



## Model – Assisted Estimation in Adaptive Sampling

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### ABSTRACT

Two problems often crop up in adaptive sampling. One, it may not be feasible to sample according to a designated sampling plan. And two, the prescribed sampling plan may result in very small selection probabilities for some units thereby giving large weights to such units in estimation. In order to ameliorate these problems, a regression procedure that combines design and model-based techniques of estimation is proposed. Adaptive sampling designs are designs in which additional units or sites for observation are selected depending on the interpretation of observations made during initial sampling. Additional sampling is driven by the observed results from an initial sample. The results from this study demonstrate that the existing Horvitz-Thompson estimator for adaptive cluster sampling can be improved using model assistance.

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### Introduction

Adaptive sampling designs are designs in which additional units or sites for observation are selected depending on the interpretation of observations made during initial sampling. Additional sampling is driven by the observed results from an initial sample. Several designs may be considered adaptive sampling designs (Thompson 1990, 1997 and Dryver, 1999, Adeleke et al 2007, Adeleke et al 2010). Adaptive cluster sampling design is implemented using the following basic algorithms: (1) Selecting the initial probability-based sample, (2) specifying a rule or criterion for performing additional sampling, and (3) defining the neighborhood of a sampling unit (Chambers, 2003). To draw an adaptive sample, a grid is placed over a geographical area of interest (target population) where each grid square is a potential (primary) sampling unit (Thompson et al, 1992). The final sample consists of clusters of selected (observed) units around the initial observed units. Each cluster is bounded by a set of observed units that do not exhibit the characteristic of interest. These are called *edge units*. A cluster without its edge units is called a *network*. Any observed unit, including an edge unit, that does not exhibit the characteristic of interest is a network of size one. Hence, the final sample can be partitioned into non-overlapping networks. These definitions are important in understanding the estimators for statistical parameters.

A Horvitz-Thompson (HT) and Hansen-Hurwitz (HH)-type estimators of the mean and variance (of the sampled population) based on the final sample, as proposed by Thompson (1990) are typically used with adaptive cluster sampling. A modified version of the HT-type and HH-type estimators using the Rao-Blackwell theorem was proposed (Thompson 1991, 1992, 1997, 2006) as a correction for the problems of selection probabilities that cannot be determined for all the units in the final sample. The usual unbiased estimators in adaptive cluster sampling are very simple but do not necessarily utilize all the information gathered. A more efficient estimator that utilizes this information of a repeat selection was discussed by Dryver

(1999). Improvements have also been made in the case when an initial sample is taken without replacement. In particular, the values of edge units are utilized in the estimators only for edge units that were picked in the initial sample. Estimators that can incorporate this information can be obtained using the Rao-Blackwell method conditioning on the minimal sufficient statistic (Thompson 1997, Philippi 2005).

The Horvitz-Thompson (HT) estimator (Horvitz and Thompson, 1952) that was used to derive the estimators for adaptive sampling applies the idea of design based inference in a rather general sense but can have large variance (Adeleke, Esan and Okafor, 2007, Adeleke et al 2008,). For example, when an outlier in the sample has low selection probability, it receives a large weight. A general problem with all design unbiased estimators (which is the area researched thus far) is that they are dependent upon the design being carried out properly. When the sampling is not carried out according to the design, estimation of the parameter of interest can be affected greatly. In addition, these sampling problems may be correlated with the parameters of interest in a number of respects. For example, a researcher may not have enough fund to sample the entire network if it is too large, whereas design unbiased adaptive sampling estimators require the entire network to be observed. Model-assisted estimators for adaptive sampling designs and the use of auxiliary variables in adaptive sampling designs have not been explored. In this research we propose an approach that introduces models into design based estimation frame-work when adaptive sampling design is in use.

