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Exponential dual to Ratio and dual to Product-type Estimators for Finite Population Mean in Double Sampling

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ABSTRACT

This paper presents exponential dual to ratio and dual to product-type estimators for estimating finite population mean using auxiliary information in double sampling. The expressions for bias and mean square error of the proposed estimators have been derived for two different cases up to the first order approximation. Comparisons have been made with other estimators *viz*. simple mean per unit estimator, usual ratio estimator Cochran and product estimator Murthy , dual to ratio estimator Kumar and Bahl and dual to product estimator Singh and Choudhury estimator, exponential ratio and product estimators Singh and Vishwakarma in double sampling. Empirical studies have also been carried out to show the merits of the proposed estimators over the existing estimators. It is concluded that the use of proposed estimators should be preferred in practice.

1. Introduction:

It is well established fact that use of auxiliary information in sample survey increases the precision of the estimate of population mean of study variate. Cochran [2] used auxiliary information and proposed usual ratio estimator for estimating population mean of study variate Y. Robson [8] and Murthy [5] worked independently and proposed usual product estimator. Srivenkataramana [11] first proposed dual to ratio estimator to estimate finite population mean. With known population mean $\bar{X}$, Bahl and Tuteja [1] were the first to suggest an exponential ratio and product-type estimators for estimation of population mean of the study variable Y .

In estimation of ratio and product estimators, it is assumed that the auxiliary information is known in advance. However, there are some situations; where the population mean of auxiliary information $\bar{X}$ is not known in advance. In such a situation, two phase (or double) sampling Neyman [7] can be used for getting ratio or product estimators. Kumar and Bahl [4] proposed dual to ratio estimator in double sampling. Singh and Vishwakarma [10] suggested exponential ratio and product-type estimator in double sampling.

Let us consider a finite population $U=\{U_1,U_2,...U_N\}$ of size N units and the value of the variables on the ith unit $U_i, i=1,2,...,N$, be (Y_i, X_i). Let $\bar{Y}=\sum_{i=1}^{N}\frac{Y_i}{N}$ and $\bar{X}=\sum_{i=1}^{N}\frac{X_i}{N}$ be the population means of the study variable Y and the auxiliary variable X respectively. For estimating the population mean $\bar{Y}$ of y, a simple

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random sample of size n is drawn without replacement from the population U. The classical ratio and product type estimators are defined by

$$\hat{\bar{Y}}_R = \bar{y}\frac{\bar{X}}{\bar{x}},\ \text{if}\ \bar{x} \neq 0\ \text{and}\ \hat{\bar{Y}}_P = \bar{y}\frac{\bar{x}}{\bar{X}},$$

where $\bar{y}$ and $\bar{x}$ are the sample means of y and x respectively ,based on a sample of size n from a population of size N units and $\bar{X}$ is the population mean of x.

If the population mean $\bar{X}$ of the auxiliary variable x is not known before start of the survey, a first-phase sample of size n_1 is drawn from the population, on which only the auxiliary variable x is observed. Then, a second phase sample of size n is drawn, on which both study variable y and auxiliary variable x are observed. Let $\bar{x}_1 = \sum_{i=1}^{n_1}\frac{x_i}{n_1}$ denotes the sample mean of size n_1 based on the first phase sample and $\bar{y} = \sum_{i=1}^{n}\frac{y_i}{n}$ and $\bar{x} = \sum_{i=1}^{n}\frac{x_i}{n}$ denote the sample means of variables y and x respectively, obtained from the second phase sample of size n. The ratio and product estimators in double sampling for estimating population mean $\bar{Y}$ are respectively given by $\bar{y}_R^d = \left(\frac{\bar{y}}{\bar{x}}\right)\bar{x}_1$ and $\bar{y}_p^d = \left(\frac{\bar{y}}{\bar{x}_1}\right)\bar{x}$. Singh and Vishwakarma [10] suggested the exponential ratio and the product-type estimators for estimating $\bar{Y}$ in double sampling respectively as,

$$\bar{y}_{\mathrm{Re}}^d = \bar{y}\exp\left(\frac{\bar{x}_1 - \bar{x}}{\bar{x}_1 + \bar{x}}\right) \quad \text{and} \quad \bar{y}_{Pe}^d = \bar{y}\exp\left(\frac{\bar{x} - \bar{x}_1}{\bar{x} + \bar{x}_1}\right)$$

