



Exact solution of unsteady a flow past parabolic started isothermal vertical plate with variable mass diffusion

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ABSTRACT

The theoretical solution of flow past a parabolic starting motion of the infinite vertical plate with isothermal vertical plate and variable mass diffusion has been studied. The temperature of the plate is raised uniformly and species concentration level near the plate is made to rise linearly with time. The dimensionless governing equations are solved by using the Laplace-transform technique. The effect of velocity profiles are studied for different physical parameters like thermal Grashof number, mass Grashof number, Schmidt number, and time. It is observed that the velocity increases as the value of the thermal Grashof number or mass Grashof number increase. The trend is just reversed with respect to the Schmidt number.

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Introduction

There are many industrial applications like purification of crude oil, molten plastics, pulps, paper industry, textile industry; the cooling of threads or sheets is of importance in the process industries.

Natural convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method by Gupta *et al* [3]. Kafousias and Raptis [5] extended this problem to include mass transfer effects subjected to variable suction or injection. Soundalgekar [8] studied the mass transfer effects on flow past a uniformly accelerated vertical plate. Mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh [6]. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar[7]. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo[4]. Mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion was studied by Basant Kumar Jha *et al* [2]. The effect of a transverse magnetic field on unsteady free convective flow and mass diffusion of an electrically conducting elasto-viscous fluid past a parabolic starting motion of the infinite vertical plate was analyzed by Agrawal *et al* [1]. The governing equations are tackled using Laplace transform technique.

It is proposed to study the effects on flow past a parabolic started an infinite isothermal vertical plate with variable mass diffusion. The dimensionless governing equations are solved by using the Laplace-Transform Technique. The solutions are in terms of exponential and complementary error function.

Analysis

Here the unsteady flow of a viscous incompressible fluid past an infinite vertical plate with variable temperature and variable mass diffusion has been considered. The x' -axis is taken along the plate in the vertically upward direction and the y -axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature T_∞ and concentration C_∞ . At time $t' > 0$, the plate is started with a velocity $u = u_0 t'^2$ in its own plane against gravitational field. The plate temperature is raised uniformly and the mass is diffused from the

plate to the fluid is made to raise linearly with time t . Since the plate is infinite in length all the terms in the governing equations will be independent of x' and there is no flow along y-direction. Then under usual Boussinesq's approximation for unsteady parabolic starting motion is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \quad (1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} \quad (3)$$

With the following initial and boundary conditions:

$$u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad \text{for all } y, t' \leq 0$$

$$t' > 0: u = u_0 \cdot t'^2, \quad T = T_w, \quad C' = C'_\infty + (C'_w - C'_\infty) A t' \quad \text{at } y = 0 \quad (4)$$

$$u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C' \rightarrow C'_\infty \quad \text{as } y \rightarrow \infty$$

Where $A = \frac{u_0^2}{\nu}$.

On introducing the following non-dimensional quantities:

$$U = u \left(\frac{u_0}{\nu^2} \right)^{1/3}, \quad t = \left(\frac{u_0^2}{\nu} \right)^{1/3} t', \quad Y = y \left(\frac{u_0}{\nu^2} \right)^{1/3}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}$$

$$Gr = \frac{g\beta(T - T_\infty)}{(\nu \cdot u_0)^{1/3}}, \quad Gc = \frac{g\beta(C' - C'_\infty)}{(\nu \cdot u_0)^{1/3}}, \quad Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{\nu}{D} \quad (5)$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} \quad (6)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} \quad (7)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} \quad (8)$$

The initial and boundary conditions in non-dimensional quantities are

$$U = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all } Y, t \leq 0$$

$$t > 0: U = t^2, \quad \theta = 1, \quad C = t \quad \text{at } Y = 0 \quad (9)$$

$$U \rightarrow 0, \quad \theta \rightarrow 0, \quad C \rightarrow 0 \quad \text{as } Y \rightarrow \infty$$

