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Semi compatibility and common random fixed point for random multivalued operators

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ABSTRACT

The purpose of this paper is to establish a common random fixed point theorem for five random multivalued operators satisfying a rational inequality using the concept of weak compatibility, semi compatibility and commutativity of random multivalued operators in polish space.

Introduction: The study of random fixed point forms a central topic in this area. Bharucha – Reid [8] have been given various ideas associated with random fixed point theory area are used to form a particularly elegant approach for the solution of non linear random system. In the recent years a vast amount of mathematical activity has been carried out to obtain many remarkable results showing the existence of ransom fixed point of single and mutivalued random operators given by spacek [13], Hans [9], Itoh [10], Beg [5], Beg and Shahzad [7], Badshah and Sayyed [2, 3], Badshah and Gagrani [1], Beg and Abbas [6], Xu, H. K. [15], Tan and Yuan [14], O'Regan [11], Plubteing and Kumar [12], and others.

Preliminaries: We begin with establishing some preliminaries by (Ω, Σ). We denote a measurable space with the Σ a sigma algebra of subsets of Ω. Let (x, d) be a polish space i.e. a separable complete metric space.

Let 2^X be the family of all subsets of X and CB (X) denote the family of all non-empty bounded closed subsets of X.

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A mapping $T: \Omega \to 2^X$ is called measurable if for any open subset C of X

$$T^{-1}(C) = \{\omega \in \Omega : T(\omega) \cap C \neq \phi\} \in a$$

A mapping $\xi: \Omega \to X$ is called measurable selector of measurable mapping $T: \Omega \to 2^X$, if ξ is measurable and for any $\omega \in \Omega$, $\xi(\omega) \in T(\omega)$.

A mapping $T: \Omega \times X \to CB(X)$ is called a random multivalued operator if for every $x \in X$, T(., X) is measurable.

A measurable mapping $f: \Omega \times X \to X$ is called a random operator if for any $x \in X$, $f(.,x)$ is measurable.

A measurable mapping $\xi : \Omega \to X$ is called random fixed point of random multivalued operator

$T: \Omega \times X \to CB(X)$ ($f: \Omega \times X \to X$), if for every $\omega \in \Omega$, $\xi(\omega) \in T(\omega, \xi(\omega)), (f(\omega), \xi(\omega)) = \xi(\omega)$.

Definition 2.1 [7] Let X is a Polish space i.e. a separable complete metric space. Mappings f, g: $X \to X$ are compatible if

$$\lim_{n\to\infty} d(fg(x_n), gf(x_n)) = 0$$

provided that $\lim_{n\to\infty} f(x_n)$ and $\lim_{n\to\infty} g(x_n)$ exist in X and

$$\lim_{n\to\infty} f(X_n) = \lim_{n\to\infty} g(X_n).$$

Random operators $S, T: \Omega \times X \to X$ are compatible if $S(\omega,.)$ and $T(\omega,.)$ are compatible for each $\omega \in \Omega$.

Definition 2.2 Let X is a Polish space. Random of $S, T: \Omega \times X \to X$ are weakly compatible if $T(\omega, \xi(\omega)) = S(\omega, \xi(\omega))$ for some measurable mapping $\xi : \Omega \to X$ and $\omega \in \Omega$, then $T(\omega, S(\omega,\xi(\omega)) = s(\omega,T(\omega,\xi(\omega))$ for every $\omega \in \Omega$.

Definition 2.3 Let X is a Polish space. Random operators $S, T : \Omega \times X \to X$ are said to be commutative if $S(\omega,.)$ and $T(\omega,.)$ are commutative for each $\omega \in \Omega$.

