# Single Network structure for Stuck-at and bridging fault Analysis and Diagnosis for Exclusive-OR Sum of Products in Reed-Muller Canonical Circuits 

Geetha V, Devarajan N and Neelakantan P. N
Department of Electrical Engineering, Government College of Technology, Coimbatore, India.

## ARTICLE INFO

## Article history:

Received: 10 January 2013;
Received in revised form:
17 March 2013;
Accepted:3 April 2013;

## Keywords

Reed-Muller Canonical Form,
Exclusive-OR Sum of Products, Testable Realization,
Single Stuck-at,
Double stuck-at,
AND-bridging fault,
OR-bridging fault.


#### Abstract

In this paper, a testable design with good fault identification capability is used for analysis and diagnosis of stuck-at and bridging faults in Exclusive-OR Sum of Product Reed-Muller canonical circuits, independent of the function for a given number of inputs. Factors of identifiability and distinguishability have been defined and determined. Further, a compact method of representing the circuit outputs has been adopted for ease of tabulation and comparison. Simulations of Single stuck-at, Double stuck-at, OR-bridging and ANDbridging faults for a few random functions have been carried out through MATLAB coding. From the test results, it was found that the fault detection for the set of random functions was more than $95 \%$ for most of the functions except few cases, with just $n+5$ test vectors compared to $2^{\mathrm{n}}$ test vectors required for conventional testing. The location of the fault can also be diagnosed through the output sets.


## 1. Introduction

The faults in digital circuits can be classified broadly as Single stuck-at-faults, Multiple stuck-at-faults, Stuck-open faults, stuck-on faults, Bridging faults, Path delay faults, Transient faults etc. Any arbitrary logic function, in general, can be expressed in Reed-Muller Canonical (RMC) form as
$\mathrm{F}=\left(\mathrm{a}_{0} \oplus \mathrm{a}_{1} \mathrm{x}_{1} * \oplus \mathrm{a}_{2} \mathrm{x}_{2} * \oplus \ldots \oplus \mathrm{a}_{\mathrm{n}} \mathrm{x}_{\mathrm{n}} * \oplus \mathrm{a}_{n+1} \mathrm{x}_{1} * \mathrm{x}_{2} * \oplus \ldots \oplus \mathrm{a}_{\mathrm{m}} \mathrm{x}_{1} *\right.$ $\mathrm{x}_{2}{ }^{*} \ldots \mathrm{x}_{\mathrm{n}}{ }^{*}$ )
where, $\mathrm{x}_{\mathrm{n}}{ }^{*}$ can be $\mathrm{x}_{\mathrm{n}}$ or its complement, $\mathrm{a}_{\mathrm{n}}$ is either 0 or 1 and m $=2^{\mathrm{n}}-1$. However, there can be variations in such forms. The different types are Fixed Polarity RMC (FPRM), Positive Polarity RMC (PPRM), Generalised RMC (GRM) and Exclusive-OR Sum-of-Products (ESOP). The FPRM has a restriction that the variables in any of the product terms have to be of the same type namely complementary or noncomplementary. For PPRM, the complementary form of variables is not allowed. The GRM may contain both complementary and non-complementary type but the combination of the variables should be unique. The ESOP form does not have any such restriction. Also the ESOP form has the least number of product terms and hence needs the least number of AND gates and is very much suitable for hardware implementation.

Extensive research has been carried out in the field of testing of digital circuits to reduce the number of input vectors. The cardinality of the test vectors proposed by many authors becomes prohibitively excessive for large number of input variables. It was demonstrated that Single stuck-at fault detection can be achieved with only $n+5$ test vectors [6]. The same structure was extended for OR-bridging fault analysis [15] and [16]. In this paper, it is shown through Matlab simulations for a few specific functions that Single stuck-at, Double stuck-at,

OR-bridging and AND-bridging fault detection and diagnosis could also be achieved with the same $\mathrm{n}+5$ test vectors considering all input lines, control lines and intermediate gate outputs.

Two quantitative indices, called identifiability factor and distinguishability factor are considered for comparison of the testability nature of given circuits. The identifiability factor is defined as the ratio of the number of faults correctly identified by the test set to the total number of possible faults of the type considered. The existence of faults can be recognized from the set of outputs measured which will be different from the faultfree circuit. The distinguishability factor pertains to the identical set of outputs among different faults, but the output set of each being very much different from the non-faulty case. The existence of even a large percentage of indistinguishability may not mean the circuit is not reliable, since it is still possible to identify the faulty condition of the circuit and take appropriate remedial action. The set of binary values for an output is converted into its decimal equivalent for convenience in comparison and ease of tabulation.

## 2. Literature Survey

A PPRM network for detection of stuck-at faults with a universal test set of size $n+4$, $n$ being the number of data inputs, was proposed in [1]. Though quite good for self-testing, the method is economical only for the specified form, which obviously has more number of product terms than the other forms in most cases. Multiple stuck-at fault detection for ESOP circuits was carried out in [2]. However, since the cardinality is $2 \mathrm{n}+6+\sum \mathrm{nC}_{\mathrm{e}}, \mathrm{e}=0$ to j , the order of ESOP expression, the test set is not universal and also is too large to be practical for large input functions.

