ & 3000 \end{aligned}$ | $\begin{aligned} & \hline 2000 \text { to } \\ & 6000= \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 4 \times 1000= \\ & 4000 \end{aligned}$ | $\begin{aligned} & 2000 \text { to } \\ & 7000= \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 5 \times 1000= \\ & 5000 \end{aligned}$ | $\begin{aligned} & 2000 \text { to } \\ & 8000= \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 6 \times 1000= \\ & 6000 \end{aligned}$ | $\begin{aligned} & \hline 2000 \text { to } \\ & 9000= \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 7 \times 1000= \\ & 7000 \end{aligned}$ |  |  |
| $\begin{aligned} & 3000 \text { to } \\ & 4000= \\ & 1000 \end{aligned}$ | $\begin{aligned} & 3000 \text { to } \\ & 5000= \\ & 1000+ \\ & 1000= \\ & 2 \times 1000= \\ & 2000 \end{aligned}$ | $\begin{aligned} & 3000 \text { to } \\ & 6000= \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 3 \times 1000= \\ & 3000 \end{aligned}$ | $\begin{aligned} & 3000 \text { to } \\ & 7000= \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 4 \times 1000= \\ & 4000 \end{aligned}$ | $\begin{aligned} & 3000 \text { to } \\ & 8000= \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 5 \times 1000= \\ & 5000 \end{aligned}$ | $\begin{aligned} & \hline 3000 \text { to } \\ & 9000= \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 6 \times 1000= \\ & 6000 \\ & \hline \end{aligned}$ |  |  |  |

Table 4 continued

| Movement $1$ | Movement 2 | Movement <br> 3 | Movement 4 | Movement 5 | Movement 6 | Movement 7 | Movement 8 | Movement 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 4000 \text { to } \\ & 5000= \\ & 1000 \end{aligned}$ | $\begin{aligned} & 4000 \text { to } 6000=\quad 1000+ \\ & 1000=2 \times 1000=2000 \end{aligned}$ | $\begin{aligned} & 4000 \text { to } \\ & 7000= \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 3 \times 1000= \\ & 3000 \end{aligned}$ | $\begin{aligned} & \hline 4000 \text { to } 8000 \\ & = \\ & 1000+ \\ & 1000+1000 \\ & +1000= \\ & 4 \times 1000= \\ & 4000 \end{aligned}$ | $\begin{aligned} & 4000 \text { to } \\ & 9000= \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 5 \times 1000= \\ & 5000 \\ & \hline \end{aligned}$ |  |  |  |  |
| $\begin{aligned} & 5000 \text { to } \\ & 6000= \\ & 1000 \end{aligned}$ | $\begin{aligned} & 5000 \text { to } 7000=\quad 1000+ \\ & 1000=2 \times 1000=2000 \end{aligned}$ | $\begin{aligned} & \hline 5000 \text { to } \\ & 8000= \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 3 \times 1000= \\ & 3000 \end{aligned}$ | $\begin{aligned} & \hline 5000 \text { to } 9000 \\ & = \\ & 1000+ \\ & 1000+1000 \\ & +1000= \\ & 4 \times 1000= \\ & 4000 \\ & \hline \end{aligned}$ |  |  |  |  |  |
| $\begin{aligned} & \hline 6000 \text { to } \\ & 7000= \\ & 1000 \end{aligned}$ | $\begin{aligned} & 6000 \text { to } 8000=\quad 1000+ \\ & 1000=2 \times 1000=2000 \end{aligned}$ | $\begin{aligned} & \hline 6000 \text { to } \\ & 9000= \\ & 1000+ \\ & 1000+ \\ & 1000= \\ & 3 \times 1000= \\ & 3000 \end{aligned}$ |  |  |  |  |  |  |
| $\begin{aligned} & 7000 \text { to } \\ & 8000= \\ & 1000 \end{aligned}$ | $\begin{aligned} & 7000 \text { to } 9000=1000+ \\ & 1000=2 \times 1000=2000 \end{aligned}$ |  |  |  |  |  |  |  |
| $\begin{aligned} & 8000 \text { to } \\ & 9000= \\ & 1000 \end{aligned}$ |  |  |  |  |  |  |  |  |

Fig (18): Movement Eight


Movement Eight (Jump eight steps from zero to the right and or vice versa)
00 to $80=00$ to 10 and 10 to 20 and 20 to 30 and 30 to 40 and 40 to 50 and 50 to 60 and 60 to 70 and 70 to $80=10+10+10+$ $10+10+10+10+10=8 \times 10=80$
10 to $90=10$ to 20 and 20 to 30 and 30 to 40 and 40 to 50 and 50 to 60 and 60 to 70 and 70 to 80 and 80 to $90=10+10+10+$ $10+10+10+10+10=8 \times 10=80$
NB: $8 \times 10=10+10+10+10+10+10+10+10$, means ten (10) has been repeated eight times

Fig (19): Movement Nine


Movement Nine (Jump nine steps from zero to the right and or vice versa)
00 to $90=00$ to 10 and 10 to 20 and 20 to 30 and 30 to 40 and 40 to 50 and 50 to 60 and 60 to 70 and 70 to 80 and 80 to $90=$ $10+10+10+10+10+10+10+10+10=9 \times 10=90$
NB: $90=10+10+10+10+10+10+10+10+10=9 \times 10=$ 90 , means ten (10) has been repeated nine times
Mathematical proves of the Ten(10's = $\mathrm{n}_{\text {\#10 }}$ ) Place Value
Fig 20: Diagram of Mathematical proves of the Ten's (10's = $n_{10}$ ) Place Value


