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Estimation in Censored Sample Using Asymmetric Loss Function

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ABSTRACT

Pandey (1977) considered the exponential distribution which is generally used in life testing experiment. Epstein & Sobel (1953) and Bhattacharya & Srivastava (1974) considered the censoring procedure in life testing problem. Pandey and Malik (1994) considered the improved estimator for θ^2 in exponential distribution. In this paper proposed an estimator for θ^2 in case of exponential distribution and studied its properties under Linex loss function.

1. Introduction

In life testing experiment, it is a common practice to terminate the experiment when certain number of items has failed or a stipulated time has elapsed. In order to overcome this situation we terminate most of the life tests before all the items fail. To avoid this many life tests are terminated before all the items fail. If we considered the first n ordered statistics $x_{(1)}$, $x_{(2)}$,……, $x_{(n)}$ in a sample of size (r), in this case the sample is censored on the right which is also called as Type II censoring. Similarly we may have censoring on the left is called Type I censoring. For example, in many biological experiments, r samples from each living things are tested for antibodies after a certain period of time , only 'n' of these samples contain measurable amounts while remaining (n-r) of the animal develop the antigen at a level too low for measurements. Cohen(1965) and Srivastava (1976) have obtained the likelihood estimates of the parameters in case of continuous distributions such as exponential, normal ,log-normal and logistic when samples are progressively censored under the assumption that the parameters of the distribution under consideration might change at each stage of censoring. In such experiments after first stage, the experiment is continued with the remaining surviving items. The idea of removing some items at every stage of censoring stamps from the fact that these items might be required for use somewhere else for related experimentation. Bhattrchaya and Srivastava (1974) considered the type I and type II censoring process.

Let $x_1, x_2,........, x_n$ be a random sample of size n from normal distribution with mean θ and variance σ^2 .In some practical situations the prior values of mean and variance are θ_0 and σ_0^2 may be available. The preliminary testimator for the mean was suggested by Bancroft (1944) as: Take θ_0 as the estimate of θ if $H_0: \theta = \theta_0$ is accepted, otherwise the usual estimator $\bar{x}$ or in other words

$$\hat{\theta}_{PT} = \begin{bmatrix} \theta_0 \text{ if } H_0: \theta = \theta_0 \text{ is accepted} \\ \bar{x}, otherwise \end{bmatrix} \quad (1.1)$$

The mean squared error criterion as

$MSE(\theta_0) = E(\theta_0 - \theta)^2$ and $MSE(\bar{x}) = \frac{\sigma^2}{n}$.

Katti(1962) suggested that $MSE(\bar{x}) = \frac{\sigma^2}{n} \le MSE(\theta_0) = E(\theta_0 - \theta)^2$ if $\bar{x} \in R$ where R is determined by the testing of hypothesis of $\theta = \theta_0$ .We know that

$$P\left[-\in_1 \le \frac{\bar{x} - \theta_0}{\frac{\sigma}{\sqrt{n}}} \le \in_1 \right] = 1 - \alpha \quad (1.2)$$

where α is the level of significance. Equation (1.2) can be written as

$$P\left[\theta_0 - \in_1 \frac{\sigma}{\sqrt{n}} \le \bar{x} \le \theta_0 + \frac{\sigma}{\sqrt{n}} \in_1 \right] = 1 - \alpha \quad (1.3)$$

The distribution of $\bar{x}$ must be known.Pandey(1977)considered the exponential distribution which is generally used in Life testing distributions, it has mean θ and variance θ^2 .If we considered the improved estimator for θ as $P_1 = k\bar{x} + (1-k)\theta_0, 0 \le k \le 1$.The vale of k for which risk will be minimum is

$$k_{\min} = \frac{(\theta - \theta_0)^2}{(\theta - \theta_0)^2 + \frac{\theta^2}{n}} = \frac{\left(1 - \frac{\theta_0}{\theta}\right)^2}{\left(1 - \frac{\theta_0}{\theta}\right)^2 + \frac{1}{n}}$$

which is less than one.