Definition 2.4 Let X is a Polish space. Random operators $S, T : \Omega \times X \to X$ are said to be semi compatible if

$$d\Big(S\big(\omega, T(\omega, \xi_n(\omega))\big), T(\omega, \xi(\omega))\Big) \to 0 \text{ as } n \to \infty$$

Whenever $\{\xi_n\}$ is a sequence of measurable mapping from $\Omega \times X \to X$ such that

$$d\big(S(\omega, \xi_n(\omega)), \xi(\omega)\big) \to 0, \ d\big(T(\omega, \xi_n(\omega)), \xi(\omega)\big) \to 0 \quad \text{as } n \to \infty \text{ for each } \omega \in \Omega.$$

Definition 2.5 Let $S, T: \Omega\times X\to CB(X)$ be continuous random multivalued operators. S and T are said to weak compatible if they commute at their coincidence points i.e. $S(\omega, \xi(\omega))= T(\omega,\xi(\omega))$ implies that $ST(\omega, \xi(\omega)) = TS(\omega, \xi(\omega))$.

S and T are said to be compatible if $d(ST(\omega,\xi_n(\omega)), TS(\omega,\xi_n(\omega)))\to 0$ as $n \to \infty$.

S and T are called semi compatible if $d\left(ST\left(\omega, \xi_n(\omega)\right), T(\omega, \xi(\omega))\right) \to 0$ as $n\to\infty$ whenever $\xi_n: \Omega\to X, n > 0$

is a measurable mapping such that $d\left(T\left(\omega, \xi_n(\omega)\right), \xi(\omega)\right)\to 0$, $d\left(S\left(\omega, \xi_n(\omega)\right), \xi(\omega)\right) \to 0$ as $n\to\infty$

Clearly if the pair (S, T) is semi compatible then they are weak compatible.

Main Result:

Theorem: Let X be a polish space and A, B, S, T, J: $\Omega \times X \to CB(X)$ be random multivalued operators satisfying

$AB(\omega, X) \subset J(\omega, X)$ and $ST(\omega, X) \subset J(\omega, X)$ and for every $\omega\in \Omega$,

$$H(AB(\omega,x), ST(\omega,y)) \leq \frac{\alpha(\omega)\left[d\left(J(\omega,x), AB(\omega,x)\right)\right]\left[d\left(J(\omega,y), ST(\omega,y)\right)\right]}{d\left(J(\omega,x), J(\omega,y)\right)} + \beta(\omega)\, d\left(J(\omega,x), J(\omega,y)\right) \quad \ldots\ldots \quad (3.1)$$

For every $\omega\in\Omega$ and $x, y \in X$ with $\alpha, \beta: \Omega \to X$ are measurable mappings such that $\alpha(\omega) + \beta(\omega) < 1$.

If either (AB, J) are semi compatible, J or AB is continuous and (ST, J) weakly compatible or (ST, J) are semi compatible, J or ST is continuous and (AB, J) are weakly compatible.

Then AB, ST and J have a unique common random fixed point. Further more if the pairs (A, B), (A, J), (B, J), (S, T), (S, J) and (T, J) are commuting mappings then A, B, S, T and J have a unique common random fixed point.

Proof: let $\xi_0, \xi_1, \xi_2: \Omega \to X$ be three measurable mappings such that

$$AB\left(\omega, \xi_0(\omega)\right) = J\left(\omega, \xi_1(\omega)\right) \quad \text{and} \quad ST\left(\omega, \xi_1(\omega)\right) = J\left(\omega, \xi_2(\omega)\right)$$

In general we can choose sequences $\{\xi_n\}$ and $\{\eta_n\}$ of measurable mappings such that

$$AB\left(\omega, \xi_{2n}(\omega)\right) = J\left(\omega, \xi_{2n+1}(\omega)\right) = \eta_{2\eta}(\omega)$$

$$ST\left(\omega, \xi_{2n+1}(\omega)\right) = J\left(\omega, \xi_{2n+2}(\omega)\right) = \eta_{2\eta+1}(\omega) \qquad \forall\, n = 0,1,2 \ldots\ldots\ldots\ldots \omega \in \Omega$$