### Model-based-cum-Design-based inference

In situations where the HT model is not reasonable (Little and Vartivarian, 2003), a model-assisted modification is to predict the non-sampled values using a more suitable model as proposed below, and then apply the HT estimator to the residuals from that model. Specifically, the generalized regression estimator of  $\hat{Y}$  takes the form:

$$\hat{Y} = \sum_{i=1}^N \hat{y}_i + \sum_{i=1}^n (y_i - \hat{y}_i) / \pi_i \tag{1}$$

where  $\hat{y}$  is the prediction from a linear regression model relating  $y$  to the covariates. The second term on the right side of (1) conveys it with the useful property of design consistency (Brewer 1979, Isaki and Fuller 1982), which means informally that the estimator converges to the population quantity being estimated as the sample size increases, in a manner that maintains the features of the sample design. Design-based statisticians usually weight cases by the design weights  $w_i$  when computing this regression, but the estimator is also design consistent if the regression is variance weighted. For discussions of generalized regression estimator and alternatives, see for example Cassel, Särndal and Wretman (1977), Särndal, Swensson, and Wretman (1992). Another general approach to design-based inference incorporate models by basing inference on “pseudo- likelihoods” that reflect survey design features (Binder, 1983; Godambe and Thompson, 1986).

**Proposed Model-Assisted Estimators**

The design-based approach to survey inference has a number of strengths that makes it popular among its practitioners: it automatically takes into account features of the survey design, and it provides reliable inferences in large samples, without the need for strong modeling assumptions. On the other hand, it is essentially asymptotic, and yields limited guidance for small-sample adjustments. It lacks a theory for optimal estimation (Godambe 1955) and estimates from the approach are potentially inefficient. The Horvitz-Thompson (HT) estimator (Horvitz and Thompson 1952) that was used to derive the estimators for adaptive sampling applies this idea of design based inference more generally.

On the other hand, model-assisted survey estimation (Särndal *et al.* 1992) is a well-known approach for incorporating auxiliary information in design-based survey estimation. It assumes the existence of a “super-population model” between the auxiliary variables and the variable of interest for the population to be sampled (Opsomer *et al.* 2007). The estimation of population quantities of interest is then performed in such a way that the design properties of the estimators can be established. This is in contrast to purely model-based estimation, for which no design-based inference is possible. While model-assisted estimation has the potential to improve the precision of survey estimators when appropriate auxiliary information is available, it typically requires that these models be linear or at least have a known parametric shape. Breidt *et al.* (2005) introduced local polynomial regression.

Consider inference about the population total:

$$Y = y_1 + y_2 + \dots + y_n$$

and any sample design with positive inclusion probability

$$\pi_i = E(I_i / y) > 0 \text{ for units } i, i = 1, 2, \dots, N$$

the HT estimator is then

$$\hat{Y}_{HT} = \sum_{i=1}^n y_i / \pi_i = \sum_{i=1}^n I_i y_i / \pi_i \tag{2}$$

and is design unbiased for  $Y$ , since

$$E(\hat{Y}_{HT} / y) = \sum_{i=1}^N E(I_i / y) \frac{y_i}{\pi_i}$$

$$\begin{aligned} &= \sum_{i=1}^N \frac{\pi_i y_i}{\pi_i} \\ &= \sum_{i=1}^N y_i = Y \end{aligned}$$

The unbiasedness of equation (2) under mild conditions conveys robustness of modeling assumptions, and makes it a mainstay of the design-based approach but has a major deficiency as an outlier with small selection probability receives a large weight Basu’s (1971). In the light of this problem, we propose a model-assisted modification to the HT estimator to predict the non-sampled values using a more suitable model as proposed below, and then apply the HT estimator to the residuals from that model. Specifically, the generalized regression estimator of  $\hat{Y}$  takes the form:

$$\hat{Y} = \sum_{i=1}^N \hat{y}_i + \sum_{i=1}^n (y_i - \hat{y}_i) / \pi_i \tag{3}$$

where  $\hat{y}$  is the prediction from a linear regression model relating  $y$  to the covariates. The second term on the right side of equation (3) conveys it with the useful property of design consistency (Brewer 1979, Fuller 2002, Breidt *et al.* 2005, Kim, 2004), which means informally that the estimator converges to the population quantity being estimated as the sample size increases, in a manner that maintains the features of the sample design.

**Model Assumptions**

- i. The population consists of  $N$  units made up of  $N'$  inside the networks and  $N - N'$  outside the networks
- ii. There are  $Y_i, i = 1, 2, 3, \dots, n$  observations in the sample. There are  $n'$  observations in the networks and  $(n - n')$  observations outside the networks.
- iii. Initial observations are randomly selected. Subsequent observations in each network are not randomly selected. They are dependent on the initial selection in the network
- iv.  $y_i$  in “inside the network” is distributed as multivariate  $N(\mu, \sigma^2 A)$  while  $y_i$  from “outside the network” is i.i.d  $N(\mu, \sigma^2)$

**Implementation Procedure**

The analysis is conceived in the context of two sub-samples:

- Units inside the networks ( $IN$ )
- Units outside the networks ( $ON$ )

We estimate separately in each of these networks and combine the estimates. This is done because estimation in  $ON$  is fairly easy due to random selection of sample units there.