Using the transformation $\bar{x}^{*d} = \frac{n_1\bar{x}_1 - n\bar{x}}{n_1 - n}$, Kumar and Bahl [4] has obtained dual to ratio estimator as $\bar{y}_R^{*d} = \bar{y}\frac{\bar{x}^{*d}}{\bar{x}_1}$,where $\bar{x}^{*d}$ is an unbiased estimate of $\bar{X}$ and correlation between $\bar{y}$ and $\bar{x}^{*d}$ is negative. Utilizing the transformation of $\bar{x}^{*d}$, Singh & Choudhury [9] proposed dual to product estimator in double sampling as $\bar{y}_P^{*d} = \bar{y}\frac{\bar{x}_1}{\bar{x}^{*d}}$.

In the present paper, under SRSWOR, we have proposed an exponential dual to ratio and product-type estimators in double sampling. Numerical illustrations are given to show the performance of the proposed estimators over other estimators.

2. Proposed Estimators:

Following the estimators of Singh & Vishkarma [10] and using the transformation of Kumar & and Bahl [4], we propose an exponential dual to ratio and product-type estimators, respectively, as follows

$$\bar{y}_{\mathrm{Re}}^{*d} = \bar{y}\exp\left(\frac{\bar{x}^{d*} - \bar{x}_1}{\bar{x}^{d*} + \bar{x}_1}\right) \quad (1) \quad \text{and}$$

$$\bar{y}_{\mathrm{Pe}}^{*d} = \bar{y}\exp\left(\frac{\bar{x}_1 - \bar{x}^{*d}}{\bar{x}_1 + \bar{x}^{*d}}\right) \quad (2)$$

The bias and MSE of the proposed estimators are obtained for the following two cases.

Case I: When the second phase sample of size *n* is a subsample of the first phase of size n_1.

Case II: When the second phase sample of size *n* is drawn independently of the first phase sample of size n_1.

Case: I

To obtain Bias (B) and MSE (M) of the proposed estimators $\bar{y}_{\mathrm{Re}}^{*d}$ and $\bar{y}_{Pe}^{*d}$, let us write,

$$e_0 = \left(\bar{y} - \bar{Y}\right)/\bar{Y},\ e_1 = \left(\bar{x} - \bar{X}\right)/\bar{X},\ e_1' = \left(\bar{x}_1 - \bar{X}\right)/\bar{X}$$

Such that

$$\left.\begin{aligned}
&E(e_0) = E(e_1) = E(e_1') = 0\\
&E(e_0^2) = \frac{1-f}{n}C_y^2,\ E(e_1^2) = \frac{1-f}{n}C_x^2,\ E(e_1'^2) = \frac{1-f_1}{n_1}C_x^2\\
&E(e_0e_1) = \frac{1-f}{n}CC_x^2,\ E(e_0e_1') = \frac{1-f_1}{n_1}CC_x^2,\ E(e_1e_1') = \frac{1-f_1}{n_1}C_x^2
\end{aligned}\right\} \quad (3)$$

where,

$$f = \frac{n}{N},\ f_1 = \frac{n_1}{N},\ C_y = \frac{S_y}{\bar{Y}},\ C_x = \frac{S_x}{\bar{X}},\ C = \rho_{yx}\frac{C_y}{C_x},\ \rho_{yx} = \frac{S_{yx}}{S_yS_x}$$

$$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2,\ S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \bar{X}\right)^2,\ S_{yx} = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)\left(x_i - \bar{X}\right)$$