Let
$\mathrm{n}_{\mathrm{th}}=$ all Place Values,
$y_{\text {nth }}=$ answer for all Place Values,
$\mathrm{n}\left({ }_{10}\right)=10$ for Ten's (10's) Place value,
$\mathrm{c}=$ movement ( s ) within the Ten's (10's) Place Value
$>y_{c}=\mathrm{an}_{(10)}$
$>y_{c}=\mathrm{cn}_{(10)}$, fig 10
For movement one on the Ten's $\left(\mathrm{n}_{(10)}\right)$ Place Value, $>\mathrm{a}_{1}=1$ and n
${ }_{(10)}=10$
$\rightarrow \mathrm{y}_{\mathrm{c}}=\mathrm{cn}_{(10)}=\mathrm{y}_{\mathrm{c}}=\mathrm{y}_{\mathrm{l}}=1 \mathrm{n}_{(10)}=1 \times 10=10$
:. $y_{1}=10$
For movement two on the Ten's $\left(\mathrm{n}_{(10)}\right)$ Place Value, $>\mathrm{a}_{2}=2$ and
$\mathrm{n}_{(10)}=10$
$>\mathrm{y}_{\mathrm{c}}=\mathrm{cn}_{(10)}=\mathrm{y}_{\mathrm{c}}=\mathrm{y}_{2}=2 \mathrm{n}_{(10)}=2 \times 10=10+10=20$
:. $\mathrm{y}_{2}=20$
For movement three on the Ten's $\left(\mathrm{n}_{(10)}\right)$ Place Value, $>\mathrm{a}_{3}=3$ and
$\mathrm{n}_{(10)}=10$
$>\mathrm{y}_{\mathrm{c}}=\mathrm{cn}_{(10)}=\mathrm{y}_{\mathrm{c}}=\mathrm{y}_{3}=3 \mathrm{n}_{(10)}=3 \times 10=10+10+10=30$
$\therefore \mathrm{y}_{3}=30$
For movement four on the Ten's $\left(\mathrm{n}_{(10)}\right)$ Place Value, $>\mathrm{a}_{4}=4$ and
$\mathrm{n}_{(10)}=10$
) $\mathrm{y}_{\mathrm{c}}=\mathrm{cn}_{(10)}=\mathrm{y}_{\mathrm{c}}=\mathrm{y}_{4}=4 \mathrm{n}_{(10)}=4 \times 10=10+10+10+10=40$
:. $y_{4}=40$
For movement five on the Ten's $\left(\mathrm{n}_{(10)}\right)$ Place Value, $>\mathrm{a}_{5}=5$ and n (10) $=10$
) $\mathrm{y}_{\mathrm{c}}=\mathrm{cn}_{(10)}=\mathrm{y}_{\mathrm{c}}=\mathrm{y}_{5}=5 \mathrm{n}_{(10)}=5 \times 10=10+10+10+10+10=$ 50
$\therefore y_{5}=50$
For movement six on the Ten's $\left(\mathrm{n}_{(10)}\right)$ Place Value, > $\mathrm{a}_{6}=6$ and n (10) $=10$
) $\mathrm{y}_{\mathrm{c}}=\mathrm{cn}_{(10)}=\mathrm{y}_{\mathrm{c}}=\mathrm{y}_{6}=6 \mathrm{n}_{(10)}=6 \times 10=10+10+10+10+10+$ $10=60$
:. $\mathrm{y}_{6}=60$
For movement seven on the Ten's $\left(\mathrm{n}_{(10)}\right)$ Place Value, $>\mathrm{a}_{8}=8$ and $\mathrm{n}_{(10)}=10$
$>y_{c}=\mathrm{cn}_{(10)}=\mathrm{y}_{\mathrm{c}}=\mathrm{y}_{8}=8 \mathrm{n}_{(10)}=8 \times 10=10+10+10+10+10+$ $10+10=70$
$\therefore \mathrm{y}_{7}=70$
For movement eight on the Ten's $\left(\mathrm{n}_{(10)}\right)$ Place Value, $>\mathrm{a}_{7}=7$ and $\mathrm{n}_{(10)}=10$
$>y_{c}=\mathrm{cn}_{(10)}=\mathrm{y}_{\mathrm{c}}=\mathrm{y}_{8}=8 \mathrm{n}_{(10)}=8 \times 10=10+10+10+10+10+$ $10+10+10$
$=80$
$\therefore \mathrm{y}_{8}=80$
For movement nine on the One's $\left(\mathrm{n}_{(10)}\right)$ Place Value, > $\mathrm{b}_{9}=9$ and $\mathrm{n}_{(10)}=10$
$>\mathrm{y}_{\mathrm{c}}=\mathrm{cn}_{(1)}=\mathrm{y}_{\mathrm{c}}=\mathrm{y}_{8}=9 \mathrm{n}_{(1)}=9 \times 1=10+10+10+10+10+$ $10+10+10+$
$10=90$
:. $\mathrm{y}_{9}=90$
Ten Place Value Solution-Boxes
Solution Box 2a: Ten's Place Value

| 0 | 0 | 1 | 0 | 2 | 0 | 3 | 0 | 4 | 0 | 5 | 0 | 6 | 0 | 7 | 0 | 8 | 0 | 9 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Solution Box 2b: reverse Ten's Place Value

| 9 | 0 | 8 | 0 | 7 | 0 | 6 | 0 | 5 | 0 | 4 | 0 | 3 | 0 | 2 | 0 | 1 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Example 1

How will you demonstrate 89 to; (1) a Classes (4-6) pupil and (JHS) Junior High School (1-3) pupil using the P-Value Solution-Box?

## Solution 1A

Demonstrating 89 by the use of $\mathbf{P}$-Value Solution-Box to Classes (4-6)
(1) Mathematical, 89 can be broken into 80 , and 9 i.e. $89=80+$ 9.
(2) Here, $89=80+9=$ Hundreds + Tens + Ones. This implies that, one has to use two (2) P Value Solution-Boxes i.e. Ten's P Value Solution-Box, and One's P Value Solution-Box.
(3)

Ten's P-Value Solution-Box

| , | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

One's P-Value Solution-Box

| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

$89=80+9=(10+10+10+10+10+10+10+10)+(1+1$
$+1+1+1+1+1+1+1)=8 \times 10+1 \times 9=80+9=89$.
Solution 1B
Demonstrating 89 by the use of P-Value Solution-Box to JHS (1-3)
(4) Mathematical, 89 can be broken into 80 , and 9 i.e. $89=80+$ 9.
(5) Here, $89=80+9=$ Tens + Ones. This implies that, one has to use two (2) P Value Solution-Boxes i.e. Ten's P Value Solution-Box, and One's P Value Solution-Box.
(6)

Ten's P-Value Solution-Box


One's P-Value Solution-Box
$89=80+9=(80)+(9)=8 \times 10+1 \times 9=80+9=89$.

