Available online at www.elixirpublishers.com (Elixir International Journal)

Mechanical Engineering

Elixir Mech. Engg. 60 (2013) 16609-16613

ISSN: 2229-712X

Optimal Design of Longitudinal Fin with different Profiles based on the least Material

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ARTICLE INFO

Article history:

Received: 28 May 2013;

Received in revised form: 15 July 2013;

Accepted: 24 July 2013;

Keywords

Longitudinal fin,
Radiation,
Convection,
Optimum dimensions.

ABSTRACT

Radiation is a volumetric phenomenon, and all solids, liquids, and gases emit, absorb or transmit radiation. However, radiation is usually considered to be a surface phenomenon for solids that are opaque to thermal radiation such as metals, woods and rocks since the radiation emitted by the interior regions of such material can never reach the surface, and the radiation incident on such bodies is usually absorbed within a few microns form the surface. Finned surfaces are used widely in industries, which due to their high temperature transferring heat by radiation is important. One of the this industry is space application, where the heat is transferred on in the mode of radiation. So, having the least material in very important in this industry. In this paper, Longitudinal fins with rectangular, triangular, concave, and convex profiles have been studied to establish the optimum dimensions based on the least material. The results showed that the concave parabolic has the best performance among other types of fins.

Introduction

Radiation is the energy emitted by matter in the form of electromagnetic waves (or photons) as a results of the changes in the electronic configurations of the atoms or molecules. Unlike conduction and convection, the transfer of energy by radiation does not require the presence of an interesting medium. In fact, energy transfer by radiation is fastest (at the speed of light) and it suffers no attenuation in a vacuum. Thermal radiation emitted by bodies is differs from other forms of electromagnetic radiation such as X-rays, gamma rays, microwaves, radio waves, and television waves that are not related to temperature. All bodies at a temperature above absolute zero emit thermal.

Many researchers have studied heat transfer by radiation in finned surfaces. Donovan and Rohrer [1] formulated a set of nonlinear integrodifferential equations pertaining to heat dissipation by radiation in an array of longitudinal fins of rectangular profile on a plane surface. These equations were solved numerically, and the results revealed that the fins are most effective when the spacing between them are relatively large and when shorter fins (smaller fin height) of higher thermal conductivity are employed they considered mutual irradiation and observed that this had an important overall effect on the heat exchange process.

Campo and Wolko [2] investigated the conduction-radiation interplay for a longitudinal fin of rectangular profile dissipating heat to the surrounding at a constant equivalent temperature. They illustrated their mathematical scheme for obtaining the heat transferred by radiation from the fins. Schnurr et.al, [3] used a nonlinear optimization approach to determine the minimum mass design for radiating finned arrays used in space. They considered straight and circular (Longitudinal and Radial) fins and included fin-to-fin and fin-to-base interactions in their analysis. The results were presented in graphical form of and gave optimum geometries for the profiles considered in terms of the dimensionless parameters which they proposed. Chiou and Na [4] developed an initial value method for the solution of nonlinear two-point boundary value problems that pertain to the analysis of radiating fins. This method is noniterative, computationally efficient , and gives good agreement with the solutions of identical problems solved by more convential methods. Mehta et.al, [5] obtained minimum mass designs for radiating finned arrays which he called heat sinks, using a direct search procedure using pseudorandom numbers. This analysis included fin-to-fin and fin-to-base interactions. Crawford et.al, [6] compared three methods of calculating the heat transfer by radiation from fins of arbitrary shape, Karam and Eby [7] showed that the differential equation for the temperature profile when radiation and convection are both present could be simplified considerably if the temperature to the fourth power in the radiation term is replaced by a linear expression about a term known as the mean temperature. Solution of the linearized steady-state equation was provided and a method was indicated in which the mean temperature was optimized as a function of the fin properties in order to minimize the errors introduced by the process of linearization.