Expressing $\bar{y}_{\mathrm{Re}}^{*(d)}$ and $\bar{y}_{Pe}^{*d}$ in terms of e’s, we have

$$\bar{y}_{Re}^{*d} = \bar{Y}\left(1+e_0\right)\exp\left[\frac{1}{2}g'\left(e_1' - e_1\right)\left\{1 + \frac{(2+g')e_1' - g'e_1}{2}\right\}^{-1}\right]$$

and

$$\bar{y}_{Pe}^{*d} = \bar{Y}\left(1+e_0\right)\exp\left[\frac{1}{2}g'\left(e_1-e_1'\right)\left\{1+\frac{\left(2+g'\right)e_1'-g'e_1}{2}\right\}^{-1}\right]$$

Expanding the right hand sides of $\bar{y}_{\text{Re}}^{*(d)}$ and $\bar{y}_{Pe}^{*d}$, multiplying out and neglecting terms of e's greater than two, we get

$$\bar{y}_{\text{Re}}^{*d} - \bar{Y} \cong \bar{Y}\left\{e_0 + \frac{1}{2}g'\left(e_1' - e_1 + e_0e_1' - e_0e_1 + e_1e_1' - e_1'^2\right) + \frac{1}{4}g'^2\left(e_1e_1' - e_1'^2 - e_1^2\right) + \frac{1}{8}g'^2\left(e_1'^2 + e_1^2\right)\right\} \tag{4}$$

and

$$\bar{y}_{Pe}^{*d} - \bar{Y} \cong \bar{Y}\left\{e_0 + \frac{1}{2}g'\left(e_1 - e_1' + e_0e_1 - e_0e_1' - e_1e_1' + e_1'^2\right) + \frac{1}{4}g'^2\left(e_1'^2 + e_1^2 - 2e_1e_1'\right) + \frac{1}{8}g'^2\left(e_1'^2 + e_1^2\right)\right\} \tag{5}$$

Therefore, the bias of the estimators $\bar{y}_{\text{Re}}^{*(d)}$ and $\bar{y}_{Pe}^{*d}$ can be obtained by using the results of equation (3) in equations (4) and (5) as

$$B\left(\bar{y}_{\text{Re}}^{*d}\right)_I = \bar{Y}\left(\frac{3}{8}g'^2\frac{1-f}{n}C_x^2 + \frac{1}{8}g'^2\frac{1-f_1}{n_1}C_x^2 - \frac{1}{2}g'\frac{1-f^*}{n}CC_x^2\right)$$

and

$$B\left(\bar{y}_{\text{Pe}}^{*d}\right)_I = \bar{Y}\left\{\frac{3}{8}g'^2\frac{1-f}{n}C_x^2 - \frac{1}{8}g'^2\frac{1-f_1}{n_1}C_x^2 + \frac{1}{2}g'\frac{1-f^*}{n}CC_x^2\right\}$$

where $f^* = n/n_1$ and $g' = n/\left(n_1 - n\right)$.

From equation (4) and (5), we write

$$\bar{y}_{\text{Re}}^{*d} - \bar{Y} \cong \bar{Y}\left\{e_0 + \frac{1}{2}g'\left(e_1' - e_1\right)\right\} \tag{6}$$

and

$$\bar{y}_{\text{Pe}}^{*d} - \bar{Y} \cong \bar{Y}\left\{e_0 + \frac{1}{2}g'\left(e_1 - e_1'\right)\right\} \tag{7}$$