## The Hundred's (100's) Place Value Concept

NB:

- The word 'and' = Addition (+)
- The movement on each place value is from both 'right' to 'left' and vice versa.
- A straight line is divided into nine (9) equal parts
- The Hundred's (100's) Place Value starts from zero (000) and ends at nine (900)


## Fig (21): Movement One



Movement One (Jump one step from zero to the right and or vice versa)
000 to $100=100$
100 to $200=100$
200 to $300=100$
300 to $400=100$
400 to $500=100$
500 to $600=100$
600 to $700=100$
700 to $800=100$
800 to $900=100$
Fig (22): Movement Two


Movement Two (Jump two steps from zero to the right and or vice versa)
000 to $200=000$ to 100 and $(+) 100$ to $200=100+100=2 \times 100$ $=200$
100 to $300=100$ to 200 and $(+) 200$ to $300=100+100=2 \times 100$ $=200$
200 to $400=200$ to 300 and $(+) 300$ to $400=100+100=2 \times 100$ $=200$
300 to $500=300$ to 400 and $(+) 400$ to $500=100+100=2 \times 100$ $=200$

400 to $600=400$ to 500 and (+) 500 to $600=100+100=2 \times 100$ $=200$
500 to $700=500$ to 600 and $(+) 600$ to $700=100+100=2 \times 100$ $=200$
600 to $800=600$ to 700 and (+) 700 to $800=100+100=$ $2 \times 100=200$
700 to $900=700$ to 800 and $(+) 800$ to $900=100+100=$ $2 \times 100=200$
NB: $2 \times 10=10+10$, means one hundred (100) has been repeated two times.

Fig (23): Movement Three


Movement Three (Jump three steps from zero to the right and or vice versa)
000 to $300=000$ to 100 and 100 to 200 and 200 to $300=100+$ $100+100=3 \times 100=300$
100 to $400=100$ to 200 and 200 to 300 and 300 to $400=100+$ $100+100=3 \times 100=300$
200 to $500=200$ to 300 and 300 to 400 and 400 to $500=100+$ $100+100=3 \times 100=300$
300 to $600=300$ to 400 and 400 to 500 and 500 to $600=100+$ $100+100=3 \times 100=300$
400 to $700=400$ to 500 and 500 to 600 and 600 to $700=100+$ $100+100=3 \times 100=300$
500 to $800=500$ to 600 and 600 to 700 and 700 to $800=100+$ $100+100=3 \times 100=300$
600 to $900=600$ to 700 and 700 to 800 and 800 to $900=100+$ $100+100=3 \times 100=300$
NB: $3 \times 100=100+100+100$, means one hundred (100) has been repeated three times

Fig (24): Movement Four


Movement Four (Jump four steps from zero to the right and or vice versa)
000 to $400=000$ to 100 and 100 to 200 and 200 to 300 and 300 to $400=100+100+100+100=4 \times 100=400$
100 to $500=100$ to 200 and 200 to 300 and 300 to 400 and 400 to $500=100+100+100+100=4 \times 100=400$
200 to $600=200$ to 300 and 300 to 400 and 400 to 500 and 500 to $600=100+100+100+100=4 \times 100=400$
300 to $700=300$ to 400 and 400 to 500 and 500 to 600 and 600 to $700=100+100+10+100=4 \times 100=400$
400 to $800=400$ to 500 and 500 to 600 and 600 to 700 and 700 to $800=100+100+100+100=4 \times 100=400$
500 to $900=500$ to 600 and 600 to 700 and 700 to 800 and 800 to $900=100+100+100+100=4 \times 100=400$

NB: $4 \times 100=100+100+100+100$, means one hundred (100) has been repeated four times.

Fig (25): Movement Five



Movement Five (Jump five steps from zero to the right and or vice versa)
000 to $500=000$ to 100 and 100 to 200 and 200 to 300 and 300 to 400 and 400 to $500=100+100+100+100+100=5 \times 100=$ 500
100 to $600=100$ to 200 and 200 to 300 and 300 to 400 and 400 to 500 and 500 to $600=100+100+100+100+100=5 \times 100=$ 500
200 to $700=200$ to 300 and 300 to 400 and 400 to 500 and 500 to 600 and 600 to $700=100+100+100+100+100=5 \times 100=$ 500
300 to $800=300$ to 400 and 400 to 500 and 500 to 600 and 600 to 700 and 700 to $800=100+100+100+100+100=5 \times 100=$ 500
400 to $900=400$ to 500 and 500 to 600 and 600 to 700 and 700 to 800 and 800 to $900=100+100+100+100+100=5 \times 100=$ 500
NB: $5 \times 100=100+100+100+100+100$, means one hundred (100) has been repeated five times.

Fig (26): Movement Six


Movement Six (Jump six steps from zero to the right and or vice versa)
000 to $600=000$ to 100 and 100 to 200 and 200 to 300 and 300 to 400 and
400 to 500 and 500 to $600=100+100+100+100+100+100$ $=6 \times 100=600$
100 to $700=100$ to 200 and 200 to 300 and 300 to 400 and 400 to 500 and
500 to 600 and 600 to $700=100+100+100+100+100+100$ $=6 \times 100=600$
200 to $800=200$ to 300 and 300 to 400 and 400 to 500 and 500 to 600 and
600 to 700 and 700 to $800=100+100+100+100+100+100$ $=6 \times 100=600$
300 to $900=300$ to 400 and 400 to 500 and 500 to 600 and 600 to 700 and
700 to 800 and 800 to $900=100+100+100+100+100+100$ $=6 \times 100=600$
NB: $6 \times 100=100+100+100+100+100+100$, means one hundred (100) has been repeated six times

Fig (27): Movement Seven


Movement Seven (Jump seven steps from zero to the right and or vice versa)
000 to $700=000$ to 100 and 100 to 200 and 200 to 300 and 300 to 400 and 400 to 500 and 500 to 600 and 600 to $700=100+$ $100+100+100+100+100+100=7 \times 100=700$
100 to $800=100$ to 200 and 200 to 300 and 300 to 400 and 400 to 500 and 500 to 600 and 600 to 700 and 700 to $800=100+$ $100+100+100+100+100+100=7 \times 100=700$
200 to $900=200$ to 300 and 300 to 400 and 400 to 500 and 500 to 600 and 600 to 700 and 700 to 800 and 800 to $900=100+$ $100+100+100+100+100+100=7 \times 100=700$
NB: $7 \times 100=100+100+100+100+100+100+100$, means one hundred (100) has been repeated seven times.