The dimensionless governing equations (6) to (8) and the corresponding initial and boundary conditions (9) are tackled using Laplace transform technique.

$$\theta = \operatorname{erfc}(\eta\sqrt{pr}) \quad (10)$$

$$C = t[(1+2\eta^2 Sc) \operatorname{erfc}(\eta\sqrt{Sc}) - \frac{2\eta}{\sqrt{\pi}} e^{-\eta^2 Sc}] \quad (11)$$

$$\begin{aligned} u = & \frac{t^2}{6} \left(2 - \frac{Gc}{(1-Sc)} \right) [(3+12\eta^2+4\eta^4) \operatorname{erfc}(\eta) - \frac{\eta}{\sqrt{\pi}} e^{-\eta^2}] \\ & - t \frac{Gr}{(1-Pr)} \left[\left((1+2\eta^2) \operatorname{erfc}(\eta) - \frac{2\eta}{\sqrt{\pi}} \right) e^{-\eta^2} \right] \\ & + t \frac{Gr}{(1-Pr)} \left[\left((1+2\eta^2 pr) \operatorname{erfc}(\eta\sqrt{pr}) - \frac{2\eta\sqrt{pr}}{\sqrt{\pi}} \right) e^{-\eta^2 pr} \right] \\ & + \frac{t^2}{6} \left(\frac{Gc}{(1-Sc)} \right) \left[\left[(3+12\eta^2 Sc + 4\eta^4 (Sc)^2) (\operatorname{erfc}(\eta\sqrt{Sc})) \right] - \frac{\eta\sqrt{Sc}}{\sqrt{\pi}} (10+4\eta^2 Sc) e^{-\eta^2 Sc} \right] \quad (12) \end{aligned}$$

$$\text{Where } \eta = \frac{Y}{2\sqrt{t}}$$

Results and Discussion

For physical understanding of the problem, numerical computations are carried out for the different physical parameters depend upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the value of Prandtl number ($Pr = 0.71$) is chosen such that they represent air. The numerical values of the velocity, temperature and concentration are computed for the different physical parameters like Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

Figure 1 shows the velocity profile for different values of Schmidt number ($Sc=0.16, 0.3, 0.6, 2.01$), $Gr = Gc = 5$ and $t = 0.4$. It is observed that the velocity increases with decreasing Schmidt number. The relative variation of the velocity with the magnitude of the Schmidt number is also observed.

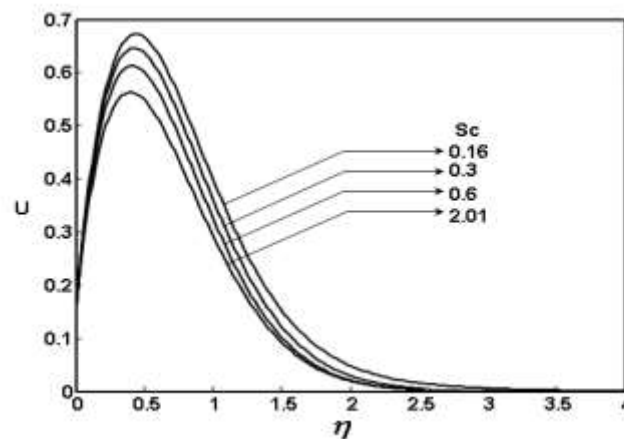


Figure 1: Velocity profile for different values of Sc

The velocity profile for different value of thermal Grashof number ($Gr=2.5$) and mass Grashof Number ($Gc=5,10$) are studied and presented in Figure 2 at the time $t=0.4$. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number. The effect of thermal Grashof number is very dominant, since the temperature of the plate is assumed to be uniform.