Nomenclature			
A	heat transfer area	ξ	profile number
B	Beta function	η	fin efficiency
k	thermal conductivity	δ	thickness
L	height	ε	emissivity
q	heat transfer rate	**Subscripts**	
T	temperature	b	base
Greek symbols		∞	ambient

Truong and Mancuso[8] treated the problem of radiation from an annular (radial) fin whose surfaces had different emissivities. The study included various profile shapes and the results were obtained by the shooting method in conjunction

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with the Runge-Kutta-Verner fifth and Sixth-order integration method. The results were plotted as a function of dimensionless parameters proposed by the authors. Rao and Naidu [9-10] analyzed the laminar natural convection heat transfer from a fin array containing a vertical base and horizontal fins. They numerically solved governing equations of mass, momentum and energy balance for the fluid in the two fin enclosure together with the heat conduction equations in the fins using ADI method. They also investigated in the heat transfer from a horizontal fin array by natural convection and radiation. They estimated the total heat transfer rate and heat transfer coefficient making use of the heat fluxes. They considered heat transfer by radiation in this analysis. Their results are obtained for a four-fin array, and then compared with the experimental data. Their results showed good agreement. A research about the role of radiation view factor in thermal performance of straight-fin heat sinks was conducted by Kang and Hung [11]. They developed three models to investigate the effects of thermal radiation and its pertinent view factor on the convection coefficient. They concluded that the practice of neglecting the radiation view factor in the thermal analysis of fin arrays should be prohibited based on the fact that the errors generated are noticeably larger than those of solely neglecting thermal radiation. Azarkish et.al, [12] focused on optimal design of a longitudinal fin array with convection and radiation heat transfer using Genetic algorithm. The effects of the base temperature, the fin length and the fin height array on the optimum geometry and on the number of fins are investigated in this study. Bhanju and Kundu [13] presented a analytical thechnique based on the decomposition method to determine the temperature distribution and thermal performance parameters of a constructed T-shape porous media. They also considered the effect of radiation on natural convective heat transfer coefficient. Predicting of the temperature in a fin cooled by radiation and natural convection was carried out by Mueller and Mulaweh [14]. In a similar study, Goria and Bakier [15] studied on the thermal analysis of natural convection and radiation in porous fins. Karabacak et.al, [16] performed an experimental study on the effects of fin parameter on the radiation and free convection heat transfer from a finned horizontal cylindrical heater. Acharya and Patankar [17] conducted a study in order to find the effect of buoyancy on laminar mixed convection in a shrouded fin array. An experimental investigation on natural convection heat transfer from horizontal fin arrays was presented by Harahap and McManus [18]

Basic thermal performance

Rectangular profile

Consider a longitudinal fin of rectangular profile with length, L, height, b, thickness , δ, thermal conductivity ,k ,and emissivity ,ε. For the boundary conditions of constant base temperature, T_b , and an insulated tip, the heat dissipation per unit length can be obtained by,

$$q=2k\delta\left[\frac{\sigma\varepsilon\left(T_b^4-T_a^4\right)}{5k\delta}\right]^{0.5} \tag{1}$$

Here T_a, the fin tip temperature, is related to the temperature distribution in the fin, which can be expressed in term of the complete beta function, $B(a,b)$ and the incomplete beta function $B_u(a,b)$ as,

$$f(\delta,T_a)=B(0.3,0.5)-B_u(0.3,0.5)-b\left(\frac{2\sigma\varepsilon T_a^3}{k\delta}\right)=0 \tag{2}$$

The optimization problem is to find δ and T_a such that q is maximized while the profile area $A_p=\delta b$ remains fixed. Liu et.al,(1960, 1961) has solved this problem using the method of Lagrange multipliers. Consequently, T_a is given by,

$$T_a=0.425.T_b\left[12-G+G^{0.5}(G+120)^{0.5}\right]^{0.2} \tag{3}$$

Where,

$$G=\frac{5k\delta}{q\varepsilon\sigma T_b^3 b^2} \tag{4}$$

Substitution of equation (3) into (2) yields a transcendental equation in G that can be solved to give the solution G=1.381. Then, by using the definition of G , it is observed that for the optimum condition, the relationship between δ and b must be,

$$\frac{\delta}{b^2}=2.486\frac{\varepsilon\sigma}{k}T_b^3 \tag{5}$$

Upon elimination of b in favor of δ using the relation $b=\frac{A_p}{\delta}$,

$$\delta_{opt}=1.355\left(\frac{\sigma\varepsilon A_p^2 T_b^3}{k}\right)^{0.333} \tag{6}$$