Fig (28): Movement Eight


Movement Eight (Jump eight steps from zero to the right and or vice versa)
000 to $800=000$ to 100 and 100 to 200 and 200 to 300 and 300 to 400 and 400 to 500 and 500 to 600 and 600 to 700 and 700 to $800=100+100+100+100+100+100+100+100=8 \times 100=$ 800
100 to $900=100$ to 200 and 200 to 300 and 300 to 400 and 400 to 500 and 500 to 600 and 600 to 700 and 700 to 800 and 800 to $900=100+100+100+100+100+100+100+100=8 \times 100=$ 800
NB: $8 \times 100=100+100+100+100+100+100+100+100$, means one hundred (100) has been repeated eight times

Fig (29): Movement Nine


Movement Nine (Jump nine steps from zero to the right and or vice versa)
000 to $900=000$ to 100 and 100 to 200 and 200 to 300 and 300 to 400 and 400 to 500 and 500 to 600 and 600 to 700 and 700 to

800 and 800 to $900=100+100+100+100+100+100+100+$
$100+100=9 \times 100=900$
NB: $900=100+100+100+100+100+100+100+100+100$
$=9 \times 100=900$, means one hundred (100) has been repeated nine times
$\underline{\text { Mathematical proves of the Hundred's }\left(100 ' s=n_{\# 100}\right) \text { Place }}$ Value
Fig 30: Diagram of Mathematical proves of the Hundred's $\left(100 ' s=n_{100}\right)$ Place Value


| 000 | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 900 | 800 | 700 | 600 | 500 | 400 | 300 | 200 | 100 | 000 |

$$
\rightarrow_{\mathrm{n}_{(100)}} \leftarrow
$$

Let
$\mathrm{n}_{\mathrm{th}}=$ all Place Values,
$y_{\mathrm{nth}}=$ answer for all Place Values,
$\mathrm{n}\left({ }_{100}=100\right.$, for Hundred's (100's) Place value,
$d=$ movement ( $s$ ) within the Hundred's (100's) Place Value
$>y_{d}=d n_{(100)}$
$>y_{d}=d n_{(100)}$
For movement one on the Hundred's $\left(\mathrm{n}_{(100)}\right)$ Place Value, $>\mathrm{d}_{1}=1$ and $\mathrm{n}_{(100)}=100$
$>y_{d}=\operatorname{dn}_{(100)}=y_{c}=y_{1}=1 n_{(100)}=1 \times 100=100$
$\therefore y_{1}=100$
For movement two on the Hundred's ( $\mathrm{n}_{(100}$ ) Place Value, > $\mathrm{d}_{2}=2$ and $\mathrm{n}_{(100)}=100$
$>y_{d}=\mathrm{dn}_{(100)}=\mathrm{y}_{\mathrm{d}}=\mathrm{y}_{2}=2 \mathrm{n}_{(100)}=2 \times 100=100+100=200$
$\therefore \mathrm{y}_{2}=200$
For movement three on the Hundred's ( $\mathrm{n}_{(100)}$ Place Value, > $\mathrm{d}_{3}=$ 3 and $\mathrm{n}_{(100)}=100$
$>y_{d}=\mathrm{dn}_{(100)}=\mathrm{y}_{\mathrm{d}}=\mathrm{y}_{3}=3 \mathrm{n}_{(100)}=3 \times 100=100+100+100=$ 300
$\therefore y_{3}=300$
For movement four on the Hundred's $\left(\mathrm{n}_{(100}\right)$ Place Value, $>\mathrm{d}_{4}=4$ and $\mathrm{n}_{(100)}=100$
$>\mathrm{y}_{\mathrm{d}}=\mathrm{dn}_{(100)}=\mathrm{y}_{\mathrm{d}}=\mathrm{y}_{4}=4 \mathrm{n}_{(100)}=4 \times 100=100+100+100+$ $100=400$
$\therefore y_{4}=40$
For movement five on the Hundred's $\left(\mathrm{n}_{(100)}\right)$ Place Value, $>\mathrm{d}_{5}=5$ and $\mathrm{n}_{(100)}=100$
$>\mathrm{y}_{\mathrm{d}}=\mathrm{dn}_{(100)}=\mathrm{y}_{\mathrm{d}}=\mathrm{y}_{5}=5 \mathrm{n}_{(100)}=5 \times 100=100+100+100+$ $100+100=500$
$\therefore y_{5}=500$
For movement six on the Hundred's ( $\mathrm{n}_{(100)}$ ) Place Value, > $\mathrm{d}_{6}=6$ and $\mathrm{n}_{(100)}=100$
$>y_{d}=\mathrm{dn}_{(100)}=y_{d}=y_{6}=6 n_{(100)}=6 \times 100=10+100+100+100$
$+100+100=600$
$\therefore \mathrm{y}_{6}=60$
For movement seven on the Hundred's ( $\mathrm{n}_{(100)}$ Place Value, > $\mathrm{d}_{7}$ $=7$ and $\mathrm{n}_{(100)}$
$=100$
$>\mathrm{y}_{\mathrm{d}}=\mathrm{dn}_{(100)}=\mathrm{y}_{\mathrm{d}}=\mathrm{y}_{7}=7 \mathrm{n}_{(100)}=7 \times 100=100+100+100+$ $100+100+100+100=700$
$\therefore y_{7}=700$
For movement eight on the Hundred's $\left(\mathrm{n}_{(100}\right)$ Place Value, $>\mathrm{d}_{8}=$ 8 and $\mathrm{n}_{(100)}=100$

