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# Payoff in Oligopoly Markets' A Mathematical Model

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**ABSTRACT**

Game Theory may be applied in situations in which a Decision Maker (DM) must take into account the reasoning of other Decision Makers. It has proved to be an enormously fruitful approach to the analysis of a wide range of problems. Any situation in which rivals make strategic choices, to which competitors will respond, can be assessed using game theory analysis. An Oligopoly is a market dominated by a few large suppliers. The degree of market concentration is very high (i.e. a large % of the market is taken up by the leading firms). Firms within an oligopoly produce branded products (advertising and marketing is an important feature of competition within such markets) and there are also barriers to entry. Another important characteristic of an oligopoly is interdependence between firms. This means that each firm must take into account the likely reactions of other firms in the market when making pricing and investment decisions. This creates uncertainty in such markets - which we seek to model through the use of game theory. The purpose of this paper is to develop a purely mathematical approach to determine a Payoff for oligopoly market. The model developed allows the researcher to derive Payoff Matrix in an oligopoly market using only assumption about each firm.

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## 1. Introduction

An oligopoly is an industry comprising a "few" firms. A duopoly, which is a special case of oligopoly, is an industry consisting of two firms. The distinguishing feature of oligopolistic or duopolistic market structures is the degree to which the output, pricing and other decisions of one firm affect and are affected by, the similar decision made by other firms in the industry. What is important, is the interdependence of the managerial decisions among the various firms in the industry. The analysis oligopolistic behavior may be modeled as a non-cooperative game in which the actions of one firm to increase market share will, unless countered, result in a reduction of the market share of other firms in the industry. Thus, action will be followed by reaction. This interdependence is the essence of an analysis of duopolistic or oligopolistic market structures. The characteristics of oligopoly are relatively few sellers, either standardized or differentiated products, price interdependence, and relatively difficult entry into and exit from the industry. A duopoly is an industry comprising two firms producing homogeneous or differentiated products in which entry and exit into and from the industry is difficult.

Consider duopoly , two firms with cost function

$$C_i = c_i q_i$$

$$c_i > 0$$

inverse demand function is :

$$p_i = \alpha_i - \beta_i q_i - \gamma q_j$$

Where $\gamma > 0$ meaning that the goods are substitutes.

If perfect substitutes $\alpha_1 = \alpha_2 = \alpha$ and $\gamma = \beta_1 = \beta_2$ and

$$p = \alpha - \gamma(q_1 + q_2)$$

Consider the profit functions

$$\pi_i (q_1, q_2) = p_i q_i - c_i q_i$$

$$= ( \alpha_i - \beta_i q_i - \gamma q_j ) q_i - c_i q_i$$

Oligopoly presents a problem in which decision makers must select strategies by taking into account the responses of their rivals, which they cannot know for sure in advance. A choice based on the recognition that the actions of others will affect the outcome of the choice and that takes these possible actions into account is called a strategic choice. Game theory is an analytical approach through which strategic choices can be assessed.

Among the strategic choices available to an oligopoly firm are pricing choices, marketing strategies, and product-development efforts.

An airline's decision to raise or lower its fares—or to leave them unchanged—is a strategic choice. The other airlines' decision to match or ignore their rival's price decision is also a strategic choice. IBM boosted its share in the highly competitive personal computer market in large part because a strategic product-development strategy accelerated the firm's introduction of new products.

## 2. Mathematical Derivation

### 2.1 Normal form game

A game in normal form consists of

1. A (finite) number of firms $M = \{a_1, a_2, a_3, \ldots, a_n\}$

2. A strategy set $S_i$ assigned to each player $i \in M$ the combination of all sets of strategies $S = \prod_{i \in M} S_i$ is called strategy space.

3. A utility /payoff function $u_i : S \to \Re$, assigned to each firm $i \in M \Rightarrow \forall s \in S : u_i(s) \in \Re$

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**2.2 Nash equilibrium**

**Notations:**

1. $s = S = \prod_{i \in M} S_i$ ; $s = (s_1, s_2, s_3, \bullet\bullet\bullet, s_M)$; $s_i \in S_i$
2. $s_{-i} = (s_1, s_2, s_3, \bullet\bullet\bullet, s_{i-i}, s_{i+1}, \bullet,\bullet,\bullet\ s_M)$; $(s_i, s_{-i}) = s$ ; $(s_{-i}, s_i) = s$
3. $S_{-i} = \prod_{i \in M,\ j \neq i} S_j$ ; $S_i \times S_{-i} = S$; $(s_i, s_{-i}) \in S_i \times S_{-i}$

A combination of strategies $s^* \in S$ is called a Nesh Equilibrium iff :

$$\forall i \in M \ \forall s_i \in S_i \ ; u_i(s^*) = u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*)$$

To understand the behavior of non-collusive oligopolists (*non-collusive meaning a few firms that do NOT cooperate on output and price*), economists have employed a mathematical tool called Game Theory. The assumption is that large firms in competition will behave similarly to individual players in a game such as poker. Firms, which are the "players" will make "moves" (referring to economic decisions such as whether or not to advertise, whether to offer discounts or certain services, make particular changes to their products, charge a high or low price, or any other of a number of economic actions) based on the predicted behavior of their competitors.