**Inside the network (IN)**

In this phase, there is the real possibility of spatial correlation among the  $y_i$  in the network, especially those close to each other and hence the need to consider a variance-covariance structure.

Thus, total of  $Y$  inside the networks is estimated as

$$\hat{Y}_{IN} = \sum_{i=1}^{N'} \hat{y}_i + \sum_{i=1}^{n'} (y_i - \hat{y}_i) / \pi_i \tag{4}$$

**Outside the networks (ON)**

The procedure is to use ordinary least squares to estimate  $\hat{Y}_{ON}$  since sampling is random. If there is evidence of heteroscedacity (assuming normality and independence assumptions hold), then use generalized least squares.

Thus,

$$\hat{Y}_{ON} = \sum_{i=1}^{N-N^1} \hat{y}_i + \sum_{i=1}^{n-n^1} (y_i - \hat{y}_i) / \pi_i \tag{5}$$

Eventually,  $\hat{Y}_{IN}$  and  $\hat{Y}_{ON}$  are combined thus,

$$\hat{T} = w_1 \hat{Y}_{IN} + w_2 \hat{Y}_{ON} \tag{6}$$

where  $w_1$  and  $w_2$  are weights that minimize the variance of  $\hat{T}$ .

The expectations and variance of the estimators are derived to give:

$$E(\hat{T}) = w_1 \sum_{i=1}^{N'} y_i + w_2 \sum_{k=1}^{N-N'} y_k$$

$$V(\hat{T}) = w_1^2 \sigma_{IN}^2 + w_2^2 \sigma_{ON}^2$$

Weights  $w_1$  and  $w_2$  that minimize the variance of the estimator

$$\hat{T} = w_1 \hat{Y}_{IN} + w_2 \hat{Y}_{ON}$$

are

$$w_1 = \frac{\rho_{\hat{Y}_{IN}\hat{Y}_{ON}} - \sigma_{\hat{Y}_{ON}}^2}{2\rho_{\hat{Y}_{IN}\hat{Y}_{ON}} - \sigma_{\hat{Y}_{IN}}^2 - \sigma_{\hat{Y}_{ON}}^2} \text{ and } w_2 = \frac{\rho_{\hat{Y}_{IN}\hat{Y}_{ON}} - \sigma_{\hat{Y}_{IN}}^2}{2\rho_{\hat{Y}_{IN}\hat{Y}_{ON}} - \sigma_{\hat{Y}_{IN}}^2 - \sigma_{\hat{Y}_{ON}}^2} \tag{7}$$

where  $\rho_{\hat{Y}_{IN}\hat{Y}_{ON}}$  is the covariance between  $\hat{Y}_{IN}$  and  $\hat{Y}_{ON}$

while  $\sigma_{\hat{Y}_{IN}}^2$  and  $\sigma_{\hat{Y}_{ON}}^2$  are their variances;

if  $\hat{Y}_{ON}$  and  $\hat{Y}_{IN}$  are correlated. Otherwise, that is, if  $\hat{Y}_{ON}$  and  $\hat{Y}_{IN}$  are uncorrelated, these reduce to

$$w_1 = \frac{\sigma_{\hat{Y}_{ON}}^2}{\sigma_{\hat{Y}_{IN}}^2 + \sigma_{\hat{Y}_{ON}}^2} \text{ and } w_2 = \frac{\sigma_{\hat{Y}_{IN}}^2}{\sigma_{\hat{Y}_{IN}}^2 + \sigma_{\hat{Y}_{ON}}^2}$$

The estimates  $\hat{w}_1$  and  $\hat{w}_2$  of the weights  $w_1$  and  $w_2$  are valid only if  $-1 < r < 1$ . Where  $r$  is the correlation coefficient between  $\hat{Y}_{IN}$  and  $\hat{Y}_{ON}$ . That is,  $\hat{T}$  is invalid if  $r = 1$ .

