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Applied Mathematics

Elixir Appl. Math. 59 (2013) 15543-15546

ISSN: 2229-712X

First order chemical reaction on flow past a parabolic started vertical plate with variable temperature and uniform mass diffusion

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ARTICLE INFO

Article history:
Received: 30 April 2013;
Received in revised form:
3 June 2013;
Accepted: 7 June 2013;

Keywords
Parabolic,
Homogeneous,
Heat and mass transfer,
Chemical reaction,
First order,
Vertical plate.

ABSTRACT

An exact solution of unsteady flow past a parabolic starting motion of an infinite vertical plate with variable temperature and uniform mass diffusion, in the presence of homogeneous chemical reaction of first order has been studied. The plate temperature is raised linearly with time and the concentration level near the plate is raised uniformly. The solutions for the velocity, temperature and concentration fields are solved using Laplace-transform technique. The effect of velocity profiles are studied for the physical parameters like chemical reaction parameter, thermal Grashof number, mass Grashof number, Schmidt number and time. It is observed that the velocity increases with increasing values the thermal Grashof number or mass Grashof number. The trend is just reversed with respect to the chemical reaction parameter or the Schmidt number.

Introduction

Chemical reactions can be divided in to two groups-homogeneous and heterogeneous. In the former case the reaction occurs in one phase only and the system is uniform throughout. In a heterogeneous reaction, the reaction occurs on the surface of a catalyst or the walls of a container and the mixture is not uniform throughout. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of first order, if the rate of reaction is directly proportional to the concentration itself. Chambre and Young [4] have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das et al [5] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das et al [6]. The dimensionless governing equations were solved by the usual Laplace-transform technique.

Natural convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method by Gupta *et al* [7]. Kafousias and Raptis [8] extended this problem to include mass transfer effects subjected to variable suction or injection. Soundalgekar [12] studied the mass transfer effects on flow past a uniformly accelerated vertical plate. Mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh [11]. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar [10]. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo [8]. Mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion was studied by Basant Kumar Jha *et al* [3]. Agrawal et al [1] studied free convection due to thermal and mass diffusion in laminar flow of an accelerated infinite vertical plate in the presence of magnetic filed. Agrawal et al [2] further extended the problem of unsteady free convective flow and mass diffusion of an electrically conducting elasto-viscous fluid past a parabolic starting motion of the infinite vertical plate with transverse magnetic plate. The governing equations are tackled using Laplace transform technique.

It is proposed to study the effects of on flow past an infinite isothermal vertical plate subjected to parabolic motion with uniform mass diffusion, in the presence of chemical reaction of first order. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

Mathematical Formulation

The unsteady flow of a viscous incompressible fluid past an infinite vertical plate with variable temperature and uniform diffusion, in the presence of chemical reaction of first order has been considered. The x'-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature T_∞ and concentration C'_∞ .At time $t' > 0$, the plate is started with a velocity $u = u_0 . t'^2$ in its own plane against gravitational field and the temperature from the plate is raised to T_w and the concentration level near the plate are also raised to C'_w. A chemically reactive species which transforms according to a simple reaction involving the conecntration is emitted from the plate and diffuses into the fluid. The reaction is

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assumed to take place entirely in the stream. Then under usual Boussinesq's approximation for unsteady parabolic starting motion is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \quad (1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} - k_l(C' - C'_\infty) \quad (3)$$

With the following initial and boundary conditions:

$$u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad \text{for all } y, t' \le 0$$
$$t' > 0: \; u = u_0 . t'^2, \; T = T_\infty + (T_w - T_\infty)At', \quad C' = C'_w \quad \text{at } y = 0 \quad (4)$$
$$u \to 0 \quad T \to T_\infty, \quad C' \to C'_\infty \quad \text{as } y \to \infty$$

On introducing the following non-dimensional quantities:

$$U = u\left(\frac{u_0}{\nu^2}\right)^{1/3}, \quad t = \left(\frac{u_0^{\,2}}{\nu}\right)^{1/3} t', \quad Y = y\left(\frac{u_0}{\nu^2}\right)^{1/3}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty},$$
$$C = \frac{C' - C'_\infty}{C'_w - C'_\infty}$$
$$Gr = \frac{g\beta(T - T_\infty)}{(u_0\nu)^{1/3}},$$
$$Gc = \frac{g\beta(C' - C'_\infty)}{(u_0\nu)^{1/3}}, \quad K = K_l\left(\frac{\nu}{u_0^{\,2}}\right)^{\frac{1}{3}}, \quad \Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{\nu}{D} \quad (5)$$