Then for each $\omega \in \Omega$

$$d\left(\eta_{2n}(\omega), \eta_{2n+1}(\omega)\right) = d\left(AB(\omega, \xi_{2n}(\omega)), ST(\omega, \xi_{2n+1}(\omega))\right)$$

$$\leq \alpha(\omega) \frac{\left[d\left(J\left(\omega, \xi_{2n}(\omega)\right), AB(\omega, \xi_{2n}(\omega))\right)\right]\left[d\left(J\left(\omega, \xi_{2n+1}(\omega)\right), ST(\omega, \xi_{2n+1}(\omega))\right)\right]}{d\left(J\left(\omega, \xi_{2n}(\omega)\right), J(\omega, \xi_{2n+1}(\omega))\right)}$$

$$+ \beta(\omega)\ d\left(J\left(\omega, \xi_{2n}(\omega)\right), J(\omega, \xi_{2n+1}(\omega))\right)$$

$$\leq \alpha(\omega) \frac{\left[d\left(J\left(\omega, \xi_{2n}(\omega)\right), J(\omega, \xi_{2n+1}(\omega))\right)\right]\left[d\left(J\left(\omega, \xi_{2n+1}(\omega)\right), J(\omega, \xi_{2n+2}(\omega))\right)\right]}{d\left(J\left(\omega, \xi_{2n}(\omega)\right), J(\omega, \xi_{2n+1}(\omega))\right)}$$

$$+ \beta(\omega)\ d\left(J\left(\omega, \xi_{2n}(\omega)\right), J(\omega, \xi_{2n+1}(\omega))\right)$$

$$\leq \alpha(\omega)\ d\left(\eta_{2n}(\omega),\ \eta_{2n+1}(\omega)\right) + \beta(\omega)\ d\left(\eta_{2n-1}(\omega),\ \eta_{2n}(\omega)\right)$$

$$(1-\alpha(\omega))\ d\left(\eta_{2n}(\omega),\ \eta_{2n+1}(\omega)\right) \leq \beta(\omega)\ d\left(\eta_{2n-1}(\omega),\ \eta_{2n}(\omega)\right)$$

$$d\left(\eta_{2n}(\omega),\ \eta_{2n+1}(\omega)\right) \leq \frac{\beta(\omega)}{1-\alpha(\omega)}\ d\left(\eta_{2n-1}(\omega),\ \eta_{2n}(\omega)\right)$$

Where $k = \frac{\beta(\omega)}{1-\alpha(\omega)} < 1$

Similarly we can prove

$$d\left(\eta_{2n+1}(\omega), \eta_{2n+2}(\omega)\right) \leq k\, d\left(\eta_{2n}(\omega), \eta_{2n+1}(\omega)\right)$$

$$\leq k.k\left(\eta_{2n-1}(\omega), \eta_{2n}(\omega)\right)$$

Similarly proceeding in the same way by induction we get

$$d\left(\eta_{2n+1}(\omega), \eta_{2n+2}(\omega)\right) \leq k^{2n+1}\left(\eta_0(\omega), \eta_1(\omega)\right)$$

Furthermore for m > n we have

$$d\left(\eta_{2n}(\omega), \eta_{2m}(\omega)\right) \leq d\left(\eta_{2n}(\omega), \eta_{2n+1}(\omega)\right) + d\left(\eta_{2n+1}(\omega), \eta_{2n+2}(\omega)\right) + \dots\dots\dots + d\left(\eta_{2m-1}(\omega), \eta_{2m}(\omega)\right)$$

$$\leq k^{2n}\, d\left(\eta_0(\omega), \eta_1(\omega)\right) + k^{2n+1}\, d\left(\eta_0(\omega), \eta_1(\omega)\right) + \dots\dots\dots + k^{2m-1}\, d\left(\eta_0(\omega), \eta_1(\omega)\right)$$

$$\leq [k^{2n} + k^{2n+1} \dots\dots\dots\dots.. + k^{2m-1}]\, d\left(\eta_0(\omega), \eta_1(\omega)\right)$$

$$\leq \frac{k^{2n}}{(1-k)} \ d\left(\eta_0(\omega), \eta_1(\omega)\right)$$

$$d\left(\eta_{2n}(\omega), \eta_{2m}(\omega)\right) \leq \frac{k^{2n}}{(1-k)} \ d\left(\eta_0(\omega), \eta_1(\omega)\right) \to 0 \ \text{ as } \ n, m \to \infty$$

Thus it follows that sequence $\{\eta_{2n}(\omega)\}$ is a Cauchy sequence. Since X is a separable complete metric space there exists a measurable mapping $\xi: \Omega \to X$ such that $\{\eta_{2n}(\omega)\}$ and its subsequences converges to $\xi(\omega)$.