**Comparison with Existing Methods**

In this section we show several numerical examples of how the components of the Horvitz-Thompson estimators are calculated in adaptive cluster sampling and the computation of various components of the proposed model-assisted estimators. A comparison of the two types of estimators are made. We chose to compare with Horvitz-Thompson's since Thompson, (1992) has shown empirically that it is generally more efficient than the Hansen-Hurwitz type estimators.

Table 1 illustrates the computation of the proposed estimators for a given sample and shows for a small population, the relative properties of the different types of estimators (proposed and existing). The population consists of N=8 units. The initial sample is a simple random sample of n = 2 units. Neighbouring (adjacent) units are added whenever the condition  $y_i \geq 10$  is satisfied.

**Table 1: An illustrative Computation**

Units	1	2	3	4	5	6	7	8
$y_i$	2	15	16	14	9	8	1	4
$x_i$	3	31	32	29	17	17	1	9
$m_i$	1	2	2	2	1	1	1	1
$w_i$	2	15.5	15.5	15	9	8	1	4
Network k	1	2	2	2	3	4	5	6
$y_k^*$	2	31	31	30	9	8	1	4
$\alpha_k$	1/4	13/28	13/28	13/28	1/4	1/4	1/4	1/4

The first row is the unit labels; the second row their associated values while the third row is the associated covariates. The subsequent rows of Table 1 are necessary components for calculating various estimators in adaptive cluster sampling, with n=2 and a condition  $y_i \geq 10$  (Recall:  $m_i$  being the number of units in network  $i$ ,  $w_i$  represents the average value of a unit in that network which contains unit  $i$ ,  $y_k^*$  being the sum of units in network  $i$ , and  $\alpha_k$  are the inclusion probabilities)

**Table 2: Computation of means of possible samples using various estimators**

The sample	$\hat{\mu}_{xy}$	$\hat{\mu}_{HT}$	$\hat{\mu}_{HT+}$
2,15; 16,14,9	13.153	13.1154	14.8654
2,16;15,14,9	13.153	13.1154	14.8654
2,14;16,9	7.893	9.0769	10.8269
2,9	5.5	5.5	5.5
2,8	5	5	5
2,1	1.5	1.5	1.5
2,4	3	3	3
15,16;14,9,2	14.393	15.5769	12.4103
15,14;16,9,2	14.393	15.5769	12.4103
15,9;16,14,8,2	16.653	16.6154	15.4487
15,8;16,14,9,2	16.153	16.1154	15.2821
15,1;16,14,9,2	12.653	12.6154	14.1154
15,4;16,14,9,2	14.153	14.1154	14.6154
16,14;15,2,9	15.743	15.3462	12.5128
16,9;15,2	13.243	12.8462	11.0962
16,8;15,2	12.743	12.3462	10.8462
16,1;15,2	9.243	8.8462	9.0962
16,4;15,2	10.743	10.3462	9.8462
14,9;16,15,2	16.653	16.6154	14.8654
14,8;16,15,2	16.153	16.1154	14.6154
14,1;16,15,2	12.653	12.6254	12.8654
14,4;16,15,2	14.153	14.1154	13.6254
9,8	8.5	8.5	8.5
9,1	5	5	5
9,4	6.5	6.5	6.5
8,1	4.5	4.5	4.5
8,4	6	6	6
MEAN	10.71941	8.625	8.625
BIAS	2.09441	0.0000	0.0000
MEAN SQUARE ERROR	28.13806	50.72843	48.79968

This table consists of all possible initial samples and a few possible associated estimates of the population mean  $\mu$ . In the first column, the number immediately preceding the semi-colon represents the initial sample and numbers after the semi-colon represent adaptively added units, where  $\hat{\mu}_{xy}$  is the proposed estimator of the population mean  $\mu$ ,  $\hat{\mu}_{HT}$  the Horvitz-Thompson estimator and  $\hat{\mu}_{HT+}$ , the improved estimator proposed by Dryver and Thompson (2003, 2005)