And , the optimum height can be expressed as,

$$b_{opt}=0.738\left(\frac{kA_p}{\varepsilon\sigma T_b^3}\right)^{0.333} \tag{7}$$

Also using G=1.381 the tip temperature for the fin with optimum fin is determined as,

$$T_{a,opt}=0.799.T_b \tag{8}$$

The heat dissipation from the optimum fin is obtained as

$$q_{opt}=0.855\left[k(\varepsilon\sigma)^2 A_p T_b^9\right]^{0.333} \tag{9}$$

And finally the fin efficiency for the optimum conditions can be obtained as ,

$$\eta_{opt}=0.579 \tag{10}$$

Wilkins et.al, (1960) used a numerical method to find the optimum dimensions, when the heat dissipation, q , is specified rather than the profile area, A_p. He has shown that the optimum dimensions in terms of the heat dissipation , q , are given by,

$$b_{opt}=0.8844\left(\frac{q}{\varepsilon\sigma T_b^4}\right) \tag{11}$$

$$\delta_{opt}=1.8484\left(\frac{q^2}{k\varepsilon\sigma T_b^5}\right) \tag{12}$$

$$A_{opt} = 1.6347\left(\frac{q^3}{k\varepsilon^2\sigma^2 T_b{}^9}\right) \quad (13)$$

Triangular profile

The optimization problem for the longitudinal fin of triangular profile has been the subject of several studies. Wilkins et.al, (1960) formulated a novel similarity transformation and used it to replace the nonlinear fin differential equation with an expression for the profile area in terms of integral. The minimum of this integral establishes the optimum dimensions of the fin. Kern and Kraus (1972) tackled the same problem using a numerical procedure to solve for the efficiency of the fins as function of the profile number $\left(\xi = \frac{2\varepsilon\sigma b^2 T_b{}^3}{k\delta_b}\right)$ the same procedure has been used by Chung and Nguyen (1986) and Nilson and Curry (1960) to establish the optimum dimensions of a triangular fin. Smith (1992) also reported the results for the optimum triangular fin. The results of all of these studies can be expressed in the forms:

$$\delta_{b,opt} = C_1\left(\frac{q^2}{\sigma\varepsilon\, kT_b{}^5}\right) \quad (14)$$

$$b_{opt} = C_2\left(\frac{q}{\sigma\varepsilon T_b{}^4}\right) \quad (15)$$

Where the values of the constants C_1 and C_2 taken from the different studies appear in Table of (1). If $\delta_{b,opt}$ and b_{opt} are desired in terms of the profile area, A_p, the expressions are:

$$\delta_{b,opt} = C_3\left(\frac{\sigma\varepsilon A_p{}^2 T_b{}^3}{k}\right)^{0.333} \quad (16)$$

$$b_{opt} = C_4\left(\frac{kA_p}{\sigma\varepsilon T_b{}^3}\right)^{0.333} \quad (17)$$

Where C_3 and C_4 are also provided in Table of (1).