Squaring both sides of the equation (6) and (7), taking expectations and using the results of (3), we get the MSE of the estimator $\bar{y}_{\text{Re}}^{*(d)}$ and $\bar{y}_{Pe}^{*d}$ to the first order approximation as

$$M\left(\bar{y}_{\text{Re}}^{*d}\right)_I = \bar{Y}^2\left\{\frac{1-f}{n}C_y^2 + g'\frac{1-f^*}{n}C_x^2\left(\frac{1}{4}g' - C\right)\right\} \tag{8}$$

and

$$M\left(\bar{y}^{*d}_{Pe}\right)_I = \bar{Y}^2\left\{\frac{1-f}{n}C_y^2 + g'\frac{1-f^*}{n}C_x^2\left(\frac{1}{4}g'+C\right)\right\} \tag{9}$$

3. Efficiency Comparisons

(a) Efficiency comparison of exponential dual to ratio-type estimator $\bar{y}^{*d}_{\mathrm{Re}}$ in double sampling

(i) with sample mean per unit estimator $\bar{y}$

The MSE of sample mean per unit estimator is $M\left(\bar{y}\right) = \bar{Y}^2\frac{1-f}{n}C_y^2$ (10)

From equations (8) and (10), we have

$$M\left(\bar{y}\right) - M\left(\bar{y}^{*d}_{\mathrm{Re}}\right)_I = \bar{Y}^2 g'\frac{1-f^*}{n}C_x^2\left(C-\frac{1}{4}g'\right).$$

Therefore, the proposed estimator is more efficient than the sample mean per unit estimator

if $C > 0.25g'$.

(ii) with ratio estimator in double sampling

The MSE of ratio estimator in double sampling is

$$M\left(\bar{y}^{d}_{R}\right)_I = \bar{Y}^2\left\{\frac{1-f}{n}C_y^2 + \frac{1-f^*}{n}C_x^2\left(1-2C\right)\right\} \tag{11}$$

From equations (8) and (11), we have

$$M\left(\bar{y}^{d}_{R}\right)_I - M\left(\bar{y}^{*d}_{\mathrm{Re}}\right)_I = \bar{Y}^2\frac{1-f^*}{n}C_x^2\left(1-2C-\frac{1}{4}g'^2+g'C\right)$$

Therefore, the proposed estimator is more efficient than the ratio estimator in double sampling

if $1+g'C > 2C+0.25g'^2$.

(iii) with dual to ratio estimator in double sampling

The MSE of dual to ratio estimator in double sampling is

$$M\left(\bar{y}^{*d}_{R}\right)_I = \bar{Y}^2\left\{\frac{1-f}{n}C_y^2 + g'\frac{1-f^*}{n}C_x^2\left(g'-2C\right)\right\} \tag{12}$$

From equations (8) and (12), we have

$$M\left(\bar{y}_R^{*d}\right)_I - M\left(\bar{y}_{\text{Re}}^{*d}\right)_I = \bar{Y}^2 g' \frac{1-f^*}{n} C_x^2 \left(\frac{3}{4}g' - C\right).$$

Therefore, the proposed estimator is more efficient than dual to ratio estimator in double sampling if $3g' > 4C$.

(iv) with exponential ratio-type estimator in double sampling

The MSE of exponential ratio-type estimator in double sampling is

$$M\left(\bar{y}_{\text{Re}}^{d}\right)_I = \bar{Y}^2 \left\{\frac{1-f}{n} C_y^2 + \frac{1-f^*}{n} C_x^2 \left(\frac{1}{4} - C\right)\right\} \quad (13)$$

From equations (8) and (13), we have

$$M\left(\bar{y}_{\text{Re}}^{d}\right)_I - M\left(\bar{y}_{\text{Re}}^{*d}\right)_I = \bar{Y}^2 \frac{1-f^*}{n} C_x^2 g'\left(1-g'\right)\left\{0.25\left(1+g'\right) - C\right\}$$

Therefore, the proposed estimator is more efficient than exponential ratio estimator in double sampling if

$n_1 > 2n$ and $0.25\left(1+g'\right) > C$.