The value of k depends on n and $\frac{\theta_0}{\theta}$ .If $\frac{\theta_0}{\theta}$ =1, k_{min} =0 and

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estimator is θ_0.The magnitude of relative efficiency will be maximum at $\frac{\theta_0}{\theta}=1$ and level of significance α is small and smaller value of n.

Epstein and Sobel (1953) and Bhattacharya and Srivastava (1974) considered the censoring procedure in life testing problem and proposed the estimator as

$$\hat{\theta}_{1PT} = \begin{bmatrix} \theta_0 \text{ if } H_0 : \theta = \theta_0 \text{ is accepted} \\ cT_r, otherwise \end{bmatrix}$$

.We know that $\frac{2n\bar{x}}{\theta_0}$ follows a chi-square distribution with 2n degrees of freedom. Also if $x_{(1)} \le x_{(2)} \le ------ \le x_{(r)} ----x_{(n)}$ and r is the censored data of type II censoring. Then,

$$T_r = \sum_{i=1}^{r} x_{(i)} + (n-r)x_{(r)} . \tag{1.4}$$

Since $\frac{2nT_r}{\theta}$ follows a chi-square distribution with 2r degrees of freedom i.e.,

$$E\left[\frac{2T_r}{\theta}\right] = 2r, V\left[\frac{2T_r}{\theta}\right] = 4r, E\left[\frac{T_r}{r}\right] = \theta, V[T_r] = \frac{\theta^2}{r} .$$

If two populations are considered with common mean, the unbiased estimator for common mean in exponential distribution is

$$T_1 = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} \text{ if } \sigma_1^2 = \sigma_2^2 = \sigma^2 .$$

Pandey and Malik (1994) considered the improved estimator for θ^2 in exponential distribution as

$$T_2 = \frac{n_1\bar{x} + n_2\bar{y}}{(n_1+n_2)^2 + 5(n_1+n_2) + 6}$$

which is in the form of improve estimator of θ^2 in exponential distribution suggested by Pandey and Singh (1977) as

$$T_1^{'} = \frac{n^2\bar{x}^2}{(n+2)(n+3)} .$$

If we considered the displaced exponential distribution having density function

$$f(x, A, \theta) = \frac{1}{\theta} \bar{e} \frac{(x-A)}{\theta}, \quad x > A, \ \theta > 0 \tag{1.5}$$

Here A is location and θ is scale parameters. The maximum likelihood estimators for θ and A are $(\bar{x} - x_{(1)})$ and $x_{(1)}$ respectively.

We know that $\frac{2n(\bar{x} - x_{(1)})}{\theta}$ follows a chi-square distribution with 2(n-1) degrees of freedom. The other research scholars have considered the preliminary estimators/adaptive estimators/ conditional specifications. (see Han etal (1988), Hogg (1974), Pandey and Singh(1977),Hirano(1973)). In section 2, proposed an estimator for θ^2 in case of exponential distribution as

$$Y_2 = c\left[\frac{T_{r_1} + T_{r_2}}{r_1 + r}\right]^2$$

which provide

$$Y_3 = w_1T_{r_1}^2 + w_2T_{r_{21}}^2 + w_3T_{r_1}T_{r_2}$$

and studied their properties under Linex loss function.

2. Estimation of θ^2 under Linex loss function

Pandey and Singh (1977) considered the improved estimator for θ^2 as

$$\hat{\theta} = \begin{bmatrix} \theta_0 \text{ if } H_0 : \theta = \theta_0 \text{ is accepted} \\ \frac{1}{a}\left(1 - e^{-\frac{a}{r+1}}\right)T_r, otherwise \end{bmatrix} .$$