So

$$AB\left(\omega, \xi_{2n}(\omega)\right) \to \xi(\omega), \ \ J\left(\omega, \xi_{2n+1}(\omega)\right) \to \xi(\omega) \qquad \text{--------------------------} \qquad (3.2)$$

and

$$ST\left(\omega, \xi_{2n+1}(\omega)\right) \to \xi(\omega), \ \ J\left(\omega, \xi_{2n+2}(\omega)\right) \to \xi(\omega) \ \ \text{for each } \ \omega \in \Omega \qquad \text{----------------------} \qquad (3.3)$$

Case I: If J is continuous

In this case, we have

$$J\left(AB\left(\omega, \xi_{2n}(\omega)\right)\right) \to J\left(\omega, \xi(\omega)\right), \qquad J^2\left(\omega, \xi_{2n}(\omega)\right) \to J\left(\omega, \xi(\omega)\right)$$

and semi compatibility of the pair (AB, J) gives $(AB)\,J\left(\omega, \xi_{2n}(\omega)\right) \to J\left(\omega, \xi(\omega)\right)$ for each $\omega \in \Omega$.

Step 1: for each $\omega \in \Omega$

$$H(AB)\left(J\left(\omega, \xi_{2n}(\omega)\right)\right), ST(\omega, \xi_{2n+1}(\omega)))$$

$$\leq \alpha(\omega) \frac{\left[d\left(J\left(\omega, \xi_{2n}(\omega)\right)\right), (AB)\,J\left(\omega, \xi_{2n}(\omega)\right)\right] \left[d\left(J\left(\omega, \xi_{2n+1}(\omega)\right)\right), (ST)J\left(\omega, \xi_{2n+1}(\omega)\right)\right]}{d\left(J(J((\omega, \xi_{2n}(\omega))), J\left(\omega, \xi_{2n+1}(\omega)\right)\right)}$$

$$+ \ \beta(\omega)\, d\left(J\,.\,J\left((\omega, \xi_{2n}(\omega)\right), J\left(\omega, \xi_{2n+1}(\omega)\right)\right)$$

$$d(J\left(\omega, \xi(\omega)\right), \xi(\omega)) \ \leq \ \alpha(\omega) \frac{\left[d\left(J\left(\omega, \xi_{2n}(\omega)\right)\right), J\left(\omega, \xi(\omega)\right)\right] \ [d(\xi(\omega), \xi(\omega))]}{d\left(J\left(\omega, \xi_{2n}(\omega)\right), \xi(\omega)\right)} + \ \beta(\omega)\, d\left(J^2\left(\omega, \xi_{2n}(\omega)\right), \xi(\omega)\right)$$

$$d\left(J\left(\omega, \xi(\omega)\right), \xi(\omega)\right) \leq \ \beta(\omega)\, d\left(J^2\left(\omega, \xi_{2n}(\omega)\right), \xi(\omega)\right)$$

$$d\left(J\left(\omega, \xi(\omega)\right), \xi(\omega)\right) \leq \ \beta(\omega)\, d\left(J\left(\omega, \xi(\omega)\right), \xi(\omega)\right)$$

$$\left(1 - \beta(\omega)\right) d\left(J\left(\omega, \xi(\omega)\right), \xi(\omega)\right) \leq \ 0$$

$$\xi(\omega) = \ J\left(\omega, \xi(\omega)\right) \qquad \text{for each } \omega \in \Omega.$$

