



Analytic solution of Bloch nuclear magnetic resonance flow equation for neural communication

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ABSTRACT

A second order partial differential equation occurring frequently in applied mathematics is the wave equation. A generalization of this equation inevitably arises in many mathematical analyses of phenomena involving the propagation of waves in continuous media. Here, we applied the wave differential equation to investigate and explain neural communication using the nervous system. On application of Bloch nuclear magnetic resonance theory, a linear 1-dimensional homogeneous second order partial differential equation, as a model, is obtained which represents a nerve set in a vibrational motion. Analytical results from the solution of the differential equation show that the transmission of nerve impulses is not a flow of electrons, as in the case of electric current, but a wave of electrical activity travelling along the neurone. A three dimensional pictorial representation of the results further explains the phenomenon clearly.

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Introduction

The continuous improvement in Nuclear Magnetic Resonance (NMR) instruments and techniques are fostering new and important applications in nuclear spectroscopy and medicine. Here we apply the NMR technique in the transport of nerve pulses. We shall apply Bloch NMR theory to develop an analytical neural communication model which will explain the mode of transmission of nerve pulses.

The interstitial or intercellular fluid (tissue fluid) is the medium through which substances pass from blood to the body cells and from cells to blood. (Ross and Wilson, 2003)

Since the cerebrospinal fluid inside the neurones is made up of water, which consists of hydrogen and oxygen that have nuclei that contain odd numbers of protons or neutrons, the application of NMR technique can be used to describe the transportation of the neurones in the body system.

Theory

The wave equation is a hyperbolic partial differential equation with two or more variables. It includes a time variable t , one or more spatial variables $x_1, x_2, \dots, x_n$, and a scalar function $u = u(x_1, x_2, \dots, x_n; t)$, whose values could model the height of a wave. The wave equation for u is

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u \quad (1)$$

where ∇^2 is the (spatial) Laplacian and c is a fixed constant identified with the propagation speed of the wave. This equation is linear, as the sum of any two solutions is again a solution. The equation alone does not specify a solution; a unique solution is usually obtained by setting a problem with further conditions, such as initial conditions, which prescribe the value and velocity of the wave. (Courant and Hilbert, 1962).

Associated with each rotating object is an angular momentum and associated with each nuclear spin is a magnetic moment arising from the angular momentum of the nucleus. In

a magnetic field, B , the magnetic moment will behave like magnetic dipole and will experience a torque (Hornak, 2011).

In terms of the Bloch NMR equations for example, a nerve cell containing cerebrospinal fluid, which contains hydrogen and oxygen molecules with many unpaired electrons, spins with angular speed ω , in a coordinate system rotating with the nerve cell. When the radio frequency (rF) field, B_1 , is applied on a microscopic volume of mass m of the nerve cell, at equilibrium, the net force acting on m must be zero. The forces are the contact force, coriolis force and the centrifugal force. The coriolis and centrifugal forces seem quite real in a rotating frame (Awojoyogbe, 2003).

The fluid particle, on the atomic scale, was considered which either initially or on the average is in steady rotation. We apply a mathematical algorithm to describe the dynamical state of the flowing fluid particle starting from the NMR flow equations (Awojoyogbe, 2002).

The fluid is assumed to be magnetized by the static magnetic field, B_0 , to an equilibrium magnetization, M_0 , before entering the exit coil. The z axis in the rotating frame coincides with the laboratory Z axis; the x axis makes an angle ' ωt ' at any instant of time ' t ' with laboratory X axis. The $X=0$ position could be such that the transverse magnetic field at the end of the detector coil is negligible. The flow properties of the Bloch NMR flow equations describes the dynamics of fluid flow under the influence of rF magnetic field as follows (Awojoyogbe, 2004).

$$\frac{dM_x}{dt} = V \cdot \text{grad} M_x + \frac{\partial M_x}{\partial t} = -\frac{M_x}{T_2} \quad (2)$$

$$\frac{dM_y}{dt} = V \cdot \text{grad} M_y + \frac{\partial M_y}{\partial t} = \gamma M_z B_1(x) - \frac{M_y}{T_2} \quad (3)$$

$$\frac{dM_z}{dt} = V \cdot \text{grad} M_z + \frac{\partial M_z}{\partial t} = -\gamma M_y B_1(x) + \frac{M_0 - M_z}{T_1} \quad (4)$$