Table. (1) Constants C_1, C_2, C_3 and C_4 for eqs (14) –(17)

	C_1	C_2	C_3	C_4	η_{opt}	$\left(\frac{T_a}{T_b}\right)_{opt}$
Wilkins (1960)	2.2986	0.9598	-	-	-	-
Kern and Kraus (1972)	2.4255	0.9091	2.274	0.880	0.550	-
Chung and Nguyen(1986)	2.3104	0.9485	2.174	0.920	0.527	0.714
Nilson and Curry (1960)	2.3027	0.9546	2.162	0.925	0.524	0.711
Smith (1992)	2.4852	0.9040	-	-	-	-

Concave profile

Studies pertaining to the optimization of the Longitudinal fin of concave parabolic profile have apparently been sparse. One study by Chung and Nguyen (1986) has been done to establish the various relationships for the optimum dimensions.

$$\delta_{b,opt} = 2.621\left(\frac{\sigma\varepsilon A_p{}^2 T_b{}^3}{k}\right)^{0.333} \quad (18)$$

$$b_{opt} = 1.145\left(\frac{kA_p}{\sigma\varepsilon T_b{}^3}\right)^{0.333} \quad (19)$$

$$T_{a,opt} = 0 \quad (20)$$

$$q_{opt} = 0.998\left[k(\varepsilon\sigma)^2 A_p T_b{}^9\right]^{0.333} \quad (21)$$

$$\eta_{opt} = 0.436 \quad (22)$$

If q is specified instead of A_p, one can use equation of (21) to calculate A_p and then use eqs.(18) and (19) to find δ_{opt} and b_{opt}. Observe that the tip temperature for the optimum fin is equal to effective sink temperature.

Convex parabolic profile

The only study that relates to the optimization of this geometry is that of Chung and Nguyen (1966), who found the optimum relationships to be:

$$\delta_{b,opt} = 1.778\left(\frac{\sigma\varepsilon A_p{}^2 T_b{}^3}{k}\right)^{0.333} \quad (23)$$

$$b_{opt} = 0.843\left(\frac{kA_p}{\sigma\varepsilon T_b{}^3}\right)^{0.333} \quad (24)$$

$$T_{a,opt} = 0.766.T_b \quad (25)$$

$$q_{opt} = 0.927\left[k(\varepsilon\sigma)^2 A_p T_b{}^9\right]^{0.333} \quad (26)$$

$$\eta_{opt} = 0.55 \quad (27)$$

Case Study

For space applications, a case study has been done, where a fin must transfer 1461 W/m of heat to free space at $0\,K^\circ$. This fin is fabricated of steel with $k = 56.3$ W/m.K and $\varepsilon = 0.85$. The main assumption is that the fin base temperature is 875 K°.

Results and discussion

Table of (2) shows the results of mechanical design of different fins.

Table .(2) Optimum dimensions

	Rectangular	Triangular	Concave	Convex
δ_{opt}	2.81 mm	3.499 mm	4.01 mm	3.15 mm
b_{opt}	46.4 mm	50.3 mm	60 mm	47.7 mm

For the same heat dissipation and operational parameters, the results showed that the concave parabolic profile fin has the least profile area and consequently, is the lightest of the four shapes. It uses only 61.5 % as much material as the rectangular fin. However, the triangular fin, which uses only 9% more material than the concave parabolic fin, may be preferable for ease of fabrication.

As in the convective case, because A_p in each case is proportional to q^3, the increase in A_p is eightfold if q is to be doubled. Thus if a single fin is used to accommodate twice the heat dissipation, the fin becomes very bulky. In this case it is better to employ two identical fins instead of one. Indeed , the use of a large number of shorter and lighter fins as opposed to fewer longer and heavier fins results in a better design. Figure of (2) shows the relationship among profile area in one side and temperature and heat dissipation in other side.