[^0]$100+100+100=800$
$\therefore y_{8}=800$
For movement nine on the One's $\left(\mathrm{n}_{(100)}\right)$ Place Value, $>\mathrm{b}_{9}=9$ and $\mathrm{n}_{(100)}=100$
$>y_{d}=\mathrm{dn}_{(1)}=\mathrm{y}_{\mathrm{d}}=\mathrm{y}_{9}=9 \mathrm{n}_{(100)}=9 \times 100=100+100+100+100$
$+100+100$
$+100+100+100=900$
$\therefore y_{9}=900$
Hundred Place Value Solution-Boxes
Solution Box 3a: Hundred's Place Value

## 

Solution Box 3b: Reverse Hundred's Place Value

## 

## Example 1

How will you demonstrate 589 to; (1) a Classes (4-6) pupil and (JHS) Junior High School (1-3) pupil using the P-Value Solution-Box?
Solution 1A
Demonstrating 589 by the use of P-Value Solution-Box to Classes (4-6)
(1) Mathematical, 589 can be broken into 500, 80, and 9 i.e. $589=500+80+9$.
(2) Here, $589=500+80+9=$ Hundreds + Tens + Ones. This implies that, one has to use three (3) P Value Solution-Boxes i.e. Hundred's P Value Solution-Box, Ten's P Value Solution-Box, and One's P Value Solution-Box.
(3)

Hundred's P-Value Solution-Box
$\left.\begin{array}{|l|l|l|ll|l|l|l|l|l|l|l|l|}\hline 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1\end{array}\right)$ $+$
Ten's P-Value Solution-Box

| 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $+$

One's P-Value Solution-Box

$$
\begin{array}{|l|l|l|l|l|l|l|l|l|}
\hline 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\hline
\end{array}
$$

$589=500+80+9=(100+100+100+100+100)+(10+10$ $+10+10+10+10+10+10)+(1+1+1+1+1+1+1+1$
$+1)=5 \times 100+8 \times 10+1 \times 9=500+80+9=589$.
Solution 1B
Demonstrating 589 by the use of P-Value Solution-Box to JHS (1-3)
(1) Mathematical, 589 can be broken into 500, 80, and 9 i.e. $589=500+80+9$.
(2) Here, $589=500+80+9=$ Hundreds + Tens + Ones. This implies that, one has to use three (3) P Value Solution-Boxes i.e. Hundred's P Value Solution-Box, Ten's P Value Solution-Box, and One's P Value Solution-Box.
(3)

Hundred's P-Value Solution-Box

\section*{|  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |}

$+$
Ten's P-Value Solution-Box


One's P-Value Solution-Box

$589=500+80+9=(500)+(80)+(9)=5 \times 100+8 \times 10+1 \times 9=$ $500+80+9=589$.

## The Thousand's (1000's) Place Value Concept

 NB:- The word 'and' = Addition (+)
- The movement on each place value is from both 'right' to 'left' and vice versa.
- A straight line is divided into nine (9) equal parts
- The Thousand's (1000's) Place Value starts from zero (0000) and ends at nine (9000)

Fig (31): Movement One


## Movement One (Jump one step from zero to the right and or

 vice versa)0000 to $1000=1000$
1000 to $2000=1000$
2000 to $3000=1000$
3000 to $4000=1000$
4000 to $5000=1000$
5000 to $6000=1000$
6000 to $7000=1000$
7000 to $8000=1000$
8000 to $9000=1000$
Fig (32): Movement Two


## Movement Two (Jump two steps from zero to the right and or vice versa)

0000 to $2000=0000$ to 1000 and $(+) 1000$ to $2000=1000+$ $1000=2 \times 1000=2000$
1000 to $3000=1000$ to 2000 and $(+) 2000$ to $3000=1000+$ $1000=2 \times 1000=2000$
2000 to $4000=2000$ to 3000 and $(+) 3000$ to $4000=1000+$ $1000=2 \times 1000=2000$
3000 to $5000=3000$ to 4000 and $(+) 4000$ to $5000=1000+$ $1000=2 \times 1000=2000$
4000 to $6000=4000$ to 5000 and $(+) 5000$ to $6000=1000+$ $1000=2 \times 1000=2000$
5000 to $7000=5000$ to 6000 and $(+) 6000$ to $7000=1000+$ $1000=2 \times 1000=2000$
6000 to $8000=6000$ to 7000 and $(+) 7000$ to $8000=1000+$ $1000=2 \times 1000=2000$
7000 to $9000=7000$ to 8000 and $(+) 8000$ to $9000=1000+$ $1000=2 \times 1000=2000$
NB: $2 \times 1000=1000+1000$, means one thousand (1000) has been repeated two times.

Fig (33): Movement Three


## Movement Three (Jump three steps from zero to the right and or vice versa)

0000 to $3000=0000$ to 1000 and 1000 to 2000 and 2000 to $3000=1000+1000+1000=3 \times 1000=3000$
1000 to $4000=1000$ to 2000 and 2000 to 3000 and 3000 to $4000=1000+1000+1000=3 \times 1000=3000$
2000 to $5000=2000$ to 3000 and 3000 to 4000 and 4000 to $5000=1000+1000+1000=3 \times 1000=3000$
3000 to $6000=3000$ to 4000 and 4000 to 5000 and 5000 to $6000=1000+1000+1000=3 \times 100=3000$
4000 to $7000=4000$ to 5000 and 5000 to 6000 and 6000 to $7000=1000+1000+1000=3 \times 1000=3000$
5000 to $8000=5000$ to 6000 and 6000 to 7000 and 7000 to $8000=1000+1000+1000=3 \times 1000=3000$
6000 to $9000=6000$ to 7000 and 7000 to 8000 and 8000 to $9000=1000+1000+1000=3 \times 1000=3000$
NB: $3 \times 1000=1000+1000+1000$, means one thousand (1000) has been repeated three times.