Stuck-at and bridging faults with a universal test set for PPRM network has been reported in [3]. Multiple fault detecting GRM realizations was propounded in [4]. It was shown that $2 n+s+3$ test vectors, where $s$ is the number of product terms in the logic function are required for Single stuck-at fault detections in GRM / ESOP circuits while $2 \mathrm{n}+\mathrm{s}$ vectors are required for detection of AND/OR-bridging faults in such circuits [7]. Here too, the test set is not universal as it depends on s , the number of product terms of the function. [5] described an ESOP implementation with a universal test set of size $n+6$ for single stuck-at faults only. [6] demonstrated that Single stuck-at fault detection can be achieved with only $\mathrm{n}+5$ minimal test vectors. [9, 10] proved that a test sequence of length $(2 n+8)$ vectors is sufficient to detect all Single stuck-at and bridging faults. Two methods, each with a small modification in this scheme with ESOP RMC circuits had been proposed for analysis and diagnosis of Single stuck-at faults [11, 12]. It was proved that test vectors for multiple fault detection and diagnosis in digital circuits could be generated using Neural Network with different training algorithms [8, 13, 18]. In [15, 16] it was demonstrated how the suitable RMC forms help in the detection of various digital faults. [19] proposed a new test pattern generation algorithm using Neural Network which requires additional gates. The analysis and diagnosis of OR-bridging faults in any of the pairs of data and control lines and ORbridging faults including intermediate gate outputs of the ESOP RMC circuits was proposed in [20, 21].

This paper is an extension of [16], and discusses the analysis and diagnosis of Single stuck-at, Double stuck-at and AND-bridging faults including the intermediate gate outputs of the ESOP RMC circuits.

## 3. Materials and methods

### 3.1 Network Structure:

The network structure of the scheme is the same as that proposed in [6] and is shown in Figure.1. It comprises literal complementing XOR block, an AND block, an XOR function tree block, which implements the required logic function as also two additional outputs $\mathrm{O}_{1}$ and $\mathrm{O}_{2}$ obtained through a separate AND gate and an OR gate. The actual data inputs to the system are $\mathrm{x}_{1}, \mathrm{x}_{2} \ldots . \mathrm{x}_{\mathrm{n}}$. Additionally, the scheme requires four control inputs $c_{1}$ to $c_{4}$. The literal-complementing block produces the complements of the literals used in the function. Only those literals appearing in complemented form require an XOR gate in this block.


Figure 1. Generalized Network Structure

The literals of each product term are combined through an AND gate and hence the number of AND gates required is the same as the number of product terms in the logic function. Further, each of the AND gates of this block has an additional input from one of the control lines depending on the number of gates used in the XOR tree block producing the final function F . Finally, all the data and complementary gate outputs are applied to a separate AND gate and an OR gate, producing auxiliary outputs $\mathrm{O}_{1}$ and $\mathrm{O}_{2}$, to aid in the detection of faults which cannot be differentiated by the main function output F alone.


## Figure. 2. Control Input Determination

The required control lines are determined as illustrated above (Figure 2). Draw the XOR gate tree for the required product terms of the given function. Assign the numerals 1,2 and 3 respectively to the two inputs and the output of the final XOR gate producing the function output F. Consider each XOR gate connected to the inputs of the final XOR gate considered. Assign the outputs of these XOR gates with the same numbers as the inputs of the final XOR gate. If the output of the XOR gate considered is 1 , then assign 2 and 3 to its inputs. Else if the output is numbered 2 , assign 3 and 1 to its input. Now consider the next earlier input stage and assign the numerals in the similar manner according to the output points connected.

### 3.2 Test Vectors:

The test set has $(n+5)$ vectors; each of the vectors is $(n+4)$ long, ' $n$ ' being the number of data inputs. The first four columns of the matrix represent the control inputs $c_{1}$ to $c_{4}$ while the remaining $n$ columns that of the data inputs are $x_{1}$ to $x_{n}$. The generalized test set is shown in Table 1.

Table 1. Generalized Test Set



Figure. 3. Circuit for $F=x_{1} \oplus X_{2} x_{3} \oplus x_{1}{ }^{\prime} \mathbf{x}_{2} \mathbf{x}_{3}$
The network structure and the set of test vectors for the function $\mathrm{F}=\mathrm{x}_{1} \oplus \mathrm{x}_{2} \mathrm{x}_{3} \oplus \mathrm{x}_{1}{ }^{\prime} \mathrm{x}_{2} \mathrm{x}_{3}$ are shown in Figure. 3 and Table 2 respectively.