Step 2: for each $\omega \in \Omega$

$$H\left(AB(\omega,\xi(\omega)), ST\left(\omega,\xi_{2n+1}(\omega)\right)\right)$$
$$\leq \alpha(\omega)\frac{\left[d\left(J(\omega,\xi(\omega))\right), AB(\omega,\xi(\omega))\right]\left[d\left(J\left(\omega,\xi_{2n+1}(\omega)\right)\right), ST\left(\omega,\xi_{2n+1}(\omega)\right)\right]}{d\left(J(\omega,\xi(\omega)), J\left(\omega,\xi_{2n+1}(\omega)\right)\right)}$$
$$+\ \beta(\omega)\, d\left(J(\omega,\xi(\omega)), J\left(\omega,\xi_{2n+1}(\omega)\right)\right)$$

Taking limit n→∞ and using result of step 1 and (3.3) we get

$$d\left(AB(\omega,\xi(\omega)),\xi(\omega)\right) \leq \alpha(\omega)\frac{\left[d\left(\xi(\omega), AB(\omega,\xi(\omega))\right)\right]\ \left[d(\xi(\omega),\xi(\omega))\right]}{\left[d(\xi(\omega),\xi(\omega))\right]} + \beta(\omega)\, d(\xi(\omega),\xi(\omega))$$

$$d\left(AB(\omega,\xi(\omega)),\xi(\omega)\right) \leq 0$$

Implying thereby $AB(\omega,\xi(\omega)) = \xi(\omega)$ for each $\omega\in\Omega$.

Hence $AB(\omega,\xi(\omega)) = \xi(\omega) = J(\omega,\xi(\omega))$ for each $\omega\in\Omega$.

Therefore a measurable mapping g: $\Omega \to X$ such that $AB(\omega,\xi(\omega)) = J(\omega,g(\omega))$.

Step 3: for each $\omega\in\Omega$

$$H\left(AB\left(\omega,\xi_{2n}(\omega)\right), ST(\omega,g(\omega))\right)$$
$$\leq \alpha(\omega)\frac{\left[d\left(J(\omega,\xi_{2n}(\omega), AB(\omega,\xi_{2n}(\omega))\right)\right]\left[d(J(\omega,g(\omega), ST(\omega,g(\omega))\right]}{d\left(J\left(\omega,\xi_{2n}(\omega)\right), J(\omega,g(\omega))\right)}$$
$$+\ \beta(\omega)\, d\left(J\left(\omega,\xi_{2n}(\omega)\right), J(\omega,g(\omega))\right)$$

Taking limit as n→∞ and using the result from above steps, we obtain that

$$d\left(\xi(\omega), ST(\omega,g(\omega))\right)$$
$$\leq \alpha(\omega)\frac{\left[d\left(J(\omega,\xi_{2n}(\omega),\xi(\omega)\right)\right]\left[d(\xi(\omega), ST(\omega,g(\omega))\right]}{d\left(J\left(\omega,\xi_{2n}(\omega)\right),\xi(\omega)\right)}$$
$$+\ \beta(\omega)\, d\left(J\left(\omega,\xi_{2n}(\omega)\right),\xi(\omega)\right)$$

$$d\left(\xi(\omega), ST(\omega,g(\omega))\right) \leq \alpha(\omega)\, d\left(\xi(\omega), ST(\omega,g(\omega))\right) + \beta(\omega)\ d\left(\xi(\omega), ST(\omega,g(\omega))\right)$$

$$\left(1-\alpha(\omega)\right)d\left(\xi(\omega), ST(\omega,g(\omega))\right) \leq 0$$

Implying thereby $ST(\omega,g(\omega)) = \xi(\omega)$ for each $\omega\in\Omega$.

Therefore $ST(\omega,g(\omega)) = J(\omega,g(\omega)) = \xi(\omega)$.

Now using the weak compatibility of (ST, J) we have

$$J\,(ST)\big(\omega, g(\omega)\big) = (ST)\; J\,\big(\omega, g(\omega)\big)$$

$$i.e.\quad ST\big(\omega, \xi(\omega)\big) = J\,\big(\omega, \xi(\omega)\big) \text{ for each } \omega \in \Omega.$$

Thus $AB\,\big(\omega, \xi(\omega)\big) = ST\,\big(\omega, \xi(\omega)\big) = J\,\big(\omega, \xi(\omega)\big)$ for each $\omega \in \Omega$.