**3. Payoff Approach**

Once a firm implements a strategic decision, there will be an outcome. The outcome of a strategic decision is called a Payoff. In general, the payoff in an oligopoly firm is the change in economic profit to each firm. The firm's payoff depends partly on the strategic choice it makes and partly on the strategic choices of its rivals. Some firms in the airline industry, for example, raised their fares in 2005, expecting to enjoy increased profits as a result. They changed their strategic choices when other airlines chose to slash their fares, and all firms ended up with a payoff of lower profits, many went into bankruptcy.

If a large firm competing with other large firms understands the various "payoffs" (referring to the profits or losses that will result from a particular economic decision made by itself and its competitors) then it will be better able to make a rational, profit-maximizing (or loss minimizing) decision based on the likely actions of its competitors. The outcome of such a situation, or game, can be predicted using payoff matrixes.

A strategy $\hat{s}_i \in S_i$ is called best answer to a combination of strategies $s_{-i} \in S_{-i}$ iff :

$$\forall s_i \in S_i : (\hat{s}_i, s_{-i}) \geq u_i(s_i, s_{-i})$$

$$\Leftrightarrow u_i(\hat{s}_i, s_{-i}) = \max \{u_i(s_i, s_{-i}) \ ; \ s_i \in S_i\}$$

Which is the best answer already leads to the maximum Payoff.

Below is an illustration of a game between two "players" i.e. petroleum firms in India , Indian oil and Hindustan petroleum

The "moves" are the action the firms can take they can either advertise around a town or not.

The "payoffs" are the profits the firms will earns: Advertising increases firms costs, but can also increase revenues.

Consider the Following Table, Showing the Payoff Possibilities for Two Firms

**Considering Whether Or Not to Advertise**:

| | Indian oil: Not Advertise | Indian oil: Advertise |
|---|---|---|
| Hindustan petroleum: Not Advertise | Indian oil 's Payoff: 50<br>Firm B's Payoff: 50 | Indian oil 's Payoff: 60<br>Firm B's Payoff: 50 |
| Hindustan petroleum: Advertise | Indian oil 's Payoff: 50<br>Hindustan petroleum 's Payoff: 60 | Indian oil 's Payoff: 55<br>Hindustan petroleum 's Payoff: 55 |

In the game above, both Hindustan petroleum and Indian oil what is called a *dominant strategy.* Regardless of what its competitor does, both companies would maximize their outcome by advertising. If Hindustan petroleum were to *not advertise,* Indian oil will earn more profits (Rs 60 lac vs 50 lac) by advertising. If Hindustan petroleum were to *advertise,* Indian oil will earn more profits (Rs 60 lac vs 50 lac) by advertising. The payoffs are the same given both options for Hindustan petroleum. Since both firms will do best by advertising given the behavior of its competitor clearly, the total profits earned are more when both firms advertise than if they both did NOT advertise, We say that *advertise/advertise* is a ***"Nash Equilibrium"*** since neither firm has an incentive to vary its strategy at this point, since less profits will be earned by the firm that stops advertising.

As illustrated above, the tools of Game Theory, including the "payoff matrix", can prove helpful to firms deciding how to respond to particular actions by their competitors in oligopolistic markets. Of course, in the real world there are often more than two firms in competition in a particular market, and the decisions that they must make include more than simply to advertise or not. Much more complicated, multi-player games with several possible "moves" have also been developed and used to help make tough economic decisions a little easier in the world of competition.

**4. Conclusion**

The payoff table we use in our study reduces the complexity of the payoff structure by presenting all possible payoffs in a crystal clear way. This may help subjects to realize better what alternatives they have and what the consequences of these alternatives are. In particular, subjects may identify collusive quantities more easily. Hence, we conjecture that payoff tables should lead to more collusive behavior. On the other hand, one can think of an alternative conjecture: with payoff tables, subjects could also easily identify best-replies. This could drive the results more in the direction of Nash-equilibrium.

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