Table 2. Test vectors for $F=x_{1} \oplus x_{2} x_{3} \oplus x_{1}{ }^{\prime} x_{2} x_{3}$

$$
\mathrm{T}=\left\{\begin{array}{ccccccc}
\mathrm{c}_{1} & \mathrm{c}_{2} & \mathrm{c}_{3} & \mathrm{c}_{4} & \mathrm{x}_{1} & \mathrm{x}_{2} & \mathrm{x}_{3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 0 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 0 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

### 3.3 Algorithm

Step 1: Set up the circuit as in Figure.2.
Step 2: Determine and connect the control lines $c_{1}$ to $c_{4}$ as explained.
Step 3: Apply the test vectors as given in Table 2, one by one.
Step 4: For each test vector, determine the fault free outputs F, $\mathrm{O}_{1}$ and $\mathrm{O}_{2}$.
Step 5: Obtain the decimal equivalents of each of the above binary output sets.
Step 6: Simulate the Single stuck-at fault at the control input, data input and intermediate gate outputs and get the corresponding decimal outputs.
Step7: Compare the set of outputs with the predetermined faultfree condition outputs
Step 8: If the two output sets match exactly, it implies that a fault, if present, is not identifiable or detectable; else, the fault is a detectable one.
Step 9: Repeat steps 4 to 8 for Double stuck-at, OR-bridging and AND-bridging fault for other possible combination pairs of control inputs, data inputs and intermediate gate outputs in the network.
Step 10: For all the faults, the identifiability factor and distinguishability factor are calculated as explained above.

## 4. Results and Discussion

The following ten random functions were considered and Single stuck-at, Double stuck-at, OR-bridging and ANDbridging faults are simulated using MATLAB coding and the results are tabulated in Table 5, 6, 8 and 9.
$\mathrm{F}_{1}=\mathrm{x}_{1} \oplus \square \mathrm{x}_{2} \mathrm{x}_{3} \oplus \square \square \mathrm{x}_{1}{ }^{\prime} \mathrm{x}_{2} \mathrm{x}_{3}$
$\mathrm{F}_{2}=\mathrm{x} 1 \mathrm{x} 2 \oplus \square \mathrm{x}_{2}{ }^{\prime} \mathrm{x}_{3} \oplus \square \square \mathrm{x}_{3}{ }^{\prime} \mathrm{x}_{4} \oplus \square \mathrm{x}_{1} \mathrm{x}_{2} \mathrm{x}_{3}$
$\mathrm{F}_{3}=\mathrm{xl}^{\prime} \oplus \square \square \mathrm{x}_{2} \mathrm{x}_{3}{ }^{\prime} \mathrm{x}_{4} \oplus \square \square \mathrm{x}_{3} \mathrm{x}_{4}{ }^{\prime} \oplus \square \square \mathrm{x}_{2}{ }^{\prime} \mathrm{x}_{3} \oplus \square \square \mathrm{x}_{1} \mathrm{x}_{4} \mathrm{x}_{5}$
$\mathrm{F}_{4}=\mathrm{x}_{1} \mathrm{X}_{2}$,
$\oplus \square \square \mathrm{x}_{2} \mathrm{x}_{3} \mathrm{x}_{4} \oplus \square \mathrm{x}_{4} \mathrm{x}_{5}{ }^{\prime} \mathrm{x}_{6} \oplus \square \square \mathrm{x}_{2} \mathrm{x}_{5} \square \oplus \square \square \mathrm{x}_{2}{ }^{\prime} \mathrm{x}_{5}{ }^{\prime} \oplus \square \square \mathrm{x}_{3}{ }^{\prime} \mathrm{x}_{2} \mathrm{x}_{1} \oplus \square \square \mathrm{x}_{4} \mathrm{x}_{6}$
$\mathrm{F}_{5}=\mathrm{x}_{1}{ }^{\prime} \mathrm{x}_{2} \mathrm{x}_{3} \oplus \square \square \mathrm{x}_{4} \mathrm{X}_{5} \mathrm{x}_{6} \square \oplus \square \square \mathrm{x}_{4}{ }^{\prime} \mathrm{x}_{6}{ }^{\prime} \mathrm{x}_{7} \square \oplus \square \square \mathrm{x}_{3} \mathrm{x}_{5} \mathrm{x}_{7}$
$\mathrm{F}_{6}=\mathrm{x}_{1} \mathrm{x}_{2}{ }^{\prime} \mathrm{x}_{3} \oplus \square \square \mathrm{x}_{4}{ }^{\prime} \mathrm{x}_{5} \mathrm{x}_{6}{ }^{\prime} \square \oplus \square \square \mathrm{x}_{7} \mathrm{x}_{8}{ }^{\prime} \square \square \oplus \square \square{ }^{\prime}$
$\oplus \square \square \mathrm{x}_{1}{ }^{\prime} \mathrm{x}_{6} \square \oplus \square \square \mathrm{x}_{3}{ }^{\prime} \mathrm{x}_{4} \oplus \square \square \mathrm{x}_{1} \mathrm{X}_{5} \square$
$\oplus \square \mathrm{x}_{4} \mathrm{x}_{5}{ }^{\prime} \square \oplus \square \square \mathrm{x}_{5} \mathrm{x}_{7} \square \oplus \square \square \mathrm{x}_{8} \mathrm{x}_{3} \mathrm{x}_{1} \square \oplus \square \square \mathrm{x}_{3} \mathrm{x}_{5}{ }^{\prime} \mathrm{x}_{8}$
$\mathrm{F}_{7}$
$\mathrm{X}_{1} \mathrm{X}_{2}{ }^{\prime} \mathrm{x}_{3}{ }^{\prime}$
$\oplus \square \square \mathrm{x}_{4} \mathrm{x}_{5}{ }^{\prime} \mathrm{x}_{6} \oplus \square \square \mathrm{x}_{7}{ }^{\prime} \mathrm{x}_{8} \mathrm{x}_{9} \square \oplus \square \square \mathrm{x}_{1}{ }^{\prime} \mathrm{x}_{4}{ }^{\prime} \mathrm{xg}_{9} \square \square \oplus \square \mathrm{x}_{2} \mathrm{x}_{5}{ }^{\prime} \square \oplus \square \square \mathrm{x}_{3} \mathrm{x}_{5}$
$\mathrm{F}_{8} \quad=\quad \mathrm{x}_{1}{ }^{\prime}{\mathrm{x} 2 \mathrm{X}_{3}}{ }^{\prime} \quad \oplus \square \square \mathrm{x}_{4}{ }^{\prime} \mathrm{x}_{5}{ }^{\prime} \mathrm{X}_{6} \oplus \square \square \mathrm{x}_{7} \mathrm{x}_{8}{ }^{\prime} \mathrm{x}_{9}{ }^{\prime}$
$\oplus \square \square \mathrm{x}_{10} \square \oplus \square \square \mathrm{x}_{6}{ }^{\prime} \mathrm{x}_{7} \oplus \square \square \mathrm{x}_{8} \mathrm{x}_{10}$
F9
$\mathrm{x}_{1} \oplus \square \square \mathrm{x}_{2}{ }^{\prime} \mathrm{x}_{3} \mathrm{x}_{4}{ }^{\prime} \oplus \square \square \mathrm{x}_{5}{ }^{\prime} \mathrm{x}_{6} \mathrm{x}_{7}{ }^{\prime} \oplus \square \square \mathrm{x}_{8} \mathrm{x}_{9} \mathrm{x}_{10} \square \oplus \square \square \mathrm{x}_{10}{ }^{\prime} \mathrm{x}_{11} \square \oplus \square \square \mathrm{x}_{1} \mathrm{x}_{3} \mathrm{x}_{9}$ $\mathrm{F}_{10}=\mathrm{x}_{1}{ }^{\prime} \mathrm{x}_{2} \oplus \square \square \mathrm{x}_{3} \mathrm{x}_{4}{ }^{\prime} \mathrm{x}_{5} \oplus \square \square \mathrm{x}_{6} \mathrm{x}_{7}{ }^{\prime} \mathrm{x}_{8} \mathrm{x}_{9} \oplus \square \square \mathrm{x}_{10} \mathrm{x}_{11}{ }^{\prime} \mathrm{x}_{12} \square \oplus \square \square \mathrm{x}_{1} \mathrm{x}_{2} \mathrm{x}_{3}{ }^{\prime} \square \oplus$