Hence $\xi(\omega)$ is a common random fixed point of random multivalued operators AB, ST and J.

Case II: if AB is continuous

In this case, we have $(AB)\,J\,\big(\omega, \xi_{2n}(\omega)\big) \to AB\,\big(\omega, \xi(\omega)\big)$ and semi compatibility of the pair (AB, J)

gives $(AB)\,J\,\big(\omega, \xi_{2n}(\omega)\big) \to J\,\big(\omega, \xi(\omega)\big)$ for each $\omega \in \Omega$.

Step 1: for each $\omega \in \Omega$, we have

$$\begin{aligned} H(AB)\Big(J\,\big(\omega, \xi_{2n}(\omega)\big), ST\big(\omega, \xi_{2n+1}(\omega)\big)\Big) &\le \alpha(\omega)\,\frac{\big[d\big(J(\omega, \xi_{2n}(\omega)), (AB)\,J(\omega, \xi_{2n}(\omega))\big)\big]\,\big[d\big(J(\omega, \xi_{2n+1}(\omega)), ST(\omega, \xi_{2n+1}(\omega))\big)\big]}{d\Big(J.J\big(\omega, \xi_{2n}(\omega)\big), J\big(\omega, \xi_{2n+1}(\omega)\big)\Big)} \\ &\quad + \beta(\omega)\, d\Big(J.J\,\big(\omega, \xi_{2n}(\omega)\big), J\big(\omega, \xi_{2n+1}(\omega)\big)\Big) \end{aligned}$$

Taking limit as $n \to \infty$ and using the result from above result, we get

$$\begin{aligned} d\Big(J\,\big(\omega, \xi(\omega)\big), \xi(\omega)\Big) &\le \alpha(\omega)\,\frac{\big[d\big(J(\omega, \xi(\omega)),\ J(\omega, \xi(\omega))\big)\big]\,\big[d\big(\xi(\omega), \xi(\omega)\big)\big]}{d\Big(J\big(\omega, \xi(\omega)\big), \xi(\omega)\Big)} + \beta(\omega)\, d\Big(J\,\big(\omega, \xi(\omega)\big), \xi(\omega)\Big) \end{aligned}$$

$$\big(1 - \beta(\omega)\big)\, d\,\Big(J\,\big(\omega, \xi(\omega)\big), \xi(\omega)\Big) \le 0.$$

$$J\,\big(\omega, \xi(\omega)\big) = \xi(\omega) \text{ for each } \omega \in \Omega.$$

Step 2: for any $\omega \in \Omega$

$$\begin{aligned} H\Big(AB\,\big(\omega, \xi(\omega)\big), ST\,\big(\omega, \xi_{2n+1}(\omega)\big)\Big) &\le \alpha(\omega)\,\frac{[d(J(\omega, \xi(\omega)), AB(\omega, \xi(\omega)))]\,\Big[d\,\Big(J(\omega, \xi_{2n+1}(\omega)), ST(\omega, \xi_{2n+1}(\omega)\Big)\Big]}{d\Big(J\big(\omega, \xi(\omega)\big), J\,\big(\omega, \xi_{2n+1}(\omega)\big)\Big)} \\ &\quad + \beta(\omega)\, d\Big(J\,\big(\omega, \xi(\omega)\big), J\big(\omega, \xi_{2n+1}(\omega)\big)\Big) \end{aligned}$$

Taking limit n→∞ and using the result of step 1 of case II we get

$$d\Big(AB\,(\omega,\xi(\omega)),\xi(\omega)\Big) \le \alpha\,(\omega)\,\frac{[d(\xi(\omega),AB(\omega,\xi(\omega)))]\,\big[d\big(\xi(\omega),\xi(\omega)\big)\big]}{d\big(\xi(\omega),\xi(\omega)\big)} + \beta\,(\omega)\,d\big(\xi(\omega),\xi(\omega)\big)$$

Hence

$$AB\,(\omega,\xi(\omega)) = \xi(\omega) \quad \text{for each } \omega\in\Omega.$$

Thus $AB\,(\omega,\xi(\omega)) = J\,(\omega,\xi(\omega)) = \xi(\omega)$ for each $\omega\in\Omega$

