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A two-warehouse stock control model for perfect quality goods with quantity discounts

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ARTICLE INFO	
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Article history:	Stock control is an act of stocking goods to satisfy expected future demand. This gives rise
Received: 2 October 2012;	to the problem of "how much and when to place order for new goods when stock is going
Received in revised form:	down. Decision regarding the quantity ordered and the time at which it is ordered is based on
5 April 2013;	minimizing an appropriate cost function to balance the total cost function resulting from
Accepted: 11 April 2013;	overstocking or under stocking. Quantity discount stimulates the interest of the distributor to
	stock more goods and hence the need for additional warehouse which may increase his
Keywor ds	holding cost. This work has considered inventory using two warehouses-owners and rented
Warehouse,	warehouses. A model similar to that of Tien Yu Lin (2011) is developed. It has been shown
Stock,	that optimal order quality that minimizes the expected total cost is obtained. The
Quantity	consequence of the limit of the holding cost of the rented warehouse on the optimal order

size is also given.

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Introduction

Stock means "a supply of goods that is available for sale in a store/warehouse". The ability to keep goods in a place to make it available for sale is called stocking. The dynamism of demand makes it pertinent to keep goods in stock. The act of maintaining the stock has its associated costs and the lack of it makes the manufacturer/distributor lose. It is therefore often necessary to stock physical goods in order to satisfy demand over a specified time period.

The act of stocking goods to satisfy future demand gives birth to the problem of "when stock is going down, how much and when do we place order for new goods"?. This is the basic inventory problem. Stock control models have been developed to solve this inventory problem. A feature of stocking goods to meet future demand is that an increase in inventory also brings about a corresponding increase of cost of holding goods and the resultant effect of shortage cost (cost resulting from running out of goods) decreases.

Decisions regarding the quantity ordered and the time at which it is ordered may be based on minimization of an appropriate cost function which balances the total cost resulting from overstocking and under stocking. The major objective of stock control model is to find an inventory level that minimizes the sum of the shortage cost, the holding cost and other associated costs.

Some of the basic features of stock control model are; economic parameters, demand pattern of goods, ordering cycle, lead times, nature of stock replenishment, time horizon and the number of supply points. A good inventory model must take all or at least most of these features into consideration.

Types of Inventory Model

Inventory models could be deterministic or probabilistic. The single item static models, single item static model with price breaks, multiple item static model with storage limitation, single item N-period dynamic model and the N-period dynamic production scheduling model are the deterministic models in vogue. The probabilistic models include a continuous review single period model (instantaneous demand, no setup cost model), instantaneous demand setup cost models, Multi-period models (backlogging zero model), no backlog zero delivery lag model and no backlog, positive delivery lag model.

The basic decision criterion used with probabilistic model is the minimization of the expected costs/maximization of expected profit. To minimize shortage cost, there is need to expand the storage facility i.e. stock more goods. This has given rise to the need for an extended warehouse. It may become useful to rent additional warehouse to add to the one owned by the manufacturer/distributor. This will no doubt increase the holding cost but on the long run may maximize the profit of the business. The rented warehouse may be in the same location (city) with the owner warehouse or in another location.

This work develops a model for a two-warehouse stock control model with quantity discount for items with perfect quality. This research would use two storage facilities (owned and rented warehouses). Quantity discount is provided because it stimulates the interest to stock more goods by the buyer. This in turn reduces the holding cost of the manufacturer/distributor. Burwell et al (1997) posited that "in many cases quantity discounts can provide the buyer lower per unit purchase cost, lower order cost and decreased likekhood of shortages". Packala and Achary (1992) and Hanga (1998) reasoned that the retailer may employ rented warehouse to hold a large stock where he gets an attractive price discount for purchasing grain.