(b) Efficiency comparison of exponential dual to product-type estimator $\bar{y}_{Pe}^{*d}$ in double sampling

(i) with sample mean per unit estimator $\bar{y}$

From equations (9) and (10), we have

$$M\left(\bar{y}\right) - M\left(\bar{y}_{Pe}^{*d}\right)_I = -\bar{Y}^2 \frac{1-f^*}{n} g' C_x \left(0.25g' + C\right)$$

Therefore, the proposed estimator is more efficient than the sample mean per unit estimator if

$0.25g' + C < 0$.

(ii) with usual product estimator in double sampling

The MSE of product estimator in double sampling is

$$M\left(\bar{y}_P^{d}\right)_I = \bar{Y}^2 \left\{\frac{1-f}{n} C_y^2 + \frac{1-f^*}{n} C_x^2 \left(1+2C\right)\right\} \quad (14)$$

From equations (9) and (14), we have

$$M\left(\bar{y}_P^d\right)_I - M\left(\bar{y}_{Pe}^{*d}\right)_I = \bar{Y}^2 \frac{1-f^*}{n} C_x^2 g'\left(1+2C-0.25g'^2-g'C\right)$$

Therefore, the proposed estimator is more efficient than the product estimator in double sampling if

$$1+2C > g'\left(0.25g'+C\right).$$

(iii) with dual to product estimator in double sampling

The MSE of dual to product estimator in double sampling is

$$M\left(\bar{y}_P^{*d}\right)_I = \bar{Y}^2\left\{\frac{1-f}{n}C_y^2 + g'\frac{1-f^*}{n}C_x^2\left(g'+2C\right)\right\} \tag{15}$$

From equations (9) and (15), we have

$$M\left(\bar{y}_P^{*d}\right)_I - M\left(\bar{y}_{Pe}^{*d}\right)_I = \bar{Y}^2 g'\frac{1-f^*}{n}C_x^2\left(C+\frac{3}{4}g'\right)$$

Therefore, the proposed estimator is more efficient than the dual to product estimator in double sampling if

$$C > 0.$$

(iv) with exponential product-type estimator in double sampling

The MSE of exponential product-type estimator in double sampling is

$$M\left(\bar{y}_{Pe}^d\right)_I = \bar{Y}^2\left\{\frac{1-f}{n}C_y^2 + \frac{1}{4}\frac{1-f^*}{n}C_x^2\left(1+C\right)\right\} \tag{16}$$

From equations (9) and (16), we have

$$M\left(\bar{y}_{Pe}^d\right)_I - M\left(\bar{y}_{Pe}^{*d}\right)_I = \bar{Y}^2\frac{1-f^*}{n}C_x^2\left(1-g'\right)\left\{0.25\left(1+g'\right)+C\right\}$$

Therefore, the proposed estimator is more efficient than the exponential product-type estimator in double sampling if

$n_1 > 2n$ and $0.25\left(1+g'\right)+C > 0$.

Case: II

To obtain Bias and MSE of the proposed estimators $\bar{y}^{*d}_{\mathrm{Re}}$ and $\bar{y}^{*d}_{Pe}$, we have

$$\left.\begin{aligned} &E(e_0)=E(e_1)=E(e_1')=0\\ &E(e_0^2)=\frac{1-f}{n}C_y^2,\ E(e_1^2)=\frac{1-f}{n}C_x^2,\ E(e_1'^2)=\frac{1-f_1}{n_1}C_x^2\\ &E(e_0e_1)=\frac{1-f}{n}CC_x^2,\ E(e_0e_1')=0,\ \ E(e_1e_1')=0 \end{aligned}\right\} \tag{17}$$