Fig (34): Movement Four


Movement Four (Jump four steps from zero to the right and or vice versa)
0000 to $4000=0000$ to 1000 and 1000 to 2000 and 2000 to 3000 and 3000 to $4000=1000+1000+1000+1000=4 \times 1000$ $=4000$
1000 to $5000=1000$ to 2000 and 2000 to 3000 and 3000 to 4000 and 4000 to $5000=1000+1000+1000+1000=4 \times 1000$ $=4000$
2000 to $6000=2000$ to 3000 and 3000 to 4000 and 4000 to 5000 and 5000 to $6000=1000+1000+1000+1000=4 \times 1000$ $=40003000$ to $7000=3000$ to 4000 and 4000 to 5000 and 5000 to 6000 and 6000 to $7000=1000+1000+1000+1000=$ $4 \times 1000=4000$
4000 to $8000=4000$ to 5000 and 5000 to 6000 and 6000 to 7000 and 7000 to $8000=1000+1000+1000+1000=4 \times 1000$ $=4000$
5000 to $9000=5000$ to 6000 and 6000 to 7000 and 7000 to 8000 and 8000 to $9000=1000+1000+1000+1000=4 \times 1000$ $=4000$
NB: $4 \times 1000=1000+1000+1000+1000$, means one thousand (1000) has been repeated four times.

Fig (35): Movement Five


## Movement Five (Jump five steps from zero to the right and or vice versa)

0000 to $5000=0000$ to 1000 and 1000 to 2000 and 2000 to 3000 and 3000 to 4000 and 4000 to $5000=1000+1000+1000$ $+1000+1000=5 \times 1000=5000$
1000 to $6000=1000$ to 2000 and 2000 to 3000 and 3000 to 4000 and 4000 to 5000 and 5000 to $6000=1000+1000+1000$ $+1000+1000=5 \times 1000=5000$
2000 to $7000=2000$ to 3000 and 3000 to 4000 and 4000 to 5000 and 5000 to 6000 and 6000 to $7000=1000+1000+1000$ $+1000+1000=5 \times 1000=5000$
3000 to $8000=3000$ to 4000 and 4000 to 5000 and 5000 to 6000 and 6000 to 7000 and 7000 to $8000=1000+1000+1000$ $+1000+1000=5 \times 1000=5000$
4000 to $9000=4000$ to 5000 and 5000 to 6000 and 6000 to 7000 and 7000 to 8000 and 8000 to $9000=1000+1000+1000$ $+1000+1000=5 \times 1000=5000$
NB: $5 \times 1000=1000+1000+1000+1000+1000$, means one thousand (1000) has been repeated five times

## Fig (36): Movement Six



Movement Six (Jump six steps from zero to the right and or vice versa)
0000 to $6000=0000$ to 1000 and 1000 to 2000 and 2000 to 3000 and 3000 to 4000 and 4000 to 5000 and 5000 to $6000=$ $1000+1000+1000+1000+1000+1000=6 \times 1000=6000$ 1000 to $7000=1000$ to 2000 and 2000 to 3000 and 3000 to 4000 and 4000 to 5000 and 5000 to 6000 and 6000 to $7000=$ $1000+1000+1000+1000+1000+1000=6 \times 1000=6000$ 2000 to $8000=2000$ to 3000 and 3000 to 4000 and 4000 to 5000 and 5000 to 6000 and 6000 to 7000 and 7000 to $8000=$ $1000+1000+1000+1000+1000+1000=6 \times 1000=6000$ 3000 to $9000=3000$ to 4000 and 4000 to 5000 and 5000 to 6000 and 6000 to 7000 and 7000 to 8000 and 8000 to $9000=$ $1000+1000+1000+1000+1000+1000=6 \times 1000=6000$ NB: $6 \times 1000=1000+1000+1000+1000+1000+1000$, means one thousand (1000) has been repeated six times

Fig (37): Movement Seven


Movement Seven (Jump seven steps from zero to the right and or vice versa)
0000 to $7000=0000$ to 1000 and 1000 to 2000 and 2000 to 3000 and 3000 to 4000 and 4000 to 5000 and 5000 to 6000 and 6000 to $7000=1000+1000+1000+1000+1000+1000$ $+1000=7 \times 1000=7000$

1000 to $8000=1000$ to 2000 and 2000 to 3000 and 3000 to 4000 and 4000 to 5000 and 5000 to 6000 and 6000 to 7000 and 7000 to $8000=1000+1000+1000+1000+1000+1000$ $+1000=7 \times 1000=7000$
2000 to $9000=2000$ to 3000 and 3000 to 4000 and 4000 to 5000 and 5000 to 6000 and 6000 to 7000 and 7000 to 8000 and 8000 to $9000=1000+1000+1000+1000+1000+1000$ $+1000=7 \times 1000=7000$
NB: $7 \times 1000=1000+1000+1000+1000+1000+1000+1000$, means one thousand (1000) has been repeated seven times.

Fig (38): Movement Eight


Movement Eight (Jump eight steps from zero to the right and or vice versa)
0000 to $8000=0000$ to 1000 and 1000 to 2000 and 2000 to 3000 and 3000 to 4000 and 4000 to 5000 and 5000 to 6000 and 6000 to 7000 and 7000 to $8000=1000+1000+1000+1000$ $+1000+1000+1000+1000=8 \times 1000=8000$
1000 to $9000=1000$ to 2000 and 2000 to 3000 and 3000 to 4000 and 4000 to 5000 and 5000 to 6000 and 6000 to 7000 and 7000 to 8000 and 8000 to $9000=1000+1000+1000+1000$ $+1000+1000+1000+1000=8 \times 1000=8000$
NB: $8 \times 1000=1000+1000+1000+1000+1000+1000+1000$ +1000 , means one thousand (1000) has been repeated eight times