## - $\mathrm{XX}_{4} \mathrm{x}_{7}$

### 4.1 Single Stuck-at fault

As an illustration, a three variable function
$\mathrm{F}_{1}=\mathrm{x}_{1} \oplus \square \mathrm{x}_{2} \mathrm{x}_{3} \oplus \mathrm{x}_{1}{ }^{\prime} \mathrm{x}_{2} \mathrm{x}_{3}$ is considered. The simulated fault-free output set was found to be $\left\{\mathrm{F}, \mathrm{O}_{1}, \mathrm{O}_{2}\right\}=\{126,112,127\}$. The stuck-at-0 and stuck-at-1 faults are simulated for the given function at lines $\mathrm{c}_{1}, \mathrm{c}_{2}, \mathrm{c}_{3}, \mathrm{c}_{4}, \mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3}, \mathrm{zl}_{1}, \mathrm{za}_{1}, \mathrm{za}_{2}, \mathrm{za}_{3}, \mathrm{zx}_{1}$ and $\mathrm{zx}_{2}$ and the results are shown in Table 3 and 4.

Table 3. Stuck-at-0 for 3 variable functions

|  | $\mathrm{c}_{1}$ | $\mathrm{c}_{2}$ | $\mathrm{c}_{3}$ | $\mathrm{c}_{4}$ | $\mathrm{x}_{1}$ | $\mathrm{x}_{2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| F | 126 | 126 | 120 | 6 | 120 | 86 |
| $\mathrm{O}_{1}$ | 112 | 112 | 112 | 112 | 0 | 0 |
| O 2 | 126 | 127 | 127 | 127 | 127 | 127 |


|  | $\mathrm{x}_{3}$ | $\mathrm{z}_{1}$ | $\mathrm{za}_{1}$ | $\mathrm{za}_{2}$ | $\mathrm{za}_{3}$ | $\mathrm{zx}_{1}$ | $\mathrm{zx}_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F | 86 | 46 | 40 | 6 | 46 | 80 | 0 |
| $\mathrm{O}_{1}$ | 0 | 0 | 112 | 112 | 112 | 112 | 112 |
| $\mathrm{O}_{2}$ | 127 | 126 | 127 | 127 | 127 | 127 | 127 |