Similarly if considered the improved estimator for θ^2 as

$$\left[\frac{T_{r_1} + T_{r_2}}{r_1 + r}\right]^2 .$$

Pandey and Malik (1994) considered the improved estimator for θ^2 as $Y_2 = w_1T_{r_1}^2 + w_2T_{r_{21}}^2 + w_3T_{r_1}T_{r_2}$ .The MSE criterion will provide

$$MSE(Y_2) = \begin{bmatrix} w_1^2 r_1(1+r_1)(r_1+2)(r_1+3) + 2w_1w_2r_1(r_1+1)r_2(r_2+1) + 2w_1w_3r_1(r_1+1)r_2(r_2+1) + \\ 2w_3w_2r_1r_2(r_2+1)(r_2+2) + w_3^2 r_1(r_1+1)r_2(r_2+1) + \\ w_2^2 r_2(r_2+1)(r_2+2)(r_2+3) - 2\{w_1r_1(1+r_1) + w_2r_2(r_2+1) + w_3r_1r_2\} + 1 \end{bmatrix}\theta^4$$

Let K= $r_1(1+r_1)(r_1+2)(r_1+3)$, P = $r_2(r_2+1)(r_2+2)(r_2+3)$, Q= $r_1(r_1+1)r_2(r_2+1)$

T= $r_1r_2(r_2+1)(r_2+2)$, S= $r_1(r_1+1)r_2(r_1+2)$, V= $r_1(1+r_1)$,U= $r_2(r_2+1)$,M= r_1r_2 . We get,

$$MSE(Y_2) = \left[w_1^2K + 2w_1w_2Q + 2w_1w_3S + 2w_3w_2T + w_3^2Q + w_2^2P - 2\{w_1V + w_2U + w_3M\} + 1\right]\theta^4$$

The value of w_1, w_2 and w_3 for which MSE (Y_2) will be minimum, we get

$$w_1 = \frac{D_1}{D}, w_2 = \frac{D_2}{D} \text{ and } w_3 = \frac{D_3}{D}$$

where

$$D = K(PQ - T^2) - Q(Q^2 - ST) + S(QT - SP) , D_1 = V(PQ - T^2) - Q(UQ - MT) + S(UT - MP)$$

$$D_2 = K(UQ - MT) - V(Q^2 - ST) + S(QM - SU)$$ and

$$D_3 = K(PM - TU) - Q(QM - SU) + V(QT - SP)$$

In some cases, the over (under) estimation may exist and Varian (1975) proposed the Linex (linear-exponential) loss function which may be appropriate. The Linex loss function which rises exponentially on one side of zero and almost linearly on the other side of zero. This loss function reduces to squared error loss for value of a near to zero. The Linex loss function is

$$L(\Delta, a) = b\left[e^{a\Delta} - a\Delta - 1\right], \quad \Delta = \hat{\mu} - \mu, \ a \ne 0,$$

a and b are shape and scale parameter.

If $|a| \to 0$, the linex loss reduced to squared error.

Using Linex loss function for the estimator $Y_2^{'} = w_1T_{r_1}^2 + w_2T_{r_{21}}^2 + w_3T_{r_1}T_{r_2}$ we have,

$$L(a, \Delta) = (e^{a\Delta} - a\Delta - 1), \quad a \ne 0$$

$$L(a, \Delta) = e^{a\left[\frac{w_1T_{r_1}^2 + w_2T_{r_{21}}^2 + w_3T_{r_1}T_{r_2}}{\theta^2} - 1\right]} - a\left[\frac{w_1T_{r_1}^2 + w_2T_{r_{21}}^2 + w_3T_{r_1}T_{r_2}}{\theta^2} - 1\right] - 1$$

which has

$$\frac{2}{a^2}R(a, \Delta) = \begin{bmatrix} (1-a)\{w_1^2K + 2w_1w_2Q + 2w_1w_3S + 2w_3w_2T + w_3^2Q + w_2^2P\} + \\ (a-2)\{w_1V + w_2U + w_3M\} + 1 - \frac{a}{3} \end{bmatrix}$$