Fig (39): Movement Nine


Movement Nine (Jump nine steps from zero to the right and or vice versa)
0000 to $9000=0000$ to 1000 and 1000 to 2000 and 2000 to 3000 and 3000 to 4000 and 4000 to 5000 and 5000 to 6000 and 6000 to 7000 and 7000 to 8000 and 8000 to $9000=1000+1000$ $+1000+1000+1000+1000+1000+1000+1000=9 \times 1000=$ 9000
NB: $9000=1000+1000+1000+1000+1000+1000+1000+$ $1000+1000=9 \times 100=900$, means one thousand $(1000)$ has been repeated nine times
$\underline{\text { Mathematical proves of the Thousand's }\left(1000 ' s=n_{\# 1000}\right)}$ Place Value
Fig 40: Diagram of Mathematical proves of the Thousand's $\left(1000 ' s=n_{1000}\right)$ Place Value

| 000 | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 900 | 800 | 700 | 600 | 500 | 400 | 300 | 200 | 100 | 000 |
| $\rightarrow_{n_{(1000)}} \leftarrow$ |  |  |  |  |  |  | (f) |  |  |

Let
$\mathrm{n}_{\mathrm{th}}=$ all Place Values,
$y_{\text {nth }}=$ answer for all Place Values,
$\mathrm{n}\left({ }_{1000)}=1000\right.$, for Thousand's (1000's) Place value,
$\mathrm{e}=$ movement (s) within the Thousand's (1000's) Place Value
$>\mathrm{y}_{\mathrm{b}}=\mathrm{bn}$ (1000)
$>y_{e}=e_{(1000)}$
For movement one on the Thousand's ( $\mathrm{n}_{(1000)}$ ) Place Value, $>\mathrm{e}_{1}=$
1 and $\mathrm{n}_{(1000)}=1000$
$>\mathrm{y}_{\mathrm{e}}=\mathrm{en}_{(1000)}=\mathrm{y}_{\mathrm{e}}=\mathrm{y}_{1}=1 \mathrm{n}_{(1000)}=1 \times 1000=1000$
$\therefore \mathrm{y}_{1}=1000$
For movement two on the Thousand's ( $\mathrm{n}_{(1000)}$ ) Place Value, $>\mathrm{e}_{2}=$
2 and $\mathrm{n}_{(1000)}=1000$
$>\mathrm{y}_{\mathrm{e}}=\mathrm{en}_{(1000)}=\mathrm{y}_{\mathrm{e}}=\mathrm{y}_{2}=2 \mathrm{n}_{(1000)}=2 \times 1000=1000+1000=2000$
$\therefore \mathrm{y}_{2}=2000$
For movement three on the Thousand's $\left(\mathrm{n}_{(100)}\right)$ Place Value, $>\mathrm{e}_{3}=$ 3 and $\mathrm{n}_{(1000)}=1000$
, $\mathrm{y}_{\mathrm{e}}=\mathrm{en}_{(1000)}=\mathrm{y}_{\mathrm{e}}=\mathrm{y}_{3}=3 \mathrm{n}_{(1000)}=3 \times 1000=1000+1000+$
$1000=3000$
$\therefore \mathrm{y}_{3}=3000$
For movement four on the Thousand's $\left(\mathrm{n}_{(1000)}\right)$ Place Value, $>\mathrm{e}_{4}=$ 4 and $\mathrm{n}_{(1000)}=1000$
$>\mathrm{y}_{\mathrm{e}}=\mathrm{en}(1000)=\mathrm{y}_{\mathrm{e}}=\mathrm{y}_{4}=4 \mathrm{n}_{(1000)}=4 \times 1000=1000+1000+$ $1000+1000=4000$
$\therefore \mathrm{y}_{4}=4000$
For movement five on the Thousand's ( $\mathrm{n}_{(1000)}$ Place Value, > $\mathrm{e}_{5}=$ 5 and $\mathrm{n}_{(1000)}=1000$
$>\mathrm{y}_{\mathrm{e}}=\mathrm{en}_{(1000)}=\mathrm{y}_{\mathrm{e}}=\mathrm{y}_{5}=5 \mathrm{n}_{(100)}=5 \times 1000=1000+1000+1000$
$+1000+1000=5000$
$\therefore y_{5}=5000$
For movement six on the Thousand's ( $\mathrm{n}_{(1000)}$ Place Value, $>\mathrm{e}_{6}=$ 6 and $\mathrm{n}_{(1000)}=1000$
$>y_{\mathrm{e}}=\mathrm{en}_{(1000)}=\mathrm{y}_{\mathrm{e}}=\mathrm{y}_{6}=6 \mathrm{n}_{(100)}=6 \times 1000=1000+1000+1000$
$+1000+1000+1000=6000$
$\therefore \mathrm{y}_{6}=6000$
For movement seven on the Thousand's ( $\mathrm{n}_{(1000)}$ Place Value, > $\mathrm{e}_{7}$ $=7$ and $\mathrm{n}_{(1000)}=1000$
> $\mathrm{y}_{\mathrm{e}}=\mathrm{en}(1000)=\mathrm{y}_{\mathrm{e}}=\mathrm{y}_{7}=7 \mathrm{n}_{(100)}=7 \times 1000=1000+1000+$ $1000+1000+1000+1000+1000=7000$
$\therefore \mathrm{y}_{7}=7000$
For movement eight on the Thousand's ( $\mathrm{n}_{(1000}$ ) Place Value, > $\mathrm{e}_{8}=8$ and $\mathrm{n}_{(1000}=1000$
$>\mathrm{y}_{\mathrm{e}}=\mathrm{en}_{(1000)}=\mathrm{y}_{\mathrm{e}}=\mathrm{y}_{8}=8 \mathrm{n}_{(1000)}=8 \times 1000=1000+1000+$ $1000+1000+1000+1000+1000+1000=8000$
$\therefore \mathrm{y}_{8}=8000$
For movement nine on the One's $\left(\mathrm{n}_{(1000)}\right)$ Place Value, $>\mathrm{e}_{9}=9$ and $\mathrm{n}_{(1000)}=1000$
$>y_{\mathrm{e}}=\mathrm{en}_{(1000)}=\mathrm{y}_{\mathrm{e}}=\mathrm{y}_{9}=9 \mathrm{n}_{(1000)}=9 \times 1000=1000+1000+$ $1000+1000+1000+1000+1000+1000+1000=9000$
$\therefore y_{9}=9000$
Thousand Place Value Solution-Boxes
Solution Box 3a: Thousand's Place Value

## 

Solution Box 3b: Reverse Thousand's Place Value

## 

## Example 1

How will you demonstrate 5589 to; (1) a Classes (4-6) pupil and (JHS) Junior High School (1-3) pupil using the P-Value Solution-Box?