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where V is the fluid flow velocity. Two reasonable initial boundary conditions which may conform to the real-time experimental arrangements were chosen. These are:

- (i). $M_0 \neq M_{za}$ situation which holds when the rF $B_1(x)$ field is strong.
- (ii). before entering signal detector coil, the fluid bolus has magnetization

$$M_x = 0, M_y = 0$$

If $B_1(x)$ is large, the M_z of the fluid bolus changes appreciably from M_0 . For steady flow

$$\frac{\partial M_y}{\partial t} = 0 \quad (5)$$

Thus from equations (3) and (4) one obtains

$$\frac{d^2 M_y}{dx^2} + \frac{1}{V} \left(\frac{1}{T_1} + \frac{1}{T_2} \right) \frac{dM_y}{dx} + \frac{1}{V^2} \left(\gamma^2 B_1^2(x) + \frac{1}{T_1 T_2} \right) M_y = \frac{M_0 \gamma B_1(x)}{V^2 T_1} \quad (6)$$

It is convenient to use, as dependent variable, the departure of the stream function from its classical form and write:

$$M_y = U + \frac{\psi(x)}{x} \quad (7)$$

where $U = \frac{M_0}{\gamma T_1 B_1}$, B_1 is treated as a constant, V is the instantaneous velocity of the fluid and $\psi(x)$ is a special function of the transverse magnetization M_y which depends on the dynamical state of the fluid particle (Kreyszc, 1998). Equation (6) can be written as:

$$\frac{d^2 \psi}{dx^2} + \frac{\gamma^2 B_1^2}{V^2} \psi = 0 \quad (8)$$

provided that $x = 2VT_0$ (9)

subject to the following two conditions:

- (i) Resonance condition exists at Larmor frequency, $f_0 = \gamma B - \omega = 0$
- (ii) $\gamma^2 B_1^2 \gg \frac{1}{T_1 T_2}$, where γ denotes the gyromagnetic ratio of fluid

spins; $\omega/2\pi$ is the rF excitation frequency; and f_0/γ is the off- resonance field in the rotating frame of reference. T_1 and T_2 are the spin-lattice and spin-spin relaxation times respectively. rF B_1 is treated as constant.. Classically, equation (8) applies to a free fluid particle moving in one dimension only. We take this as the x direction, so that the trajectory is a function $x(t)$. The flow velocity V is the differential of position with respect to time. We can write equation (9) as

$$x = 2T_0 \frac{dx}{dt} \quad (10)$$

After following some mathematical procedures (Awojoyogbe, 2004) equation (8) becomes

$$\frac{d^2 \psi}{dx^2} + \frac{4\mu^2 \Omega_0^2}{h^2} \left(\frac{1}{T_1 T_2} \gamma^2 B_1^2(x) \right) \psi = 0 \quad (11)$$

The Bloch equation (11) plays a fundamental role in the search for the best possible NMR data obtainable in a fluid flow system at microscopic level. The wave function $\psi(x)$ is associated with any particle at the atomic and molecular level flowing in a conservative force field. The wave function determines everything that can be known about the flow system. (Awojoyogbe, 2004).

In our model we will show through the above Bloch theory that the transmission of nerve impulses (that is, communication between neurones) is not a flow of electrons as in the case of electric current, but is a wave of electrical activity travelling along the neurone.

Our model for the neural communication

Here we derive a partial differential equation governing small transverse vibrations of a nerve, which is assumed to be stretched to length L and then fixed at the endpoints. Suppose that the nerve is distorted and then at a certain instant, say at the time $t = 0$, is released and allowed to vibrate. The problem is to determine the vibrations of the nerve, that is, to find its

deflection $u(x, t)$ at any point x and time $t > 0$. We make the following assumptions which simplify the resulting equation:

- (i) The mass of the nerve per unit length is constant ("homogenous nerve"). The nerve is perfectly elastic and does not offer any resistance to bending.
- (ii) The tension caused by stretched nerve that is connected in the nervous system is so large that the action of gravitational force on the nerve can be neglected.
- (iii) The nerve performs a small transverse motion in a vertical plane that is, every particle of the nerve moves strictly vertically so that the deflection and the slope at every point of the nerve remain small in absolute value.