Taking expectations in equations (4) and (5) and using the results of (17), we get the bias of the estimators $\bar{y}^{*(d)}_{\mathrm{Re}}$ and $\bar{y}^{*d}_{Pe}$ to the first order approximation as

$$B\left(\bar{y}^{*d}_{\mathrm{Re}}\right)_{II}=-\bar{Y}\left\{\frac{1}{8}g'^2f^{**}C_x^2+\frac{1}{2}g'C_x^2\left(\frac{1-f_1}{n_1}+\frac{1-f}{n}C\right)\right\} \text{ and}$$

$$B\left(\bar{y}^{*d}_{\mathrm{Pe}}\right)_{II}=\bar{Y}\left\{\frac{3}{8}g'^2f^{**}C_x^2+\frac{1}{2}g'C_x^2\left(\frac{1-f_1}{n_1}+\frac{1-f}{n}C\right)\right\}$$

where $f^{**}=\dfrac{1-f}{n}+\dfrac{1-f_1}{n_1}$.

Squaring both sides of the equations (6) and (7), taking expectations and using the results of equation (17), we get the MSE of $\bar{y}^{*(d)}_{\mathrm{Re}}$ and $\bar{y}^{*d}_{\mathrm{Pe}}$ as

$$M\left(\bar{y}^{*d}_{\mathrm{Re}}\right)_{II}=\bar{Y}^2\left\{\frac{1-f}{n}C_y^2+\frac{1}{4}g'^2f^{**}C_x^2-g'\frac{1-f}{n}CC_x^2\right\} \tag{18}$$

$$M\left(\bar{y}^{*d}_{\mathrm{Pe}}\right)_{II}=\bar{Y}^2\left\{\frac{1-f}{n}C_y^2+\frac{1}{4}g'^2f^{**}C_x^2+g'\frac{1-f}{n}CC_x^2\right\} \tag{19}$$

(c) Efficiency comparison of exponential dual to ratio estimator $\bar{y}^{*(d)}_{\mathrm{Re}}$ in double sampling

(i) with sample mean per unit estimator $\bar{y}$

From equations (10) and (18), we have

$$M(\bar{y})-M\left(\bar{y}^{*d}_{Re}\right)_{II}=\bar{Y}^2g'C_x^2\left(\frac{1-f}{n}C-\frac{1}{4}g'f^{**}\right)$$

Therefore, the proposed estimator is more efficient than the sample mean per unit estimator if $C(1-f)/n > 0.25g'f^{**}$

(ii) with ratio estimator in double sampling

The MSE of ratio estimator in double sampling is

$$M\left(\bar{y}_R^d\right)_{II} = \bar{Y}^2\left\{\frac{1-f}{n}C_y^2 + f^{**}C_x^2 - 2\frac{1-f}{n}CC_x^2\right\} \tag{20}$$

From equations (18) and (20), we have

$$M\left(\bar{y}_R^d\right)_{II} - M\left(\bar{y}_{\text{Re}}^{*d}\right)_{II} = \bar{Y}^2\left\{\frac{3}{4}g'^2 f^{**}C_x^2 + (g'-2)\frac{1-f}{n}CC_x^2\right\}$$

Therefore, the proposed estimator is more efficient than the ratio estimator in double sampling if

$(g'-2)C > 0$

(iii) with dual to ratio estimator in double sampling

The MSE of dual to ratio estimator in double sampling is

$$M\left(\bar{y}_R^{*d}\right)_{II} = \bar{Y}^2\left\{\frac{1-f}{n}C_y^2 + g'C_x^2\left(g'f^{**} - 2C\frac{1-f}{n}\right)\right\} \tag{21}$$

From equations (18) and (21), we have

$$M\left(\bar{y}_R^{*d}\right)_{II} - M\left(\bar{y}_{\text{Re}}^{*d}\right)_{II} = \bar{Y}^2 g'C_x^2\left(\frac{3}{4}g'f^{**} - \frac{1-f}{n}C\right)$$

Therefore, the proposed estimator is more efficient than dual to ratio estimator in double sampling if

$0.75g'f^{**} > C(1-f)/n$.