Assumptions And Notations

For the purpose of this work in line with general assumptions of stock control models, the following assumptions are made:

I. The owner's warehouse will not be adequate to stock the needed goods required to meet demand of customers. This gives rise to a rented warehouse. (an extended werehouse)

II. The quantity discount price applies to all units in the order quantity i.e. let c_j be the unit price of j th level, y_j the ith lowest quantity $(y_{j-1} \leq y_j)$. if $y_{j-1} \leq y_j$, then the unit price is c_j

III. The demand rate for items is known and constant.

- IV. The holding cost is higher in rented warehouse (RW) than in owners warehouse (OW). Consequently, it is advisable to use owners warehouse first while stocking then the rented warehouse after the owners warehouse is filled. During dispensing of goods, the items in rented warehouse are used before those in the owners warehouse.
- V. The transportation cost incurred between the owners warehouse and the rented warehouse is included in the holding cost of the rented warehouse.
- VI. Shortages are not allowed

Notations

D: The demand rate

- K: The ordering cost per order
- J: Discount Category

W: The storage capacity of owners warehouse

Z: the storage capacity of rented warehouse

y : The order size

n: The number of times the purchase price changes for owners warehouse in the price schedule i.e. $n = \{j/y_{j-1} \le w \le y_j\}$.

 I_r : The holding cost per unit time for owner's warehouse expressed as a fraction of naira value.

 I_w :: The holding cost per unit time for owner's warehouse expressed as a fraction of naira value

 c_j : Unit purchasing price of j th level in the price schedule t_o : Is the time to use up the inventory in rented warehouse.

Y ($^{C_{j}}$): The Economic order quantity (EOQ) under the j th level of purchasing price.

 t_r : The time at which the inventory level reaches zero in rented warehouse.

 t_s : The time at which the shortage level reaches the lowest point in the replenishment cycle.

 $N_{rw}\left(y\right)\!\!:$ Number of item in Rented warehouse

 $N_{ow}(y)$: Number of item in owners warehouse

Tc(y): The total cost per cycle of EOQ is set at y units. $T_c A$

 $Tc_{2j}^{A}(y)$: The total cost per cycle of EOQ is set at y units and

at \dot{J} th level of purchase price for condition A of case II

 $Tc_{2j}^{B}(y)$: The total cost per cycle of EOQ is set at y units and

at \mathbf{j} th level of purchase price for condition B of case II.

EC $({}^{C_{j}, y})$: The expected relevant cost of EOQ is set at y units and at j th level purchase price.

 $ECTU_{1j}^{(y)}$: The expected cost per unit time if EOQ is set at y units and at the j th level of purchase price for case I.

 $ECTU_{2i}$ (y): The expected cost per unit time if EOQ is set at y

units and at the l th level of purchase price for case II.

Model Development

Salameh and Jabber (2000) considered a model with single warehouse where they defined the number of goods/item in each order as N(p.y) = y (1-p). Tien Yu Li (2011) considered a two warehouse inventory model with imperfect quality and used defined number of good item in the same way. This paper

considers a model with two warehouses and a perfect quality. Hence in line with the two earlier papers mentioned above, this paper defines the number of item in each warehouse as follows. $N_{rw}(y) = y - w$ and

$$N_{ow}(y) = w$$

This paper would consider two cases Viz, when $t_o \ge t_w + t_r$ and when $t_o \le t_r + t_w$









If the order size is less then the storage capacity of the owners warehouse, there would be no need to rent additional warehouse to store extra items.

The average set-up cost – (Expected total demand/year (setup cost/order ______

Amount ordered/cycle The following cases are hereby considered.

Case I:

If the order size is less than storage capacity of owners warehouse, the situation is equivalent to a one warehouse inventory situation i.e.

Expected cost/unit time = $\frac{1}{Ei_j} \{ C_j \frac{D}{y} + I_w C_j \frac{E_z}{(2} + D) \}$ = $\frac{1}{E_1} \{ C_j D + \frac{KD}{y} + I_w C_j \frac{E_z}{(2} + D) \}$ Case IIA: $w \leq y \leq w + Z$ Total inventory cost = purchasing cost + set up cost + Holdingcost + shortage cost. v - w

i.e. TC_i (y) = K +
$$C_j$$
 y + I_r C_j { $\frac{y}{2D}$ + (y - w)² + I_w
 C_j { $\frac{w^2 + w(2y - w)}{2D}$
Now the cycle length is T

low the cycle leng

T(CY) \therefore Average cost/unit term becomes TCU (y) = T Similarly, the cycle length is a random variable hence by the renewal theorem of Ross 1993, the expected profit/unit time is written as

ECTU (y) = ([ECT(y))/(E[T])

$$\frac{KD}{yE} + \frac{Dc_j}{E} + I_rC_j (y-w)^2 (E + 2D) + \{I_wC_j w (E + 2D) + \{V_wC_j w (E + 2D) + (V_wC_j w (V_w) + (V_wC_j w (V_wC_j w (V_w) + (V_wC_j w (V_wC_j w (V_w) + (V_wC_j w (V_wC$$

With some conditions and algebra it is seen that the total cost in cases IIA and IIB are equal i.e. $T^{C_1}(y) = T^{C_2}(y)$

Consequently, the ECTU for cases IIA and IIB are the same ie ECTU (y) is continuous on o < w + Z.