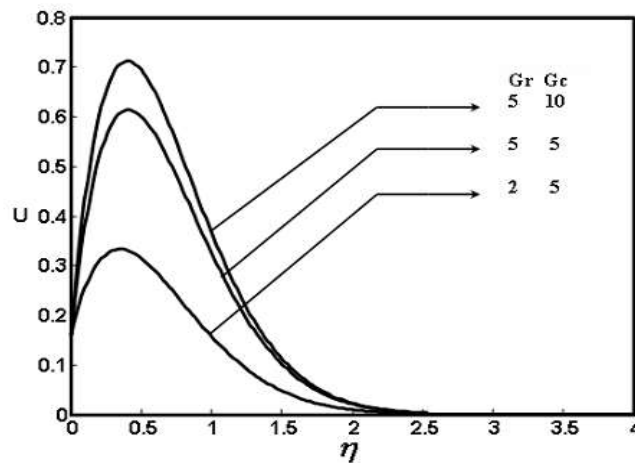


Figure 2: Velocity Profile for different values of Gr,Gc

Figure 3 illustrates that the velocity profile for different value of time ($t=0.2,0.4,0.6,0.8$) and $Gr=Gc=5$. It is observed that the velocity increases gradually with respect to time t .

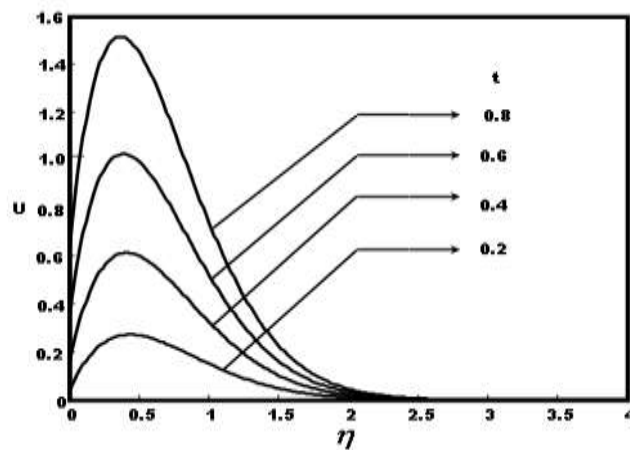


Figure 3: Velocity Profile for different values of t

Figure 4 shows that the temperature profile for different Prandtl number. The effect of temperature field for different value of time ($t = 0.2, 0.4, 0.6, 0.8$) in the presence of air. It is observed that the plate temperature increases with increasing values of the time t .

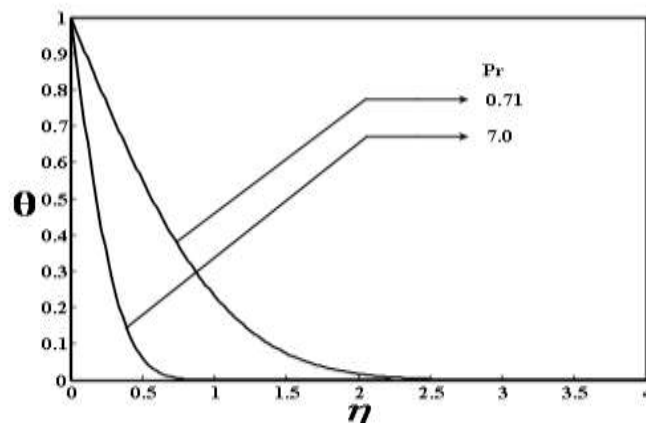


Figure 4 : Temperature Profile for different Pr

Figure 5 represents the effect of concentration profile for different time ($t=0.2, 0.4, 0.6, 0.8$) and $Sc=0.6$. The trend shows that the concentration increases with increasing values of the time t .