The equations (1) to (3) reduces to the following dimensionless form:

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} \quad (6)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} \quad (7)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - KC \quad (8)$$

The corresponding initial and boundary conditions in dimensionless form are as follows:

$$U = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all } Y, t \le 0$$
$$t > 0: \quad U = t^2, \quad \theta = t, \quad C = 1 \quad \text{at } Y = 0 \quad (9)$$
$$U \to 0, \quad \theta \to 0, \quad C \to 0 \quad \text{as } Y \to \infty$$

The dimensionless governing equations (6) to (8) and the corresponding initial and boundary conditions (9) are tackled using Laplace transform technique.

$$\theta = t\left[(1 + 2\eta^2 Pr)\, erfc(\eta\sqrt{Pr}) - \frac{2\eta}{\sqrt{\pi}}\sqrt{Pr}\, \exp(-\eta^2 Pr)\right] \quad (10)$$

$$C = \frac{1}{2}\left[\exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} - \sqrt{Kt})\right] \quad (11)$$

Results and Discussion

For physical understanding of the problem numerical computations are carried out for different physical parameters *Gr*, *Gc*, *Sc* and *t* upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the values of Prandtl number Pr are chosen such that they represent air (Pr = 0.71). The numerical values of the velocity are computed for different physical parameters like chemical reaction parameter, Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

$$U = \frac{t^2}{3}\left[(3 + 12\eta^2 + 4\eta^4)\, erfc(\eta) - \frac{\eta}{\sqrt{\pi}}(10 + 4\eta^2)\exp(-\eta^2)\right]$$
$$+ \frac{bt^2}{6}\left[\begin{array}{l}(3 + 12\eta^2 \Pr + 4\eta^4 (\Pr)^2)\, erfc(\eta\sqrt{\Pr}) - \frac{\eta\sqrt{\Pr}}{\sqrt{\pi}}(10 + 4\eta^2 \Pr)\exp(-\eta^2 \Pr) \\ -(3 + 12\eta^2 + 4\eta^4)\, erfc(\eta) + \frac{\eta}{\sqrt{\pi}}(10 + 4\eta^2)\exp(-\eta^2)\end{array}\right]$$
$$+ \frac{d}{2}\left[\begin{array}{l}2erfc(\eta) + \exp(at)\left[\begin{array}{l}\exp\left(2\eta\sqrt{Sc(K+a)t}\right) erfc\left(\eta\sqrt{Sc} + \sqrt{(K+a)t}\right) \\ + \exp\left(-2\eta\sqrt{Sc(K+a)t}\right) erfc\left(\eta\sqrt{Sc} - \sqrt{(K+a)t}\right)\end{array}\right] \\ - \exp(at)\left[\exp(2\eta\sqrt{at})\, erfc(\eta + \sqrt{at}) + \exp(-2\eta\sqrt{at})\, erfc(\eta - \sqrt{at})\right] \\ - \left[\exp(2\eta\sqrt{Sc\,K\,t})\, erfc(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{Sc\,K\,t})\, erfc(\eta\sqrt{Sc} - \sqrt{Kt})\right]\end{array}\right] \quad (12)$$

where, $a = \frac{K\,Sc}{1 - Sc}$ $b = \frac{Gr}{1 - \Pr}$ $d = \frac{Gc}{a(1 - Sc)}$ and $\eta = \frac{Y}{2\sqrt{t}}$.

Figure 1 llustrates the effect of the concentration profiles for different values of the chemical reaction parameter $(K = 0.2, 2, 5, 10)$ at $t = 0.4$. The effect of chemical reaction parameter is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the concentration increases with decreasing chemical reaction parameter.