Table 4. Stuck-at-1 for 3 variable functions

|  | $\mathrm{c}_{1}$ | $\mathrm{c}_{2}$ | $\mathrm{c}_{3}$ | $\mathrm{c}_{4}$ | $\mathrm{x}_{1}$ | $\mathrm{x}_{2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| F | 38 | 126 | 126 | 126 | 126 | 126 |
| $\mathrm{O}_{1}$ | 0 | 112 | 112 | 112 | 120 | 116 |
| O 2 | 255 | 127 | 127 | 127 | 255 | 255 |


|  | $\mathrm{x}_{3}$ | $\mathrm{z} \mathrm{l}_{1}$ | $\mathrm{za}_{1}$ | $\mathrm{za}_{2}$ | $\mathrm{za}_{3}$ | $\mathrm{zx}_{1}$ | $\mathrm{zx}_{2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| F | 126 | 118 | 215 | 249 | 209 | 175 | 255 |
| $\mathrm{O}_{1}$ | 114 | 112 | 112 | 112 | 112 | 112 | 112 |
| O 2 | 255 | 255 | 127 | 127 | 127 | 127 | 127 |

The total number of possible single stuck-at faults are $2^{*}(\mathrm{nc}+\mathrm{nx}+\mathrm{zl}+\mathrm{za}+\mathrm{zx})=2^{*} 13=26$.
Where $n x$ is the number of data inputs
nc is the number of control inputs
zl is the number of complementary functions
$z a$ is the number of AND gate outputs
zx is the number of XOR gate outputs
For stuck-at-0 fault at $\mathrm{c}_{2}$ and for stuck-at-1 fault at $\mathrm{c}_{2}, \mathrm{c}_{3}$ and $c_{4}$ the output sets obtained are same as that of fault free one. Hence, these faults are unidentifiable. The identifiability factor $=$ $(26-4) / 26100=84.62 \%$. Also, in stuck-at-0 fault the output set
$\{6,112,127\}$ is repeated two times for $\mathrm{c}_{4}$ and $\mathrm{za}_{2}$ and
$\{86,0,127\}$ is repeated two times for $x_{2}$ and $x_{3}$.

These faults are indistinguishable. The distinguishability factor for this set is $(26-4) / 26 * 100=84.62 \%$

The simulated results for the ten random functions are tabulated in Table 5.

Table 5. Simulation results for Single stuck -at fault

| $\begin{aligned} & \dot{\circ} \\ & \dot{\sim} \\ & \dot{n} \end{aligned}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\mathrm{F}_{1}$ | 3 | 26 | 84.62 | 84.62 |
| 2 | $\mathrm{F}_{2}$ | 4 | 34 | 97.06 | 82.35 |
| 3 | $\mathrm{F}_{3}$ | 5 | 44 | 95.45 | 95.45 |
| 4 | $\mathrm{F}_{4}$ | 6 | 54 | 96.30 | 92.59 |
| 5 | $\mathrm{F}_{5}$ | 7 | 42 | 97.62 | 73.81 |
| 6 | $\mathrm{F}_{6}$ | 8 | 82 | 96.34 | 97.56 |
| 7 | $\mathrm{F}_{7}$ | 9 | 62 | 98.39 | 83.87 |
| 8 | $\mathrm{F}_{8}$ | 10 | 64 | 98.44 | 78.13 |
| 9 | $\mathrm{F}_{9}$ | 11 | 62 | 98.39 | 79.03 |
| 10 | $\mathrm{F}_{10}$ | 12 | 64 | 98.44 | 78.13 |
| Average | 96.11 | 84.55 |  |  |  |

### 4.2 Double Stuck-at faults

Double Stuck-at faults can occur quite easily due to the shorting of any two of the lines, especially the adjacent lines of the circuit.

The network structure and test vectors are the same as those for the single stuck-at fault. However, in the test procedure, two lines at a time are considered and made to stuck-at-0 or stuck-at1 and simulated. Since two lines are involved, four possible combinations, viz. $(0,0),(0,1),(1,0)$ and $(1,1)$ are simulated and tabulated in Table 6.

Table 6. Simulation results for Double stuck -at fault

| $\begin{aligned} & \dot{8} \\ & \dot{n} \end{aligned}$ | $\begin{array}{r} \text { 気 } \\ 0 \\ \hline \end{array}$ |  | $\begin{aligned} & \frac{0}{0} \\ & \text { ? } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\mathrm{F}_{1}$ | 3 | 312 | 98.40 | 30.13 |
| 2 | $\mathrm{F}_{2}$ | 4 | 544 | 100 | 31.25 |
| 3 | $\mathrm{F}_{3}$ | 5 | 924 | 99.89 | 33.98 |
| 4 | $\mathrm{F}_{4}$ | 6 | 1404 | 99.93 | 33.26 |
| 5 | $\mathrm{F}_{5}$ | 7 | 840 | 100 | 29.64 |
| 6 | $\mathrm{F}_{6}$ | 8 | 3280 | 100 | 32.50 |
| 7 | $\mathrm{F}_{7}$ | 9 | 1860 | 100 | 32.80 |
| 8 | $\mathrm{F}_{8}$ | 10 | 1984 | 100 | 31.50 |
| 9 | $\mathrm{F}_{9}$ | 11 | 1860 | 100 | 33.12 |
| 10 | $\mathrm{F}_{10}$ | 12 | 1984 | 100 | 31.10 |
| Average |  |  |  | 99.82 | 31.93 |