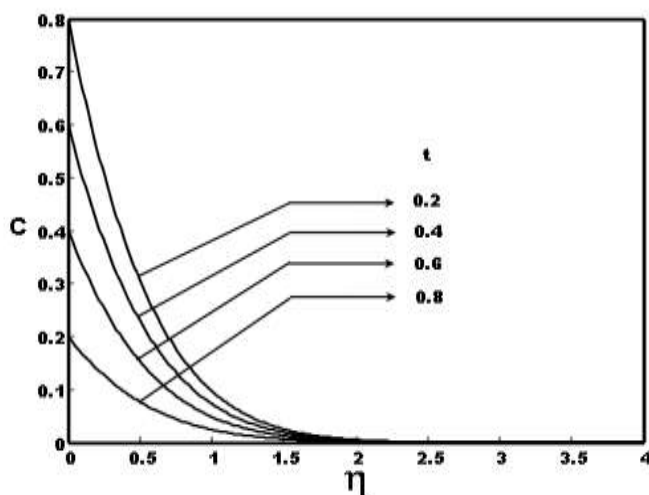


Figure 5 : Concentration Profile for different t

Figure 6 illustrate the concentration profile for different Schmidt number. The effect of concentration is important in concentration field. The profile have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the concentration increases with decreasing value of the Schmidt number ($Sc = 0.6, 0.78, 1, 2.01$) and time $t = 0.4$

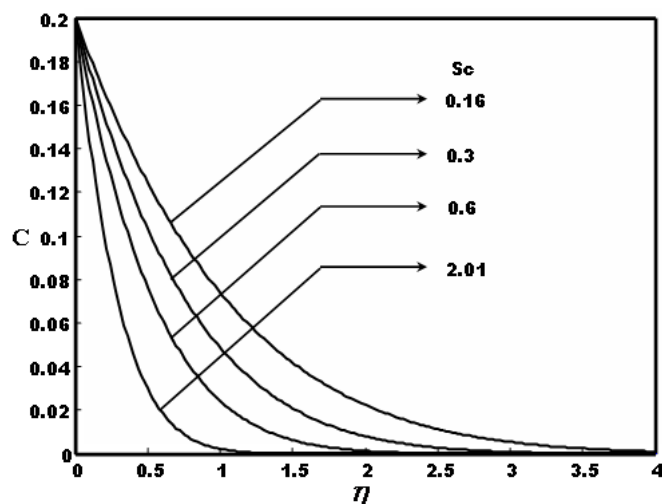


Figure 6 : Concentration Profile for different Sc

CONCLUSION

The theoretical solution of flow past a parabolic starting motion of the infinite isothermal vertical plate in the presence of variable mass diffusion has been studied. The dimensionless governing equations are solved by the usual Laplace transform technique. The effect of different physical parameters like thermal Grashof number, mass Grashof number, Schmidt number and Time are studied graphically. The conclusions of the study are as follows:

- (i) The velocity increases with increasing thermal Grashof number or mass Grashof number.
- (ii) The velocity increases with increasing values of the time t , but the trend is just reversed with respect to the Schmidt number.

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NOMENCLATURE

A	Constants
C'	species concentration in the fluid $kg\ m^{-3}$
C	dimensionless concentration
C_p	specific heat at constant pressure $J.kg^{-1}.k$
D	mass diffusion coefficient $m^2.s^{-1}$
Gc	mass Grashof number
Gr	thermal Grashof number
g	acceleration due to gravity $m.s^{-2}$
k	thermal conductivity $W.m^{-1}.K^{-1}$
Pr	Prandtl number
Sc	Schmidt number
T	temperature of the fluid near the plate K
t'	time s
u	velocity of the fluid in the x' -direction $m.s^{-1}$
u_0	velocity of the plate $m.s^{-1}$
u	dimensionless velocity
y	coordinate axis normal to the plate m
Y	dimensionless coordinate axis normal to the plate

Greek symbols

β	volumetric coefficient of thermal expansion K^{-1}
β^*	volumetric coefficient of expansion with concentration K^{-1}
μ	coefficient of viscosity $Ra.s$
ν	kinematic viscosity $m^2.s^{-1}$
ρ	density of the fluid $kg.m^{-3}$
τ	dimensionless skin-friction $kg.m^{-1}.s^2$
θ	dimensionless temperature
η	similarity parameter
$erfc$	complementary error function

Subscripts

w	conditions at the wall
∞	free stream conditions