(iv) with exponential ratio-type estimator in double sampling

The MSE of exponential ratio-type estimator in double sampling is

$$M\left(\bar{y}_{\text{Re}}^d\right)_{II} = \bar{Y}^2\left\{\frac{1-f}{n}C_y^2 + \frac{1}{4}f^{**}C_x^2 - \frac{1-f}{n}CC_x^2\right\} \tag{22}$$

From equations (18) and (22), we have

$$M\left(\bar{y}_{\text{Re}}^{d}\right)_{II} - M\left(\bar{y}_{\text{Re}}^{*d}\right)_{II} = \bar{Y}^2 C_x^2 \left(1-g'\right)\left\{\frac{1}{4}\left(1+g'\right) f^{**} - \frac{1-f}{n} C\right\}$$

Therefore, the proposed estimator is more efficient than exponential ratio estimator in double sampling if

$n_1 > 2n$ and $0.25\left(1+g'\right) f^{**} > C\left(1-f\right)/n$.

(d) Efficiency comparison of exponential dual to product estimator $\bar{y}_{Pe}^{*(d)}$

(i) with sample mean per unit estimator $\bar{y}$

From equations (10) and (19), we have

$$M\left(\bar{y}\right) - M\left(\bar{y}_{Pe}^{*d}\right)_{II} = -\bar{Y}^2 g' C_x^2 \left(\frac{1}{4} g' f^{**} + \frac{1-f}{n} C\right)$$

Therefore, the proposed estimator is more efficient than the sample mean per unit estimator

if $0.25 g' f^{**} + C\left(1-f\right)/n < 0$

(ii) with product estimator in double sampling

The MSE of product estimator in double sampling is

$$M\left(\bar{y}_{P}^{d}\right)_{II} = \bar{Y}^2 \left\{\frac{1-f}{n} C_y^2 + C_x^2 \left(f^{**} + 2C\frac{1-f}{n}\right)\right\} \qquad (23)$$

From equations (19) and (23), we have

$$M\left(\bar{y}_{P}^{d}\right)_{II} - M\left(\bar{y}_{Pe}^{*d}\right)_{II} = \bar{Y}^2 C_x^2 \left\{\frac{3}{4} g'^2 f^{**} + \frac{1-f}{n} C\left(2-g'\right)\right\}$$

Therefore, the proposed estimator is more efficient than the product estimator in double sampling if

either $C > 0$ and $2 - g' > 0$

or $C < 0$ and $2 - g' < 0$.

(ii) with dual to product estimator in double sampling

The MSE of dual to product estimator in double sampling is

$$M\left(\bar{y}_{P}^{*d}\right)_{II} = \bar{Y}^2 \left\{\frac{1-f}{n} C_y^2 + g' C_x^2 \left(g' f^{**} + 2C\frac{1-f}{n}\right)\right\} \qquad (24)$$

From equations (19) and (24), we have

$$M\left(\bar{y}_P^{*d}\right)_{II}-M\left(\bar{y}_{Pe}^{*d}\right)_{II}=\bar{Y}^2 g'C_x^2\left(\frac{3}{4}g'f^{**}+\frac{1-f}{n}C\right)$$

Therefore, the proposed estimator is more efficient than the dual to product estimator in double sampling if $C>0$.

(iv) with exponential product-type estimator in double sampling

The MSE of exponential product-type estimator in double sampling is

$$M\left(\bar{y}_{Pe}^{d}\right)_{II}=\bar{Y}^2\left\{\frac{1-f}{n}C_y^2+\frac{1}{4}f^{**}C_x^2+\frac{1-f}{n}CC_x^2\right\} \qquad (25)$$

From equations (19) and (25), we have

$$M\left(\bar{y}_{Pe}^{d}\right)_{II}-M\left(\bar{y}_{Pe}^{*d}\right)_{II}=\bar{Y}^2\left(1-g'\right)C_x^2\left\{\frac{1}{4}f^{**}\left(1+g'\right)+\frac{1-f}{n}C\right\}$$

Therefore, the proposed estimator is more efficient than the exponential product-type estimator in double sampling if

$1-g'>0$ and $C>0$.