### 4.3 AND-Bridging Faults

The bridging faults are considered as a special case of multiple faults. The AND-bridging fault is simulated by shorting two lines at a time. A detailed numerical illustration for three variable AND-bridging faults is given below.
Function considered: $\mathrm{F}_{1}=\mathrm{x}_{1} \oplus \square \mathrm{x}_{2} \mathrm{x}_{3} \oplus \square \mathrm{x}_{1}{ }^{\prime} \mathrm{x}_{2} \mathrm{x}_{3}$
Fault-free output set $\left\{\mathrm{F}, \mathrm{O}_{1}, \mathrm{O}_{2}\right\}=\{126,112,127\}$
The outputs of AND-bridging faults at lines $c_{1}$ in combination with $\mathrm{c}_{2}, \mathrm{c}_{3}, \mathrm{c}_{4}, \mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3}, \mathrm{zl}_{1}, \mathrm{za}_{1}, \mathrm{za}_{2}, \mathrm{za}_{3}, \mathrm{zx}_{1}$ and $\mathrm{zx}_{2}$ are tabulated in Table 7.

Table 7. Simulation Results for a few Random Logic Functions

|  | $\mathrm{c}_{1} \mathrm{c}_{2}$ | $\mathrm{c}_{1} \mathrm{c}_{3}$ | $\mathrm{c}_{1} \mathrm{c}_{4}$ | $\mathrm{c}_{1} \mathrm{x}_{1}$ | $\mathrm{c}_{1} \mathrm{X}_{2}$ | $\mathrm{c}_{1} \mathrm{x}_{3}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| F | 126 | 120 | 6 | 120 | 86 | 86 |
| $\mathrm{O}_{1}$ | 112 | 112 | 112 | 0 | 0 | 0 |
| $\mathrm{O}_{2}$ | 126 | 126 | 126 | 126 | 126 | 126 |


|  | $\mathrm{c}_{1} \mathrm{zl}$ | $\mathrm{c}_{1} \mathrm{zal}$ | $\mathrm{c}_{1} \mathrm{za} 2$ | $\mathrm{c}_{1} \mathrm{za3}$ | $\mathrm{c}_{1} \mathrm{zx} 1$ | $\mathrm{c}_{1} \mathrm{Zx}_{2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| F | 46 | 40 | 6 | 46 | 80 | 0 |
| $\mathrm{O}_{1}$ | 0 | 112 | 112 | 112 | 112 | 112 |
| $\mathrm{O}_{2}$ | 127 | 126 | 126 | 126 | 126 | 126 |

Control inputs: $\mathrm{c}_{1}$ to $\mathrm{c}_{4}$; Data inputs: $\mathrm{x}_{1}$ to $\mathrm{x}_{3}$
Complementary outputs: zl1; AND gate outputs: $\mathrm{za}_{1}$, za2, $\mathrm{za}_{3}$ XOR gate outputs: zx1 \& zx2
Total No. of Fault location pair combinations
$=(\mathrm{nc}+\mathrm{nx}+\mathrm{zl}+\mathrm{za}+\mathrm{zx}) \mathrm{C}_{2}=13 \mathrm{C}_{2}=78$
Total number of possible bridging faults for the given three variable function used is 78 .

When the Post fault outputs are identical as of fault free one, then those faults are termed as unidentifiable faults. From the simulation results it was found that the number of unidentifiable faults as 11 .
$\therefore$ The Identifiability Factor is $(78-11) / 78 \times 100=85.90 \%$.
When the post fault outputs are same for different combinations of faults, then those faults are termed as Indistinguishable faults. For the given example, the output sets that get repeated are as follows:

| $\{6,112,126\}$ | 2 times |
| :--- | :--- |
| $\{62,112,127\}$ | 5 times |
| $\{80,112,127\}$ | 2 times |
| $\{80,112,127\}$ | 2 times |
| $\{86,0,126\}$ | 2 times |
| $\{86,80,127\}$ | 2 times |
| $\{86,112,127\}$ | 5 times |
| $\{94,112,127\}$ | 2 times |
| $\{118,112,126\}$ | 5 times |
| $\{118,112,127\}$ | 2 times |
| $\{120,112,127\}$ | 4 times |
| $\{126,48,127\}$ | 3 times |
| $\{126,80,126\}$ | 3 times |
| $\{126,80,127\}$ | 2 times |