## Solution 1A

Demonstrating 5589 by the use of P-Value Solution-Box to Classes (4-6)
(7) Mathematical, 5589 can be broken into 5000, 500, 80, and 9 i.e. $5589=5000+500+80+9$.
(8) Here, $5589=5000+500+80+9=$ Thousands + Hundreds + Tens + Ones. This implies that, one has to use four (4) P Value Solution-Boxes i.e. thousand's P Value Solution-Box, Hundred's P Value Solution-Box, Ten's P Value Solution-Box, and One's P Value Solution-Box.
(9)

Thousand's P-Value Solution-Box

$+$
Hundred's P-Value Solution-Box

Ten's P-Value Solution-Box

| 1 | 0 | 1 | 0 | 1 | 0 |  | 1 | 0 | 1 | 0 |  |  | 0 | 1 | 0 | 1 | 0 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

One's P-Value Solution-Box

| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $5589=1000+500+80+9=(1000+1000+1000+1000+$ $1000)+(100+100+100+100+100)+(10+10+10+10+$ $10+10+10+10)+(1+1+1+1+1+1+1+1+1)=$ $5 \times 1000+5 \times 100+8 \times 10+1 \times 9=5000+500+80+9=5589$.

## Solution 1B

## Demonstrating 5589 by the use of P-Value Solution-Box to

 JHS (1-3)(10) Mathematical, 5589 can be broken into 5000, 500, 80, and 9
i.e. $5589=5000+500+80+9$.
(11) Here, $5589=1000+500+80+9=$ Thousands + Hundreds + Tens + Ones.
This implies that, one has to use four (4) P Value SolutionBoxes i.e. thousand's P Value Solution-Box, Hundred's P Value Solution-Box, Ten's P Value Solution-Box, and One's P Value Solution-Box.
(12)

Thousand's P-Value Solution-Box


Hundred's P-Value Solution-Box


Ten's P-Value Solution-Box


One's P-Value Solution-Box


## Combination of some $\mathbf{P}$-Value Models

Fig 41: Diagram of some P-Values Models (i.e. One’s, Ten's, Hundred's, Thousand's and Million's)


## Combination of Formulas of the $\mathbf{P}$-Value Models

$\mathrm{n}_{\mathrm{j}}=$ all Place Values,
$y_{j}=$ answer for all Place Values
$\mathrm{y}_{\mathrm{b}}=\mathrm{an}(1) ; \mathrm{a}=$ number of movement ( s ) in One's (1's) Place Value
$\mathrm{y}_{\mathrm{c}}=\mathrm{bn}_{(10)} ; \mathrm{b}=$ number of movement (s) in Ten's (10's) Place Value
$\mathrm{y}_{\mathrm{d}}=\mathrm{cn}(100) ; \mathrm{c}=$ number of movement (s) in Hundred's (100's) Place Value
$\mathrm{y}_{\mathrm{e}}=\mathrm{dn}(1000) ; \mathrm{d}=$ number of movement ( s ) in Thousand's (1000's) Place Value
$\mathrm{y}_{\mathrm{f}}=\mathrm{en}(1000000) ; \mathrm{e}=$ number of movement ( s ) in Million's (1000000's) Place
Value
$\mathrm{y}_{\mathrm{g}}=\mathrm{fn}{ }_{(1000000000)} ; \mathrm{f}=$ number of movement ( s ) in Billion's (1000000000's) Place
Value
$\mathrm{y}_{\mathrm{h}}=\mathrm{gn}_{(1000000000000)} ; \mathrm{g}=$ number of movement ( s ) in Trillion's (1000000000000's)
Place Value
$y_{i}=\mathrm{gn}_{(1000000000000)} ; \mathrm{i}=$ number of movement (s) in Quintillion's (1000000000000000's) Place Value
$\mathrm{y}_{\mathrm{nth}}=\mathrm{Dn}_{(\mathrm{j})} ; \mathrm{j}=$ number of movement ( s ) in Infinity's ( j 's) Place Value
Let
$\mathrm{y}_{\mathrm{n}(\mathrm{x})}=\mathrm{Dn}_{(\mathrm{x})} ; \mathrm{x}=$ specific place value i.e. one, ten, hundred, thousand, million etc
For place values from one's to Quintillion's
$\mathrm{y}_{\mathrm{n}(\mathrm{X})}=\sum_{j=1}^{n} D n_{j}=\mathrm{y}_{\mathrm{n}(1)}+\mathrm{y}_{\mathrm{n}(10)}+\mathrm{y}_{\mathrm{n}(100)}+\mathrm{y}_{\mathrm{n}(1000)}+\mathrm{y}_{\mathrm{n}(1000000)}+. .+$ $\mathrm{y}_{\mathrm{n}(1000000000000000)}$
For place values from one's to infinity's

$\mathrm{y}_{\mathrm{j}}=\sum_{j=1}^{n} D n_{j}=\mathrm{an}_{(1)}+\mathrm{bn}_{(10)}+\mathrm{cn}_{(100)}+\mathrm{dn}_{(1000)}+\mathrm{en}_{(1000000)}+$ $\ldots . . . . . .+y_{n j}$
Where, $\mathrm{D}=\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}, \ldots \ldots \ldots \mathrm{z}$ depending on the place value (s) used.

## Reference

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Appendix A

(Refer to Price, 2011, p 41)

Appendix B

(Refer to Price 2012, p 263)


[^0]:    $>\mathrm{y}_{\mathrm{d}}=\mathrm{dn}_{(100)}=\mathrm{y}_{\mathrm{d}}=\mathrm{y}_{8}=8 \mathrm{n}_{(100)}=8 \times 100=100+100+100+$ $100+100+$