Step 3: for any $\omega\in\Omega$ there exist a measurable mapping $g' : \Omega\to X$:

$$AB\,(\omega,\xi(\omega)) = J\,(\omega,\xi(\omega)) = J\,(\omega,g'(\omega)) = \xi(\omega)$$

for any $\omega\in\Omega$

$$\begin{aligned} &H\Big(AB\,(\omega,\xi_{2n}(\omega)),ST(\omega,g'(\omega))\Big) \\ &\quad\le \alpha(\omega)\,\frac{d\Big(J\,(\omega,\xi_{2n}(\omega)),AB\,(\omega,\xi_{2n}(\omega))\Big)\,d\Big(J(\omega,g'(\omega)),ST(\omega,g'(\omega))\Big)}{d\Big(J\,(\omega,\xi_{2n}(\omega)),J(\omega,g'(\omega))\Big)} \\ &\quad+ \beta(\omega)\,d\Big(J\,(\omega,\xi_{2n}(\omega)),J(\omega,g'(\omega))\Big) \end{aligned}$$

Taking limit n→∞ and using the result from above step, we obtain that

$$d\Big(\xi(\omega),ST(\omega,g'(\omega))\Big) \le \alpha(\omega)\,\frac{d\big(\xi(\omega),\xi(\omega)\big)\,d\Big(\xi(\omega),ST(\omega,g'(\omega))\Big)}{d\big(\xi(\omega),\xi(\omega)\big)} + \beta(\omega)\,d\big(\xi(\omega),\xi(\omega)\big)$$

Implying thereby $ST\,(\omega,g'(\omega)) = \xi(\omega)$ for each $\omega\in\Omega$.

Therefore $ST\,(\omega,g'(\omega)) = J\,(\omega,g'(\omega)) = \xi(\omega)$ for each $\omega\in\Omega$.

Now using the weak compatibility of (ST, J) we have

$$J\,(ST)\,(\omega,g'(\omega)) \to (ST)\,J\,(\omega,g'(\omega))$$

$$i.e.\ ST\,(\omega,\xi(\omega)) \to J\,(\omega,\xi(\omega)) \text{ for each } \omega\in\Omega.$$

$$thus\ \ AB(\omega,\xi(\omega)) = ST(\omega,\xi(\omega)) = J(\omega,\xi(\omega)) = \xi(\omega) \quad \text{for each } \omega\in\Omega.$$

Hence $\xi(\omega)$ is a common random fixed point of random multivalued operators AB, ST and J.

If the mapping ST of J is continuous instead of AB or J then this proof that $\xi(\omega)$ is common random fixed point of AB, ST and J is similar.

Uniqueness: Let h: Ω→X is another common random fixed point of random multivalued operators AB, ST and J. Then for each ω∈Ω.

$$H\Big(AB\,(\omega,\xi(\omega)), ST(\omega,h(\omega))\Big) \le \alpha\,(\omega)\frac{\Big[d\Big(J\big(\omega,\xi(\omega)\big), AB\big(\omega,\xi(\omega)\big)\Big)\Big]\,\big[d\big(J(\omega,h(\omega)), ST(\omega,h(\omega)\big)\big]}{d\Big(J\big(\omega,\xi(\omega)\big), J\big(\omega,h(\omega)\big)\Big)} + \beta\,(\omega)\,d\Big(J\,\big(\omega,\xi(\omega)\big), J\big(\omega,h(\omega)\big)\Big)$$

Taking limit $n\rightarrow\infty$ and using the result we obtain that

$$d\big(\xi(\omega),h(\omega)\big) \le \alpha\,(\omega)\frac{\big[d\big(\xi(\omega),\xi(\omega)\big)\big]\,\big[d\big(h(\omega),h(\omega)\big)\big]}{d\big(\xi(\omega),h(\omega)\big)} + \beta\,(\omega)\,d\big(\xi(\omega),h(\omega)\big)$$

Yielding thereby

$$\xi(\omega) = h(\omega)$$

Hence $\xi(\omega)$ is a unique common random fixed point AB, ST and J.