**Table 3: Computation of variances of possible samples using various estimators**

The sample	$\hat{v}ar(\hat{\mu}_{HT})$	$\hat{v}ar(\hat{\mu}_{xy})$	$\hat{v}ar(\hat{\mu}_{HT+})$
2,15; 16,14,9	77.36428	60.47542	74.30178
2,16;15,14,9	77.36428	60.47542	74.30178
2,14;16,9	33.57701	2.319527	30.51451
2,9	24.5	24.5	24.5
2,8	18	18	18
2,1	0.5	0.5	0.5
2,4	2	2	2
15,16;14,9,2	87.953	9.278107	77.92522
15,14;16,9,2	87.953	9.278107	77.92522
15,9;16,14,8,2	84.73447	60.47542	83.37336
15,8;16,14,9,2	82.55658	60.47542	81.86214
15,1;16,14,9,2	77.81139	60.47542	75.56139
15,4;16,14,9,2	77.59504	60.47542	77.34504
16,14;15,2,9	82.03	2.319527	74.00222
16,9;15,2	42.6408	2.319527	39.5783
16,8;15,2	40.54926	2.319527	38.29926
16,1;15,2	36.40805	2.319527	36.34555
16,4;15,2	35.9331	2.319527	35.6831
14,9;16,15,2	84.73447	60.47542	82.48447
14,8;16,15,2	82.55658	60.47542	82.49408
14,1;16,15,2	77.81139	60.47542	75.56139
14,4;16,15,2	77.59504	60.47542	77.53254
9,8	0.5	0.5	0.5
9,1	32	32	32
9,4	12.5	12.5	12.5
8,1	24.5	24.5	24.5
8,4	8	8	8
MEAN = MSE	50.72843	28.13806	48.79968

This table consists of all possible initial samples and their variances. In the first column, numbers before the semi-colon represent the initial sample and numbers after the semi-colon represent adaptively added units, where  $\hat{v}ar(\hat{\mu}_{xy})$  is the variance of the proposed estimator and  $\hat{v}ar(\hat{\mu}_{HT+})$  is the improved Horvitz-Thompson estimator.

**Simulation**

Various estimates were obtained using the Horvitz-Thompson type estimators and the proposed model-assisted estimators on simulated data from bivariate normal distribution with parameters stated below:

$$\mu_X = 3, \sigma_X = 2, \mu_Y = 2, \sigma_Y = 3, \rho = 0.9$$

Varying initial sample sizes were used. The simulated values satisfying the condition for further sample selection are used to calculate the estimates of the various types of estimators.

**Table 4: Horvitz – Thompson estimator results for mean and variance for the simulations from the bivariate normal distribution. Condition is  $y_i \geq 5$  for the simulated data**

n	Horvitz-Thompson estimator	
	Mean	Variance
5	887.47	244213.9
10	4260.05	14087071
15	10964.84	19665113
20	8010.61	8081247
30	5010.63	13041992

**Table 5: Proposed estimator of mean based on simulated data from the bivariate normal distribution. Condition is  $y_i \geq 5$  for the simulated data**

n	Mean		Weight		proposed estimator mean
	Inside	Outside	Inside	outside	
5	-3.34	25.92	0.88	0.12	22.38
10	137.74	153.51	0.21	0.79	141.01
15	589.99	606.19	0.57	0.43	599.25
20	451.76	740.73	0.36	0.64	555.92
30	50.50	640.71	0.00	1.00	51.96

**Table 6: Proposed estimator for variance based on simulated data from the bivariate normal distribution. Condition is  $y_i \geq 5$  for the simulated data**

n	Variance		Weights		proposed estimator Variance
	Inside	Outside	inside	outside	
5	593.51	4314.41	0.88	0.12	521.74
10	3188.48	834.08	0.21	0.79	661.13
15	11332.07	15125.21	0.57	0.43	6478.37
20	14175.83	7990.43	0.36	0.64	5110.06
30	14269.73	35.42	0.00	1.00	35.42

**Conclusion**

This study has investigated the model-assisted modification of the Horvitz-Thompson estimators to correct for the problems of a prescribed adaptive sampling plan that may result in very small selection probabilities for some units and thereby receiving large weights in estimation. This model-assisted modification to the HT estimator is to predict the non-sampled values using a more suitable model as proposed and then apply the Horvitz-Thompson estimator to the residuals from that model. Model-assisted estimation in adaptive sampling has shown some useful properties. Firstly, the estimates are close to Horvitz-Thompson estimates. Secondly, the gain in precision was remarkable as the estimated variances of model assisted estimators are uniformly smaller than the estimated variances of Horvitz-Thompson estimators. The results from this study demonstrate that the existing Horvitz-Thompson estimator for adaptive cluster sampling can be improved using model assistance.

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