The value of w_1, w_2 and w_3 for which MSE ($Y_2^{'}$) will be minimum, we get

$$w_1 = \frac{D_1^{'}}{D}, w_2 = \frac{D_2^{'}}{D} \text{ and } w_3 = \frac{D_3^{'}}{D}$$

where

$$D = K(PQ - T^2) - Q(Q^2 - ST) + S(QT - SP)\ , D_1^{'} = \left(\frac{1-\frac{a}{2}}{1-a}\right)\left[V(PQ - T^2) - Q(UQ - MT) + S(UT - MP)\right]$$

$$D_2^{'} = \left(\frac{1-\frac{a}{2}}{1-a}\right)\left[K(UQ - MT) - V(Q^2 - ST) + S(QM - SU)\right]$$

and

$$D_3^{'} = \left(\frac{1-\frac{a}{2}}{1-a}\right)\left[K(PM - TU) - QQM - SU) + V(QT - SP)\right]$$

Table 2.1 to 2.4, represents the relative efficiency of the estimator $Y_2^{'}$ with respect to Y_2 for a = .1(.1).4, $r_1 =$3,4,5,6, $r_2 =$3, 4, 5, 6.The figure 2.1 to 2.4 shows that for small values of a and for small values of r_1, r_2 the proposed estimator are performs better. The preliminary testimators may be proposed and properties can be studied.

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4. Appendices

Figure 2.1 Relative Efficiency of Testimator $Y_2^{'}$ w.r.to Y_2 when a=0.1

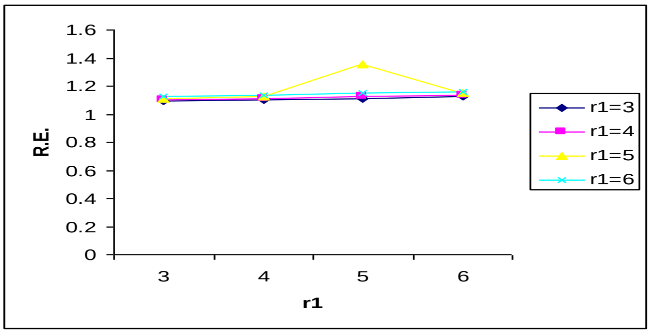

Figure 2.2 Relative Efficiency of Testimator $Y_2^{'}$ w.r.to Y_2 when a=0.2

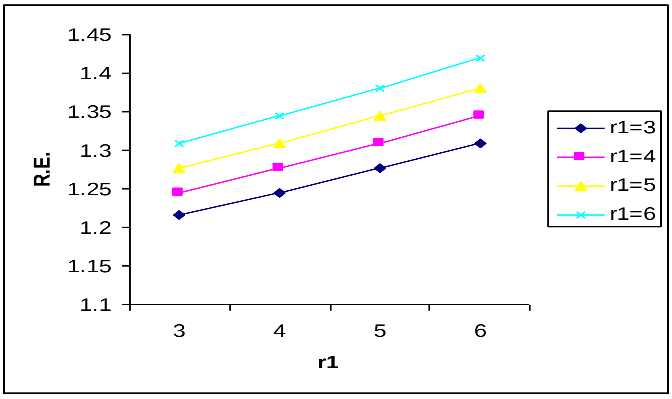

Figure 2.3 Relative Efficiency of Testimator $Y_2^{'}$ w.r.to Y_2 when a=0.3

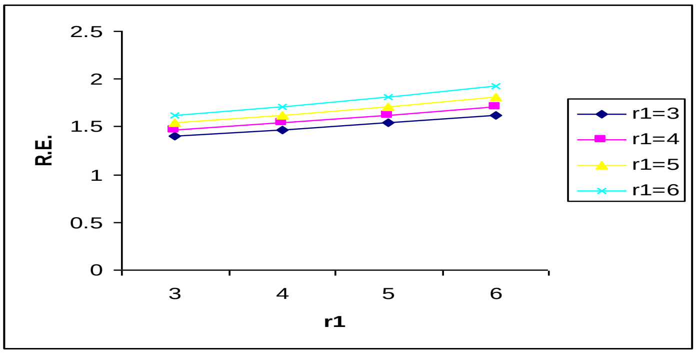

Figure 2.4 Relative Efficiency of Testimator $Y_2^{'}$ w.r.to Y_2 when a=0.4