Thus totally repetition occurs for 41 fault location combinations. Hence overall distinguishability factor is
$(78-41) / 78 \times 100=47.44 \%$
However, when the individual cases are considered the distinguishability factor can be seen to be appreciably high as seen below:
Same output set of $\{120,112,127\}$ for the following fault combinations.
AND-bridging fault at $\mathrm{c}_{3}, \mathrm{za}_{3}$ lines
AND-bridging fault at $\mathrm{za}_{1}, \mathrm{za}_{3}$ lines
AND-bridging fault at $\mathrm{za}_{2}, \mathrm{Zx}_{1}$ lines
AND-bridging fault at $\mathrm{za}_{2}, \mathrm{zx}_{2}$ lines
The distinguishability for this set is
$(78-4) / 78 \times 100=94.87 \%$.
Similarly, the output set $\{126,80,126\}$ occurs 3 times, for which the distinguishability factor is
$(78-3) / 78 \times 100=96.15 \%$.
Further, the location of fault can also be easily diagnosed from the output set. For instance if the output set is $\{120,112$, 127\} then the fault condition would be one of the four cases discussed above involving $\mathrm{c}_{3}, \mathrm{za}_{1}, \mathrm{za}_{2}, \mathrm{za}_{3}, \mathrm{Zx}_{1}, \mathrm{Zx}_{2}$ and hence those lines only need to be checked.

Similarly, the fault simulations were carried out for the remaining nine random functions and the results are tabulated in Table 8.

Table 8. Simulation results for AND-bridging faults

| $\dot{8}$ | $\begin{aligned} & \text { En } \\ & \\ & 0 \\ & \hline \end{aligned}$ | sınduI ${ }^{\text {elp }}$ Gృo ${ }^{\circ} \mathrm{N}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\mathrm{F}_{1}$ | 3 | 78 | 85.90 | 47.44 |
| 2 | F2 | 4 | 136 | 95.59 | 36.76 |
| 3 | F3 | 5 | 231 | 89.18 | 47.62 |
| 4 | $\mathrm{F}_{4}$ | 6 | 351 | 90.88 | 52.42 |
| 5 | F5 | 7 | 210 | 86.19 | 48.10 |
| 6 | F6 | 8 | 820 | 91.59 | 58.66 |
| 7 | $\mathrm{F}_{7}$ | 9 | 465 | 91.40 | 47.74 |
| 8 | F8 | 10 | 496 | 90.52 | 33.06 |
| 9 | F9 | 11 | 465 | 90.75 | 38.49 |
| 10 | $\mathrm{F}_{10}$ | 12 | 496 | 89.11 | 47.98 |
| Average |  |  |  | 90.11 | 45.83 |

### 4.4 OR-Bridging Faults

The OR-bridging fault is simulated considering two lines at a time with all possible combinations of control lines, data lines and intermediate gate outputs. The simulated results for ten random functions are given in Table 9.

Table 9. Simulation results for OR-bridging faults

| $\dot{\sim}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  | Total Possible Faults |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\mathrm{F}_{1}$ | 3 | 78 | 84.62 | 58.97 |
| 2 | $\mathrm{F}_{2}$ | 4 | 136 | 98.53 | 52.21 |
| 3 | $\mathrm{F}_{3}$ | 5 | 231 | 96.54 | 59.74 |
| 4 | F4 | 6 | 351 | 96.58 | 67.24 |
| 5 | F5 | 7 | 210 | 98.57 | 85.71 |
| 6 | F6 | 8 | 820 | 98.29 | 70.12 |
| 7 | $\mathrm{F}_{7}$ | 9 | 465 | 98.28 | 79.14 |
| 8 | $\mathrm{F}_{8}$ | 10 | 496 | 98.59 | 85.89 |
| 9 | F9 | 11 | 465 | 98.71 | 86.88 |
| 10 | F10 | 12 | 496 | 98.79 | 80.85 |
| Average |  |  |  | 96.75 | 72.68 |

From the test results as given in Table 5, 6, 8 and 9, it was found that the identifiability factor for the set of random functions tested through MATLAB simulation was more than $95 \%$ for all the functions of Single stuck-at, Double stuck-at, and OR-bridging types of faults except AND-bridging faults, with just $\mathrm{n}+5$ test vectors compared to $2^{\mathrm{n}}$ test vectors required for conventional testing. It was also observed that even though the overall distinguishabililty factor was in the range of $31-85 \%$, the individual set distinguishability factor was more than $93 \%$ as explained above.

Though the overall distinguishability is small, it does not affect the detection capability. Further, the distinguishing capability for an individual output set can be quite high, as illustrated above.

## 5. Conclusion

A test set scheme for detection of Single stuck-at, Double stuck-at, OR-bridging and AND-bridging faults for ESOP RMC logic functions have been detailed and the simulation results are shown. The results conclude that $\mathrm{n}+5$ test vectors can be used to detect Single Stuck-at, Double stuck-at, OR-bridging and ANDbridging faults in digital circuits. Further, the location can also be diagnosed through the output set. The analysis and diagnosis have been done through compact tabulation and two quantification indices. All possible combinations of the data lines, control lines and all intermediate gate outputs line pairs have been considered. Detection factor and Distinguishability factor can be further improved by modifying the network structure or using different test vectors.