These assumptions are such that we may expect that the solution $u(x, t)$ of the differential equation to be obtained will reasonably be well described by small vibrations of the physical nerve consisting of small homogenous mass under large tension.

Equation of Motion for the Model

To obtain the equation of motion we consider the forces acting on a small portion of the nerve PQ as shown in Fig.1. Since the nerve does not offer resistance to bending, the tension is tangential to the curve representing the nerve at each point.

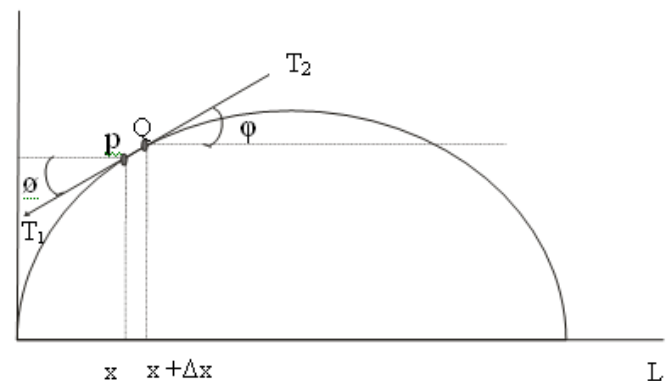


Fig. 1: Tensions T_1 and T_2 acting on a small portion, PQ, of the nerve

Let T^1 and T^2 be the tension at the endpoints P and Q of the portion. Since there is no motion in the horizontal direction, the horizontal components of the tension must be constant. Therefore we have;

$$T^1 \cos \phi = T^2 \cos \phi = T = \text{const} \quad (12)$$

In the vertical direction we have two forces namely, $-T^1 \sin \phi$ and $T^2 \sin \phi$. The resultant of the two forces is equal to the mass $\mu \Delta x$ of the portion times the acceleration $\frac{\partial^2 u}{\partial t^2}$ evaluated at some point between x and $x + \Delta x$, where μ is the mass of the

undeflected nerve per unit length and Δx is the length of the portion of the undeflected nerve.

$$\text{Hence, } T_2 \sin \phi - T_1 \sin \phi = \mu \Delta x \frac{\partial^2 u}{\partial t^2}$$

By using eqn (12), we obtain;

$$\frac{T_2 \sin \phi}{T_2 \cos \phi} - \frac{T_1 \sin \phi}{T_1 \cos \phi} = \tan \phi - \tan \phi = \left(\frac{\mu \Delta x}{T} \right) \left(\frac{\partial^2 u}{\partial t^2} \right) \quad (13)$$

Now, $\tan \phi$ and $\tan \phi$ are the slopes of the nerve at x and $x + \Delta x$.

Therefore, $\tan \phi = \left(\frac{\partial u}{\partial x} \right)_x$ and $\tan \phi = \left(\frac{\partial u}{\partial x} \right)_{x+\Delta x}$. Dividing equation (13) by Δx , and taking the limit as Δx tends to zero we obtain the linear partial differential equation:

$$\frac{\partial^2 u}{\partial t^2} = \beta^2 \frac{\partial^2 u}{\partial x^2}, \text{ where } \beta^2 = \frac{T}{\mu} \quad (14)$$

Here β is the velocity of fluid flow from neurone to neurone. Equation (14) is a one dimensional homogenous second order differential equation. It represents a nerve set in motion (when pinched by an insect, for instance) that vibrates in vertical direction at the point x at time t . The notation β^2 for physical

constant $\frac{T}{\mu}$ has been chosen to indicate that this constant is positive. Equation (14) is the wave equation, where T is the tension in the nerve and μ is the mass per unit length of the nerve. Thus β has the physical dimension of velocity. Therefore, β is the velocity of fluid flow from neurone to neurone.