Now, the aim is to minimize cost of keeping inventory. We need the expected total cost. This implies differencing ECTU (Y) i.e.

$$\frac{\partial}{\partial y} \underbrace{I_{1j}}_{(\text{ECTU}_{1}(y))} = \frac{I_{1j}}{E_{1j}} \underbrace{KD}_{y^{2}} I_{w}C_{j}(\text{E}[D]) \text{ and}$$

$$\frac{\partial^{2}}{\partial y^{2}} \underbrace{I_{W}C_{j}(\text{E}[D])}_{(\text{ECTU}(y))} = \frac{2KD}{E_{1}y^{3}}$$

Obviously, $\partial y = \text{ECTU} > 0$ for $0 < y \leq w$ Similarly

$$\frac{\partial^{2}}{\partial y} (ECTU_{2}(y)) = \frac{1}{E} \left\{ -\frac{Kr}{y^{2}} + I_{r}C_{j} (E_{2} + 2D) \left(\frac{y^{2} - w^{2}}{2^{y^{2}}} \right) - \frac{I_{w}C_{j}}{2y^{2}} (ECTU_{2}(y)) = \frac{2KD}{E_{1}y^{2}} + C_{j}w^{2} (I_{r} (E_{2} + 2D) + I_{w})$$

$$\frac{\partial^{2}}{\partial y^{2}} (ECTU_{2}(y)) = \frac{2KD}{E_{1}y^{2}} + C_{j}w^{2} (I_{r} (E_{2} + 2D) + I_{w})$$

$$\frac{\partial^{2}}{\partial y^{2}} (ECTU_{2}) > 0 \text{ for } w \leq y \leq w + Z$$

$$\frac{\partial^{2}}{\partial y^{2}} (ECTU_{2}) > 0 \text{ for } w \leq y \leq w + Z$$
This implies that $\frac{\partial^{2}}{\partial y^{2}}$ (ECTU_{1}) and $\frac{\partial^{2}}{\partial y^{2}}$ (ECTU_{2}) are positive

It is also obvious that (*) above is greater than zero because $\overline{E_1}$ \geq 1 and $I_r > I_w$. It is also pertinent to note that the unit cost of purchasing item is dependent on the ordering quantity and may hence give rise to different cost curve. Consequently y* could be calculated from

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$$\frac{\partial}{\partial y} \frac{\partial}{(\text{ECTU}_1) \text{ or } \partial y} \text{ (ECTU}_2).$$

$$\therefore \text{ if } y_{1j}^* = \text{Lowest point on each curve } C_j \text{ under } J < n \text{ and } y_1 2j^* (= \text{ the lowest point of each cost curve } C_j \text{ under } j \ge n$$

$$\frac{\partial}{\partial y_j} = \frac{\partial}{\partial y_j} \text{ (ECTU}_2).$$

Equating ∂y (ECTU₁ and $\partial / \partial y$ (ECTU₂) to zero gives

$$y_{1}1j^{\dagger}(\sqrt{\frac{2KD}{I_{w}C_{j}}}, (E_{2} + 2D)) \text{ where }$$

$$j = 1, 2, \dots, \dots, \dots, n.$$

$$y_{1}2j^{\dagger}(\sqrt{\frac{2KD}{I_{w}C_{j}}}, (E_{2} + 2D)) = 2D(I_{1}(r + I_{1}w)) + E_{1}2(I_{1}(r - I_{1}w))$$

$$= 2KD + C_{1}(jw^{\dagger}2) [2D(I_{1}(r + I_{1}w)) + E_{1}2(I_{1}(r - I_{1}w))]$$