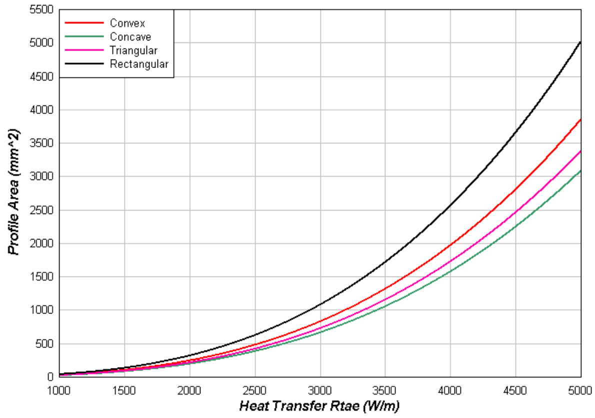

Figure.(1) Profile area for different types of Longitudinal fins

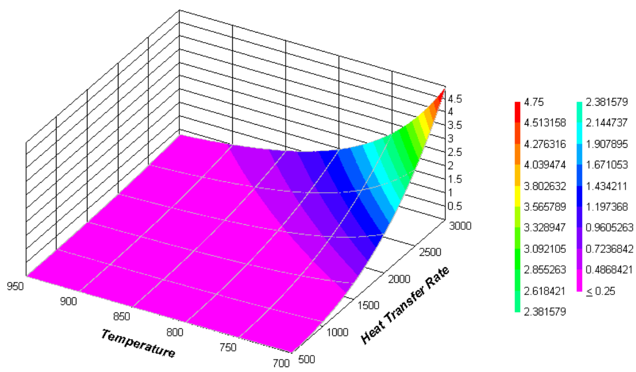

Figure.(2) Profile area for Concave profile fin

As it is clear when $500 \le q \le 1500$ the profile area is approximately constant. On the other hands, although increasing temperature base can decrease the profile area, but this decrease is not considerable, especially when $500 \le q \le 1500$ and $700 \le T \le 950$. So, there are some limitations for the heat dissipation from fin for all of the Longitudinal fins. Of course, in the many industries due to ease construction using rectangular profile is common. So, knowing temperature distribution is necessary. Applying the conservation energy requirement to the differential element of figure .(3):

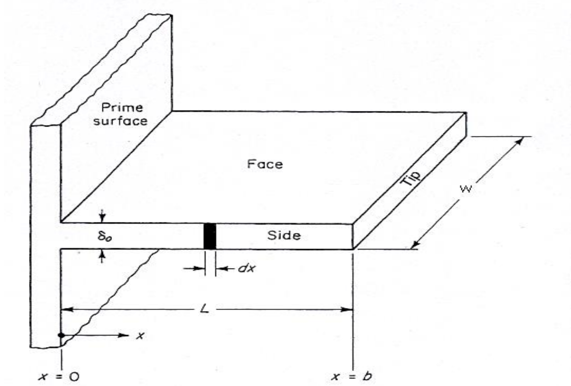

Figure.(3) Terminology and coordinate system for a radiating longitudinal fin of rectangular profile

$$q_x = q_{x+dx} + dq_{rad} \quad (28)$$

$$q_x = -kA_c \frac{dT}{dx} \quad (29)$$

$$dq_{rad} = \varepsilon\sigma dA_s \left(T^4 - T_e^{\,4}\right) \quad (30)$$

To reach a analytical solution it is assumed that $T_e \approx 0$ and let the fin be very long. So , the solution is:

$$\frac{d^2T}{dx^2} - \left(\frac{2\varepsilon\sigma W}{kA_c}\right)T^4 = 0 \quad (31)$$

Equation (31) is a nonlinear second order differential equation, which after algebraic operations the solution is:

$$T(x) = 1.813\left(\frac{\varepsilon\sigma W}{A_c k}\right)x^{-\frac{2}{3}} + C \quad (32)$$

To evaluate the constant C , it is necessary to specify appropriate boundary conditions. One such condition may be specified in terms of the temperature at the base of the fin.

$$T(x)\Big|_{x=0} = T_b \quad \Rightarrow \quad T(x) = 1.813\left(\frac{\varepsilon\sigma W}{A_c k}\right)x^{-\frac{2}{3}} + T_b \quad (33)$$

Figure of (4) demonstrates temperature distribution versus different emissivity in Rectangular profile.as we expect with high value of emissivity the heat dissipation from fin is more. Thus , to have the least material in designing a fin, not only the optimum dimensions is important but also the type of material play a key role(Figure of (4)).