Finally we need to show that $\xi(\omega)$ is a common random fixed point of random multivalued operators A, B, S, T and J. For this $\xi(\omega)$ is the unique common random fixed point of both the pair (AB, J) and (ST, J).

Then

$$A\big(\omega,\xi(\omega)\big) = A\Big(\omega, AB\big(\omega,\xi(\omega)\big)\Big) = A\Big(\omega, BA\big(\omega,\xi(\omega)\big)\Big) = AB\Big(\omega, A\big(\omega,\xi(\omega)\big)\Big)$$

$$A\big(\omega,\xi(\omega)\big) = A\Big(\omega, J\big(\omega,\xi(\omega)\big)\Big) = J\Big(\omega, A\big(\omega,\xi(\omega)\big)\Big)$$

$$B\big(\omega,\xi(\omega)\big) = B\Big(\omega, AB\big(\omega,\xi(\omega)\big)\Big) = BA\Big(\omega, B\big(\omega,\xi(\omega)\big)\Big) = AB\Big(\omega, B\big(\omega,\xi(\omega)\big)\Big)$$

$$B\big(\omega,\xi(\omega)\big) = B\Big(\omega, J\big(\omega,\xi(\omega)\big)\Big) = J\Big(\omega, B\big(\omega,\xi(\omega)\big)\Big)$$

This shows that $A\big(\omega,\xi(\omega)\big)$ and $B\big(\omega,\xi(\omega)\big)$ is a common random fixed point of (AB, J) yielding thereby

$$A\big(\omega,\xi(\omega)\big) = \xi(\omega) = B\big(\omega,\xi(\omega)\big) = J\big(\omega,\xi(\omega)\big) = AB\big(\omega,\xi(\omega)\big)$$

In the view of uniqueness of the common random fixed point of the pair (AB, J).

Similarly using the commutativity of (S, T), (S, J) and (T, J) it can be shown that

$$S\big(\omega,\xi(\omega)\big) = \xi(\omega) = T\big(\omega,\xi(\omega)\big) = J\big(\omega,\xi(\omega)\big) = ST\big(\omega,\xi(\omega)\big)$$

Now we need to show that

$$A\big(\omega,\xi(\omega)\big) = S\big(\omega,\xi(\omega)\big) \quad \text{and} \quad B\big(\omega,\xi(\omega)\big) = T\big(\omega,\xi(\omega)\big)$$

also remains a common random fixed point of both the pair (AB, J) and (ST, J).

For this

$$H\left(AB\left(\omega, A\left(\omega, \xi(\omega)\right)\right), ST\left(\omega, S\left(\omega, \xi(\omega)\right)\right)\right)$$

$$\leq \alpha(\omega) \frac{\left[d\left(J\left(\omega, A\left(\omega, \xi(\omega)\right)\right), AB\left(\omega, A\left(\omega, \xi(\omega)\right)\right)\right)\right]\left[d\left(J\left(\omega, S\left(\omega, \xi(\omega)\right)\right), ST\left(\omega, S\left(\omega, \xi(\omega)\right)\right)\right)\right]}{d\left(J\left(\omega, A\left(\omega, \xi(\omega)\right)\right), J\left(\omega, S\left(\omega, \xi(\omega)\right)\right)\right)}$$

$$+ \beta(\omega)\, d\left(J\left(\omega, A\left(\omega, \xi(\omega)\right)\right), J\left(\omega, S\left(\omega, \xi(\omega)\right)\right)\right)$$

$$d\left(A\left(\omega, \xi(\omega)\right), S\left(\omega, \xi(\omega)\right)\right) \leq 0$$

$$A\left(\omega, \xi(\omega)\right) = S\left(\omega, \xi(\omega)\right) \quad \text{for each } \omega \in \Omega$$

Similarly it can be show that

$$B\left(\omega, \xi(\omega)\right) = T\left(\omega, \xi(\omega)\right) \quad \text{for each } \omega \in \Omega$$

Thus $\xi(\omega)$ is a unique common random fixed point of random multivalued operators A, B, S, T and J.

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