4. Empirical Study

To examine the merits of the proposed estimators, we have considered three natural populations data sets. The descriptions of the populations are given below.

Population I: Source: Murthy [6]

X :Number of workers, Y : output

N = 80, n = 10, $n_1=30$, $\bar{Y}=5182.64$, $\rho_{yx}=.9150$

C_y = 0.3542, C_x = 0.9484

Population II: Source**:** Steel and Torrie [12]

X : Chlorine percentage, Y : log of leaf burn in sacs, N = 30, n = 4, $n_1=12$,

$\bar{Y}=.6860$, ρ_{yx}= -0.4996

C_y = 0.4803, C_x = 0.7493

Population III: Source: Dobson [3]

X : initial white blood cell count, Y : survival time leukaemia patient,

$N = 20$, $n = 4$, $n_1 = 8$, $\rho_{yx} = -.4074$

$C_y = 0.2017$; $C_x = 0{:}1502$

To observe the relative performance of different estimators of $\bar{Y}$, we have computed the percentage relative efficiency of the proposed estimators $\bar{y}_{\text{Re}}^{*d}$ and $\bar{y}_{Pe}^{*d}$, usual ratio, dual-to-ratio, exponential ratio estimator, exponential product estimator in double sampling and sample mean per unit estimator $\bar{y}$ with respect to usual unbiased estimator $\bar{y}$ in the two cases. The findings are presented in Table 1 and Table 2

Table 1: Percentage relative efficiencies of different estimators w. r. t. $\bar{y}$ for Case I

Estimator→	$\bar{y}$	$\bar{y}_R^d$	$\bar{y}_P^d$	$\bar{y}_R^{*d}$	$\bar{y}_P^{*d}$	$\bar{y}_{\text{Re}}^d$	$\bar{y}_{Pe}^d$	$\bar{y}_{\text{Re}}^{*d}$	$\bar{y}_{Pe}^{*d}$
Population I	100	36.65	*	200.42	*	200.23	*	245.05	*
Population II	100	*	94.48	*	115.14	*	123.61	*	122.36
Population III	100	*	103.35	*	103.37	*	111.48	*	111.48

*Data not applicable

Table 2: Percentage relative efficiencies of different estimators w. r. t. $\bar{y}$ for Case II

Estimator→	$\bar{y}$	$\bar{y}_R^d$	$\bar{y}_P^d$	$\bar{y}_R^{*d}$	$\bar{y}_P^{*d}$	$\bar{y}_{\text{Re}}^d$	$\bar{y}_{Pe}^d$	$\bar{y}_{\text{Re}}^{*d}$	$\bar{y}_{Pe}^{*d}$
Population I	100	20.09	*	200.42	*	130.02	*	303.23	*
Population II	100	*	74.56	*	115.14	*	122.22	*	125.39
Population III	100	*	86.58	*	103.37	*	112.73	*	112.73

*Data not applicable

5. Conclusion

In case I, the population I exhibits the gain in efficiency for the estimator $\bar{y}_{\text{Re}}^{*d}$, wherein Population II, the estimator $\bar{y}_{Pe}^{*d}$ shows its efficiencies over other estimators except $\bar{y}_{Pe}^{d}$, which has obtained little gain over it. In population III, the estimator $\bar{y}_{Pe}^{*d}$ shows its efficiencies over other estimators, except the estimator $\bar{y}_{Pe}^{d}$, where it is equally better.

In case II, in population I, the estimator $\bar{y}_{\text{Re}}^{*d}$ shows its high efficiency over other estimators, whereas, in population II and III, the estimator $\bar{y}_{Pe}^{*d}$ has shown its efficiency over others, except the estimator $\bar{y}_{Pe}^{d}$, which is equally better. Thus, the use of proposed estimators should be preferred in practice.

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