To describe the motion of the nerve completely it is necessary to specify suitable initial and boundary condition for the displacement $u(x, t)$. The first neurone which is set in motion is assumed to be fixed at the initial end and free at the other end, while the other neurones attached to the first neurone are assumed to be free at both ends (Fig. 2).

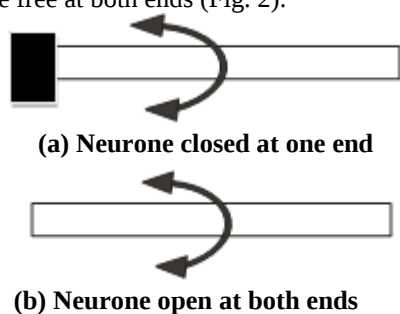


Fig. 2: Model of a Neurone (a) Closed at one end and (b) Open at both ends

Since the differential equation (14) is of the second order with respect to t , it is plausible that two initial conditions must be prescribed. There is the initial position of the neurone;

$$u(x, 0) = f(x), 0 \leq x \leq l \quad (15)$$

Where its initial velocity is:

$$u_t(x, 0) = g(x), 0 \leq x \leq l \quad (16)$$

where f and g are given functions.

We assume, as a working hypothesis that solutions for the displacement $u(x, t)$ exist as a product of a function of x alone

and a function of t alone. Using the method of separation of variables we set:

$$u(x, t) = X_{(x)} T_{(t)} \quad (17)$$

In this case, partial differentiation of $u(x, t)$ amounts to total differentiation of one or the other of the factors of $u(x, t)$ and we have

$$\frac{\partial^2 u}{\partial x^2} = X'' T \quad \text{and} \quad \frac{\partial^2 u}{\partial t^2} = X T''$$

where X'' represents second order partial derivatives with respect to position and T'' represents second order partial derivatives with respect to time. Substituting these into the wave

equation (6) and dividing by the product XT , we obtain

$$XT'' = \beta^2 X'' T \quad \text{and} \quad (18)$$

$$\frac{T''}{T} = \beta^2 \frac{X''}{X} = \sigma \quad (19)$$

where σ is a constant. Thus, we find that $X_{(x)}$ and $T_{(t)}$ satisfy ordinary differential equations.

In equation (19) the variables have been separated: the left side depends only on t while the right side depends only on x .

For equation (19) to be valid for $0 \leq x \leq l, t > 0$, it is necessary that both sides be equal to the same constant. This separation constant is indicated as σ . The partial differential equation (14) has thus been replaced by two ordinary differential equations. Each of these equations can be readily solved for any value of the separation constant σ .

$$T'' = \sigma T \quad \text{and} \quad X'' = \frac{\sigma}{\beta^2} X \quad (20)$$

Assuming that we need to consider only real values of σ , there are three cases to investigate; when $\sigma > 0$, $\sigma = 0$, $\sigma < 0$. If $\sigma > 0$, we can write $\sigma = \lambda^2$ ($\lambda > 0$). In this case the two differential equations and their solutions are:

$$T'' = \lambda^2 T, \quad X'' = \frac{\lambda^2}{\beta^2} X$$

$$T = Ae^{\lambda t} + Be^{-\lambda t}, \quad X = Ce^{\lambda x / \beta} + De^{-\lambda x / \beta} \quad (21)$$

But a solution of the form

$$u(x, t) = X_{(x)} T_{(t)} = (Ce^{\lambda x / \beta} + De^{-\lambda x / \beta})(Ae^{\lambda t} + Be^{-\lambda t}) \quad (22)$$

cannot describe the un-damped vibrations of a system because it is not periodic. Hence, although the product solution exists for $\sigma > 0$, they have no significance in relation to the physical problem we are considering. If $\sigma = 0$, the equations and their solutions are:

$$T'' = 0, \quad X'' = 0$$

$$T = A_1 + B, \quad X = C_x + D$$

But, again a solution of the form $u(x, t) = X_{(x)} T_{(t)} = (C_x + D)(A_1 + B)$ cannot describe a periodic motion. Hence the alternative $\sigma = 0$, must also be rejected. Finally, if $\sigma < 0$, we can write $\sigma = -\lambda^2$ ($\lambda > 0$). Then the component differential equations and their solutions are:

$$T'' = -\lambda^2 T, \quad X'' = \frac{-\lambda^2}{\beta^2} X$$

$$T = A \cos \lambda t + B \sin \lambda t, \quad X = (C \cos \frac{\lambda}{\beta} x + D \sin \frac{\lambda}{\beta} x)$$

In this case the solution becomes:

$$u(x, t) = X_{(x)} T_{(t)} = (C \cos \frac{\lambda}{\beta} x + D \sin \frac{\lambda}{\beta} x) (A \cos \lambda t + B \sin \lambda t) \quad (23)$$

We thus find that the free ends are characterized by the requirement that

$$E_s \frac{\partial u}{\partial x} \Big|_{end} = 0, \quad (24)$$

Since E_s is a nonzero constant of the material of the nerve. For a nerve of uniform section such as the one considered in Fig. 2, it follows that at a free end (where $x = 0$),

$$\frac{\partial u}{\partial x} = (-C \frac{\lambda}{\beta} \sin \frac{\lambda}{\beta} x + D \frac{\lambda}{\beta} \cos \frac{\lambda}{\beta} x) (A \cos \lambda t + B \sin \lambda t) = 0,$$

$$\frac{\lambda}{\beta} D (A \cos \lambda t + B \sin \lambda t) = 0, \quad \text{for all } t. \quad (25)$$

and from this we conclude that $D = 0$. Similarly, imposing the right-hand end condition by substituting $x = l$ and again equating the result to zero, we find;

$$-C \frac{\lambda}{\beta} \sin \frac{\lambda l}{\beta} (A \cos \lambda t + B \sin \lambda t) = 0, \quad \text{for all } t. \quad (26)$$

we cannot permit $C = 0$, since it leads only to a trivial solution

$$\sin \frac{\lambda l}{\beta} = 0 \quad \text{or} \quad \frac{\lambda l}{\beta} = n\pi, \quad n = 0, 1, 2, 3, \dots$$

Thus, to have the end conditions of the problem fulfilled we must be restricted to one of the discrete set of values,

$$\lambda_n = \frac{n\pi\beta}{l}, \quad n = 0, 1, 2, 3, \dots$$

Again, we construct the product solution for each admissible value of λ , getting

$$u_n(x, t) = \cos \frac{n\pi x}{l} (A_n \cos \frac{n\pi\beta t}{l} + B_n \sin \frac{n\pi\beta t}{l}) \quad (27)$$

In an attempt to form an infinite series of these solutions, we get:

$$u(x, t) = \sum_{n=1}^{\infty} u_n(x, t) = \sum_{n=1}^{\infty} \cos \frac{n\pi x}{l} (A_n \cos \frac{n\pi\beta t}{l} + B_n \sin \frac{n\pi\beta t}{l}) \quad (28)$$

which satisfies the initial displacement condition

$$u(x, 0) = f(x) \quad \text{and the initial velocity condition}$$

$$\frac{\partial u}{\partial t} \Big|_{x,0} = g(x)$$

To satisfy the initial displacement condition, we must have

$$u(x, 0) \equiv f(x) = \sum_{n=1}^{\infty} A_n \cos \frac{n\pi x}{l} \quad (29)$$

which requires that the A_n 's be the coefficients in the half-range cosine expansion of the known function $f(x)$ over the interval $(0, l)$ that is,

$$A_n = \frac{2}{l} \int_0^l f(x) \cos \frac{n\pi x}{l} dx \quad (30)$$

to satisfy the initial velocity condition, we must have;

$$\frac{\partial u}{\partial t} \Big|_{x,0} \equiv g(x) = \sum_{n=1}^{\infty} \left(\frac{n\pi\beta}{l} B_n \right) \cos \frac{n\pi x}{l}, \quad (31)$$

which requires that the quantities $\frac{n\pi\beta}{l} B_n$ be coefficients in the

half-range cosine series for $g(x)$ over the interval $(0, l)$ that

$$\frac{n\pi\beta}{l} B_n = \frac{2}{l} \int_0^l g(x) \cos \frac{n\pi x}{l} dx, \quad \text{or} \quad B_n = \frac{2}{n\pi\beta} \int_0^l g(x) \cos \frac{n\pi x}{l} dx \quad (32)$$