 $I_r C_j (E_2 + 2\mathbf{D})$

To obtain the optimal ordering quantity that minimizes ECTU(y), the objective function is redefined with respect to minimizing the expected relevant cost as

$$\underset{\substack{\text{EC} \ (C_{\mathbf{j}}j, y) \\ < y < w \text{ and } J - 1, 2, \dots, n \\ \frac{KD}{yE_{\mathbf{1}}} + \frac{DC_{j}}{E_{\mathbf{1}}} + I_{r}C_{j} \frac{(y - [w])^{2}(E_{2} + 2\mathbf{D})}{2yE_{\mathbf{1}}} + I_{r}C_{j} \frac{(y - [w])^{2}(E_{2} + 2\mathbf{D})}{2yE_{\mathbf{1}}} + I_{w}C_{j}w(E_{2}(2\mathbf{y} - \mathbf{w}) + 2\mathbf{D}\mathbf{W}) }$$

 $z_{j \in 1} = 1, n + 1, ..., m$ Observe that in the equation for the optional order size i.e.

$$\frac{y_{\mathbf{1}_{j}}}{y_{\mathbf{2}_{j}}} \sqrt{\frac{z_{KD}}{\sqrt{U_{k}C_{j}(E_{2} + 2\mathbf{D})}}}$$

where J = 1, 2 - n
$$\frac{y_{\mathbf{2}_{j}}}{\sqrt{U_{k}}} \sqrt{\frac{z_{KD}}{\left[2D\left(I_{r+I_{w}}\right) + E_{2}\left(I_{r-I_{w}}\right)\right]}}}{\frac{1_{r}C_{j}(E_{2} + 2\mathbf{D})}}{I_{r}C_{j}(E_{2} + 2\mathbf{D})}}$$

(a) As the holding cost of the rented warehouse gets small i.e. $I_r \rightarrow 0,$

$$\lim_{i.e} I_r \rightarrow {}_{0}y_{\mathbf{2}j} = \sqrt{\frac{2KD}{I_w C_j (E_2 + 2\mathbf{D})}}$$

 $\lim \mathcal{Y}_{2j} \to \infty$

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 $I_r \rightarrow 0$

The buyer tends to stock more goods

(b) If
$$I_r = \sqrt{I_w, \text{ then } \mathcal{Y}_{2j}^A} = (2KD + 4w^{\dagger}2D)/(|I_w C_j j (E_12 + 2W)^2)$$

Subsequently, as the unit purchasing price gets small i.e

$$\lim_{Lim} y_{1}ij^{\dagger}(\rightarrow \infty) \quad \lim_{Lim} y_{1}2j^{\dagger}(\rightarrow C_{i} \rightarrow \mathbf{0}_{i})$$

i.e more goods would be stocked.

Conclusion

The work has considered two warehouse stock models for perfect quality goods with quantity discount. The paper has used the work of Render et al (2003), Maddah and Jabber (2008) and the Tien T. Y (2011) as basis to derive the optimal ordering quantity \mathcal{Y}_{ij} . The behaviour of the optimal order quantity with respect to corresponding charges in holding cost and unit purchasing price was considered.

References

Burwell T. H et al (1997): Economic Lot size Model for price dependent demand under quality and freight discount, international Journal of Production Economies 48:141-155.

Chang H . C (2004) An application of Fuzzy sets theory to the EOQ model with imperfect quality production process. Journal of computer and industrial Engineering 56: 193 - 197

Flynn J and Garstka S. (1997): The Optimal review period in a dynamic Inventory model. Journal of operation research vol 45 205, 736-750, + A20) Warehouse inventory Model for item

Lin /I . Y (2011): A Wo Warehouse inventory Model for item with Imperfect quality and quantity discount pacific Asia-Pacyic. Journal of Operation research Vo. 28, No 2 147 – 16.

Maddeh B and Jaber M. Y (2008) Economic order quantity for item with imperfect quality: (Revisited), International Journal of Production Economic 808- 815

Moinzodeh K. and Aggarwel P. K (1997): An information based multiechelon inventory system with emergency orders. Journal of operation Research vol. No. 5, 694 - 701

Pakkala T. M and Achary K. K (1992): Discrete Time inventory model for deteriorating times with two warehouses. Operation research Journal 29: 891-103

Render B. et al (2003): Quantitative Analysis for management New Jersey prentice Hall Publishers

Salameh, M. K and Jaber M. Y (2000) EOQ model for items with imperfect quality. International Journal of Production, Economics 64, 59-84

Taha, H. A Operations research. An Introduction Second edition. Macmillan Publishing co. Inc New York.