## 6. References

[1]. Reddy S.M., "Easily Testable Realizations for Logic Functions", IEEE Trans. Computers, vol.21, no.11, Nov.1972, pp. 1183- 1188.
[2]. Pradhan D.K., "Universal Test Sets for Multiple Fault Detection in AND-EXOR Arrays", IEEE Trans. Computers, vol.27, no.2, Feb. 1978 pp. 181-187.
[3]. Bhattacharya B.B., Gupta B., Sarkar S and Choudhury A.K. "Testable design of RMC networks with universal tests for detecting stuck-at and bridging faults", IEE Proc. Computers and Digital Techniques, Vol.132, Part E., no.3, May 1985. pp. 155-162.
[4]. Sasao, T., "Easily Testable Realization for Generalized Reed-Muller Expressions", IEEE Trans. Computers, vol.46, no.6, Jun.1997, pp. 709-716,.
[5]. Kalay U, Hall D.V. and Petrowski M.A., "A Minimal Universal Test Set for Self-Test of EXOR-Sum-of-Products Circuits", IEEE Trans. Computers, vol.49, no.3, Mar. 2000 pp. 267-276.
[6]. Zhongliang P., "Testable realizations of ESOP expressions of logic functions", Proc. of $11^{\text {th }}$ Asian Test Symposium (ATS"02), IEEE Computer Society, 2002.
[7]. Zhongliang P.,"Bridging fault detections for testable realizations of logic functions", Proc of $16^{\text {th }}$ International Conference on VLSI Design, Jan. 2003, pp. 423-427.
[8]. Pan Zhongliang ,'Neural Network Model For Testing Stuck-at And Delay Faults In Digital Circuits' IEEE Proceedings Of The $1^{7 \text { th }}$ International Conference On VLSI Design (VLSID'04) August, 2004, pp.499-504.
[9]. Hafizur Rahaman, Debesh K.Dass,'Bridging Fault Detection in Double Fixed-Polarity Reed-Muller (DFPRM) PLA' IEEE Proceedings Asia and South Pacific Design Automation Conference (ASP-DAC'05), 2005, pp 172-177.
[10]. Hafizur Rahaman, Debesh K.Dass ,'Universal Test Set for Detecting Stuck-at and Bridging Faults in Double FixedPolarity Reed-Muller Programmable Logic Arrays' IEE Proc.Comput. Digit. Tech., Vol.153, No.2, March 2006. pp. 109-116.
[11]. Neelakantan P.N. and Ebenezer Jeyakumar A., "Stuck-at Fault Test Vectors for Exclusive-OR Sum Reed-Muller Canonical Boolean Functions", GESTS International Transactions on Computer Science and Engineering, Vol. 27, No.1, Jan. 2006 pp. 139-147.
[12]. Neelakantan P.N. and Jeyakumar A. E., "Single Stuck-At Fault Diagnosing Circuit of Reed-Muller Canonical ExclusiveOr Sum of Product Boolean Expressions", Journal of Computer Science, USA,Vol.2,Issue7, July 2006, pp. 595-599.
[13]. Pan Zhongliang , Chen Ling ,Liu Shouqiang, Zhang Guangzhao, 'Neural Network Approach for Multiple Fault Test
of Digital Circuit', IEEE Proceedings Of The $6^{\text {th }}$ International Conference On Intelligent Systems Design And Applications (ISDA'06), October,2006, pp.24-29.
[14]. Pan Zhongliang, Chen Guangju, "Fault Detection Test Set for Testable Realizations of Logic Functions with ESOP Expressions", Journal of Electronics (China), Vol. 24(2), 2007, pp. 238-244.
[15]. Wenjin Wu; Pengjun Wang; Xiaoying Zhang; Lingli Wang; Dai Jing "Search for the Best Polarity of Multi-Output RM Circuits Base on QGA", Second International Symposium on Intelligent Information Technology Application, Vol. 3, pp. 279 - 282, 2008.
[16]. Xiao-ying Zhang; Ling-li Wang; Xue-gong Zhou "Efficient RM conversion algorithm for large multiple output functions "Ninth International Conference on Solid-State and Integrated-Circuit Technology, 2008, pp. 2300-2303.
[17]. Ling Chen, Zhong Liang Pan, "Fault Detection of Bridging Faults in Digital Circuits by Shared Binary Decision Diagram", Key Engineering Materials, Tans Tech Publications (Volumes 439-440), June, 2010, pp. 1235-1240.
[18]. Zhong Liang Pan, Ling Chen, Guang Zhao Zhang, "Test Pattern Generation of VLSI Circuits Using Hopfield Neural Networks", Applied Mechanics and Materials, Trans Tech Publications (Volumes 29-32), August, 2010, pp. 1034-1039.
[19]. ZHAO Ying, LI Yanjuan, 'A Multiple Faults Test Generation Algorithm Based on Neural Networks and Chaotic Searching For Digital Circuits', International Conference on Computational Intelligence and Software Engineering (CiSE), December 2010.
[20]. Geetha V, Devarajan N and Neelakantan P N, "ORBridging Fault Identification and Diagnosis for Exclusive-OR Sum of Products Reed-Muller Canonical Circuits", Journal of Computer Science, USA, Issue 7(5), 2011, pp. 744-748.
[21]. Geetha V, Devarajan N and Neelakantan P N, "ORBridging Fault Identification and Diagnosis for Exclusive-OR Sum of Products Reed-Muller Canonical Circuits", European Journal of Scientific Research, USA, Issue 7(5), 2012, pp. 744748.