Therefore for the S-numbers attached at both free ends of the nerves, we have;

$$B_{n_{k_2}} = \sum_{k=1}^2 \frac{2}{kn\pi\beta} \int_0^l g(x) \cos \frac{kn\pi x}{l} dx \quad (33)$$

where S is the number of the attached free nerves at both ends.

The case of the nerve with one end fixed and the other free can

be disposed of quickly by taking the fixed end at $x = 0$ and the

free end at $x = l$, we have the two conditions:

$$u(0, t) = 0 \quad \text{and} \quad \frac{\partial u}{\partial x} \Big|_{l,t} = 0, \quad \text{for all } t.$$

Imposing the first of these upon the general product solution

$$(23) \text{ gives } C(A \cos \lambda t + B \sin \lambda t) = 0 \quad (34a)$$

Hence it follows that $C = 0$. Imposing the second then gives;

$$\frac{\lambda}{\beta} D \cos \frac{\lambda l}{\beta} (A \cos \lambda t + B \sin \lambda t) = 0 \quad (34b)$$

in which we conclude that;

$$\cos \frac{\lambda l}{\beta} = 0, \quad \frac{\lambda l}{\beta} = \frac{(2n-1)\pi}{2} \quad \text{and} \quad \lambda_n = \frac{(2n-1)\pi\beta}{2l}.$$

The general solution of the problem, formed by adding together

the product solutions corresponding to each λ_n , is therefore

$$u(x, t) = \sum_{n=1}^{\infty} \sin \frac{(2n-1)\pi x}{2l} (A_n \cos \frac{(2n-1)\pi\beta t}{2l} + B_n \sin \frac{(2n-1)\pi\beta t}{2l}) \quad (35)$$

To fit the initial displacement condition $u(x, 0) = f(x)$, we must have;

$$f(x) = \sum_{n=1}^{\infty} A_n \sin \frac{(2n-1)\pi x}{2l}. \quad (36)$$

This is not quite the usual half-range sine-expansion problem since the arguments of the various terms are not integral

multiples of the fundamental argument $\frac{\pi x}{l}$. It is however, the half-range sine expansion covered by the formula for the coefficients which was shown to be;

$$A_n = \frac{2}{l} \int_0^l f(x) \sin \frac{(2n-1)\pi x}{2l} dx \quad (37)$$

Similarly, to satisfy the initial velocity condition,

$$\frac{\partial u}{\partial t} \Big|_{x,0} = g(x), \quad \text{requires that} \quad B_n = \frac{4}{(2n-1)\pi\beta} \int_0^l g(x) \sin \frac{(2n-1)\pi x}{2l} dx \quad (38)$$

We use equation (35) to evaluate A_n and B_n for some special values of $f(x)$ and $g(x)$. Since it is a solution of (14) that satisfies the conditions (16) and (17). For the sake of simplicity we consider only the value when the initial velocity $g(x)$ is

identically zero. Then the B_{n_l} are zero and equation (35) reduces to:

$$u(x, t) = \sum_{n=1}^{\infty} A_n \sin \frac{(2n-1)\pi x}{2l} \cos \frac{(2n-1)\pi \beta t}{2l} \quad (39)$$

On application of trigonometric identities, equation (39) becomes:

$$u(x, t) = \frac{1}{2} \sum_{n=1}^{\infty} [A_n \sin \{ \frac{(2n-1)\pi}{2l} (x - \beta t) \} + A_n \sin \{ \frac{(2n-1)\pi}{2l} (x + \beta t) \}] \quad (40)$$