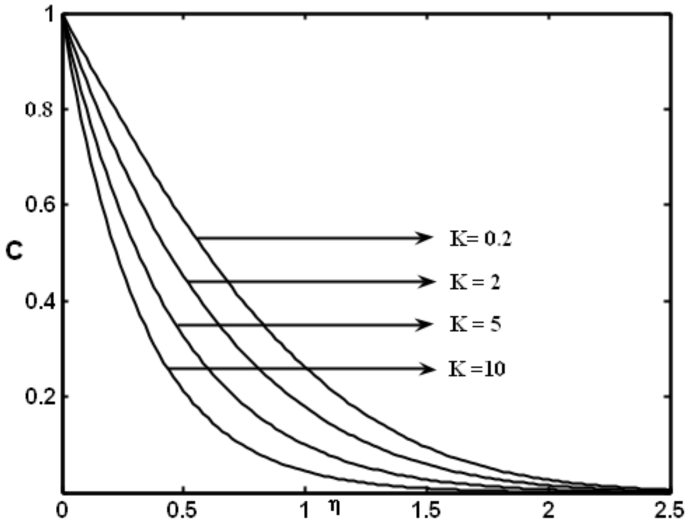

Figure 1. Concentration profiles for different values of K

The velocity profiles for different values of the chemical reaction parameter $(K = 0.2, 2, 5), Gr = 5 = Gc, \Pr = 0.71$ and $t = 0.2$ are shown in figure 2. It is observed that the velocity increases with decreasing chemical reaction parameter. Figure 3 demonstrates the effects of different thermal Grashof number (*Gr* = 2, 5), mass Grashof number (*Gc* = 5,10), K=2 and Pr=0.71 on the velocity at *t* = 0.2. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

The velocity profiles for different values of the time $(t = 0.3, 0.4, 0.6, 0.8), K = 2, Gr = 5$ and $Gc = 5$ are presented in figure 4. The trend shows that the velocity increases with incrasing values of the time t. The effect of velocity profiles for different values of the Schmidt number $(Sc = 0.16, 0.3, 0.6), Gr = 5 = Gc, \Pr = 0.71$ and $t = 0.2$ are shown in figure 5. It is observed that the velocity increases with decrasing values of the Schmidt number.