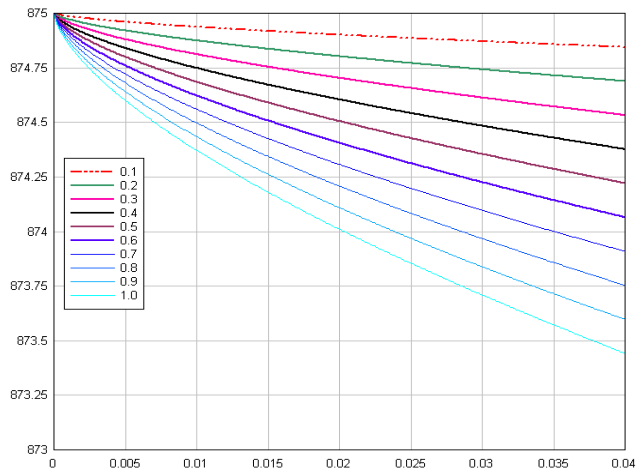

Figure.(4) Temperature distribution for Rectangular fin profile versus emissivity

To validate the results presented in the Table of (2) , studying entropy generation for four profile can be useful. Based on the concept of the entropy generation minimization that developed by Bejan [21], the rate of the entropy generation for the extended surfaces is:

$$\left(\dot{S}_{gen}\right)_{external} = \iint_A q''\left(\frac{1}{T_\infty} - \frac{1}{T_w}\right)dA + \frac{F_D U_\infty}{T_\infty} \quad (34)$$

Also, a fin generates entropy internally, because the fin nonisothermal,

$$\left(\dot{S}_{gen}\right)_{internal} = \iint_A \left(\frac{q''}{T_w}\right)dA - \frac{q_B}{T_b} \quad (35)$$

In this expression, q_B and T_b represent the base heat transfer and absolute temperature. Adding Equations of (34) and (35) side by side obtaining the entropy generation rate for a single fin is possible.

$$\dot{S}_{gen} = \frac{q_B \theta_B}{T_\infty^2} + \frac{F_D U_\infty}{T_\infty} \quad (36)$$

Where θ_B is the base-stream temperature difference $(T_b - T_\infty)$. Also, Drag coefficient for a pin fin is:

$$\begin{cases} C_D = \dfrac{F_D}{\frac{1}{2}\rho U_\infty DL} \\ C_D \cong 5.484\,\mathrm{Re}^{-0.246} \end{cases} \quad (37)$$

Table.3. Entropy generation results

	Rectangular	Triangular	Concave	Convex
$\dot{S}_{gen}$	667×10^{-3}	723×10^{-3}	862×10^{-3}	686×10^{-3}

Table of (3) dictates that rectangular profile has the minimum rate of entropy generation. On the other hands, rectangular profile among four profiles has the least length and thickness. The results showed that although concave profile is the lightest of four shape, the rate of entropy generation for this profile type is about 22% more than rectangular profile. Totally, the information presented in the Table (2) showed good agreement with experimental studies.

Conclusion

Radiation is the simplest means of heat transfer. Heat radiation is carried not by moving atoms (as in conduction or convection) but by electromagnetic waves. Radiation is the only way that heat can move through a vacuum. The biggest example of a vacuum would be space. In the space applications to enhance heat transfer making use of extended surfaces is common. In this situations, one of the important problem is that fin employed has the least material . In this paper we analyzed four common profiles of fins for optimum weight and heat transfer rate. The results showed that the fin with concave parabolic profile has the least material for the same conditions. Although the triangular fin uses only 9% more material than the concave parabolic fin, but due to ease o fabrication and maintenance many designers prefer to use them. Finally, temperature distribution of rectangular profiles versus emissivity is presented. The results denoted that when the emissivity is between 0.1 to 0.3, increasing the optimum height of fin is 10 mm , and increasing its height did not any effects on the cooling of the fin. Nevertheless, for the emissivity more than 0.5 there is direct relation between height of the fin and cooling.

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