Since the movement of the impulse in the neurones is along the positive direction, equation (40) reduces to:

$$u(x, t) = \frac{1}{2} \sum_{n=1}^{\infty} [A_n \sin \{ \frac{(2n-1)\pi}{2l} (x - \beta t) \}] \quad (41)$$

Taking the initial displacement $f(x)$ to be unity from equation (37), with $l = 1$, we get:

$$A_n = \frac{2}{l} \int_0^l \sin \frac{(2n-1)\pi x}{2l} dx = \frac{4}{(2n-1)\pi} \quad (42)$$

Putting equation (37) into (41) and using equation (42), we get

$$u(x, t) = \frac{1}{2} \sum_{n=1}^{\infty} [(\frac{4}{(2n-1)\pi})^2 \sin \frac{(2n-1)\pi x}{2l} \sin \{ \frac{(2n-1)\pi}{2l} (x - \beta t) \}] \quad (43)$$

Equation (43) is the general solution of the equation of motion for the neural communication. A computer program designed in visual basic has been used to illustrate the solution.

Results And Discussion

The following constants have been considered in computing the numerical results for the neural communication:

β - the velocity of fluid flow from one neurone to another neurone.

l - the length of each neuron and

n - the number of neurons that are involved in the communication.

The value of $u(x, t)$ is displayed for each value of n , starting from $n=1$ to, say M , depending on the number of neurons considered. We assume for simplicity, that $\beta = 120 \text{ ms}^{-1}$ and $l = 1 \text{ m}$.

By considering the range of values of x and T as depicted in Table 1, corresponding values of u are generated. The results can be clearly seen in a 3-dimensional graph of u against X, T (Fig. 3) in which one observes a gradual folding with time along the y -axis.

Three cases of the separation constant σ were investigated and it was observed that when the separation constant $\sigma > 0$ or $\sigma = 0$ the equation of motion cannot describe the un-damped vibrations of the system because it is not periodic and has no significance in relation to the physical problem under consideration. It is only when separation constant $\sigma < 0$ that the equation describes the periodic motion. This observation is the reason why the separation gap between the neurons is not a wide gap. The $\sigma < 0$ condition leads us to the general solution of the equation of motion. Computed results plotted as a three dimensional graph shows a form of sinusoidal movement. This sinusoidal wave is the transmission of nerve impulses from one neurone to the next. This explains why the transmission of nerve impulses is not a flow of electrons as in the case of electric current.

Table 1: The Numerical Results of Data Plotted on the Graph

T-Axis Values	4.857	3.571	2.286
X-Axis Values	0.750	1.500	2.250
U-Axis Values	-52.750	-0.500	51.750

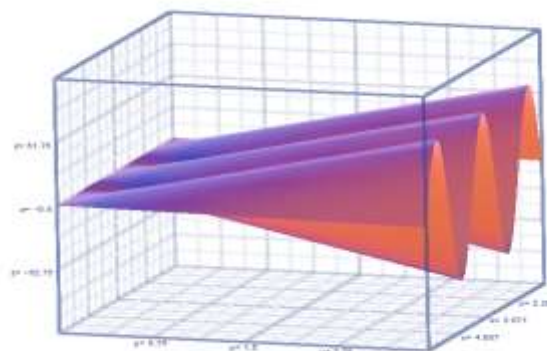


Fig 3: A Three Dimensional Graph of U against X and T in z, x, y coordinates respectively.

Conclusion

A second order partial differential equation occurring frequently in applied mathematics is the wave equation. A generalization of this equation inevitably arises in many mathematical analyses of phenomena involving the propagation of waves in continuous media, for example, the studies of acoustic waves, water waves, and such others are all based on this equation. Here, the wave differential equation has been applied to investigate and explain neural communication using the nervous system as a case study. On application of Bloch nuclear magnetic resonance theory, a model of linear one-dimensional homogeneous second order partial differential equation is derived which represents a nerve set in a vibrational motion. Results from the solution of the differential equation show that the transmission of nerve impulses is not a flow of electrons, like in the case of electric current, but a wave of electrical activity travelling along the neurone.

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