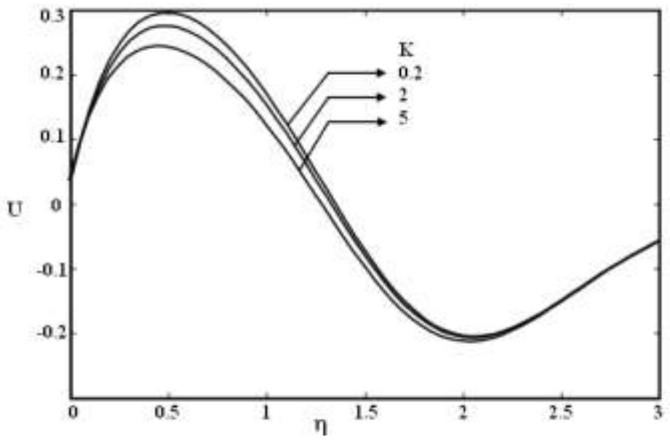

Figure 2. Velocity profiles for different K

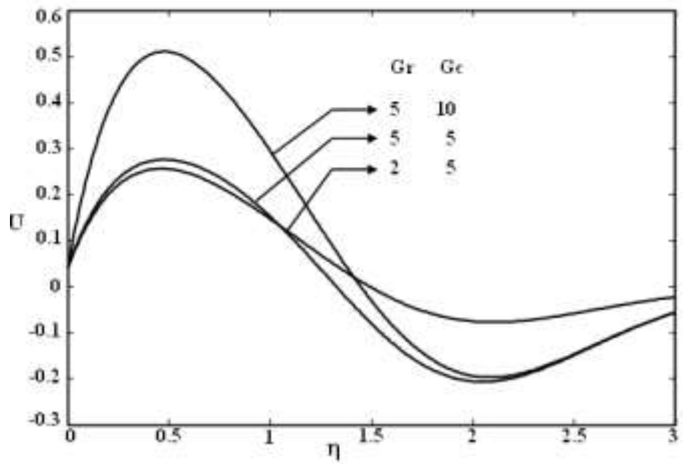

Figure 3. Velocity profiles for different Gr and Gc

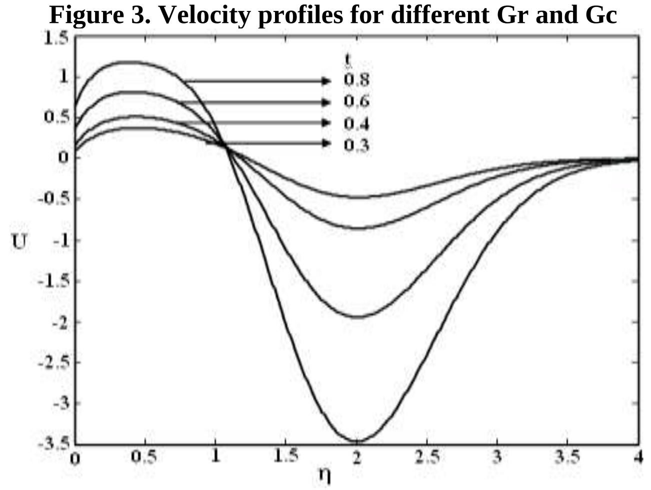

Figure 4. Velocity profiles for different t

Conclusion

An exact solution of flow past a parabolic starting motion of the infinite vertical plate with variable temperature and uniform mass diffusion, in the presence of chemical reaction of first order has been studied. The dimensionless governing equations are solved by the usual Laplace transform technique. The effect of the temperature, the concentration and the velocity fields for the physical parameters like chemical reaction parameter, thermal Grashof number and mass Grashof number are studied graphically. The conclusions of the study are as follows:

(i) The velocity increases with increasing thermal Grashof number or mass Grashof number, but the trend is just reversed with respect to the chemical reaction parameter.

(ii) The temperature of the plate increases with decreasing values of the Prandtl number.

(iii) The plate concentration increases with decreasing values of the chemical reaction parameter.

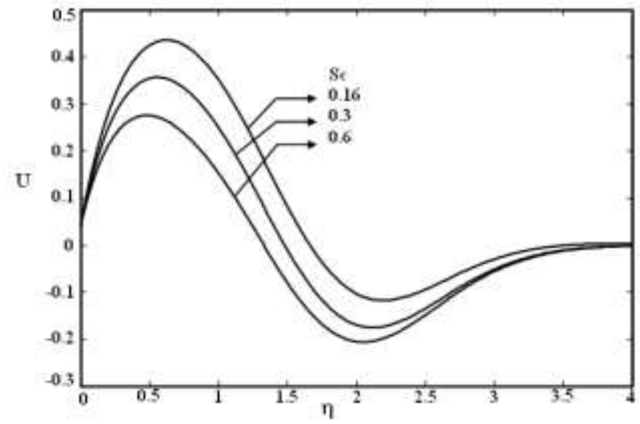

Figure 5. Velocity profiles for different Sc

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Nomenclature

A Constants
C' species concentration in the fluid $kg\,m^{-3}$
C dimensionless concentration
C_p specific heat at constant pressure $J.kg^{-1}.k$
D mass diffusion coefficient $m^2.s^{-1}$
Gc mass Grashof number
Gr thermal Grashof number
g acceleration due to gravity $m.s^{-2}$
k thermal conductivity $W.m^{-1}.K^{-1}$
Pr Prandtl number
Sc Schmidt number
T temperature of the fluid near the plate K
t' time s
u velocity of the fluid in the x'-direction $m.s^{-1}$
u_0 velocity of the plate $m.s^{-1}$
u dimensionless velocity
y coordinate axis normal to the plate m
Y dimensionless coordinate axis normal to the plate

Greek symbols

β volumetric coefficient of thermal expansion K^{-1}
β^* volumetric coefficient of expansion with concentration K^{-1}
μ coefficient of viscosity $Ra.s$
ν kinematic viscosity $m^2.s^{-1}$
ρ density of the fluid $kg.m^{-3}$
τ dimensionless skin-friction $kg.m^{-1}.s^2$
θ dimensionless temperature
η similarity parameter
$erfc$ complementary error function

Subscripts

w conditions at the wall
∞ free stream conditions