Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Appl. Math. 60 (2013) 16260-16265

ISSN: 2229-712X

Some special types of compactness

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ARTICLE INFO

Article history:

Received: 1 May 2013;

Received in revised form:

24 June 2013;

Accepted: 8 July 2013;

Keywords

Nearlycompact space ,

δ - continuous maps ,

SC - compact space ,

Anti compact space .

ABSTRACT

The main focus of this paper is to introduce the properties of some special types of compact spaces.

Introduction

In 1991 Chattopadhyay and Bandyopadhyay [1] introduced δ set . A subset A of a topological space X is called δ closed if int (cl (A)) $\subset$ cl (int (A)) .

G.L.Garg , D.Sivaraj introduced SC - compact space in 1984 .Levine introduced the notion of semi open sets in topological spaces .

The main focus of this paper is to introduce the properties of some special types of compact spaces .

1. Preliminaries

Throughout this paper X and Y always represent nonempty topological spaces (X , τ) and (Y , σ) ..The interior and the closure of a subset A of a topological space (X , τ) are denoted by int (A) and cl (A) respectively.Now we shall require the following known definitions are prerequisites

Definition 1.1 - A subset of A a topological space (X , τ) is called

(1) a regular open set if A = int [cl(A)]

(2) a regular closed set if A = cl [int(A)]

Definition 1.2 - A subset A of a topological space (X , τ) is called

(1) a semi - open set if A $\subseteq$ cl [int(A)]

(2) a semi - closed set if int [cl(A)] $\subseteq$ A .

Definition 1.3 - A topological space (X , τ) is called **compact ,** if every open covering A of X contains a finite sub collection that also covers X .

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Definition 1.4 :

Let f : (X , τ) $\rightarrow$ (Y , σ) be a function . Then f is said to be δ - continuous if for each x $\in$ X and each open neighbourhood of f (x) there is an open neighbourhood U of x such that f ($\alpha\sigma$) $\subset\alpha$ V .

2. Nearly Compact

In this section we introduce the properties of nearly compact spaces .

Definition 2.1 - (X , τ) is said to be nearly compact if (X , τ_S) is compact , where τ_S = { all regular open sets } .

Definition 2.2 - Let f : (X , τ) $\rightarrow$ (Y , σ) be a function . Then f is said to be δ - continuous if f : (X , τ_S) $\rightarrow$ (Y , σ_S) is continuous .

If f is

i) δ - continuous

ii) f^{-1} is δ - continuous

iii) f is bijective

then f is called a δ - homeomorphism .

Note 2.1 :

1. f^{-1} is δ - continuous $\Leftrightarrow$ f is a δ - open map .
2. f is δ - open if image of every regular open set is regular open .

Theorem 2.2 : Let X be a nearly compact and Y be a Almost Hausdorff . Let f : X $\rightarrow$ Y be a δ - continuous bijection . Then f is a δ - homeomorphism .

Proof : Let A $\subset$ X be regularly closed .

$\Rightarrow$ f (A) is regularly closed , because f is δ - continuous .

$\Rightarrow$ f (A) is regularly closed in the almost Hausdorff space .

$\Rightarrow$ f (A) is regularly closed .

Thus , f amps regularly closed sets .

$\Rightarrow$ f maps regularly open sets into regularly open sets .

$\Rightarrow$ f is a δ - open map .

$\Rightarrow$ f is a δ - homeomorphism .

Theorem 2.3 : Suppose that (X , τ) is nearly compact . Let A be a regularly closed subset of X . Then A is nearly compact .

Proof : Let $\mathcal{C}$ be any regularly open over of A .

Put = $\mathcal{C} \cup (X - A)$.

Then $\mathcal{D}$ is a regularly open cover of X .

But X is nearly compact .

Hence there exists a finite sub cover $\mathcal{D}'$ of $\mathcal{D}$ of X .

This implies that $\mathcal{D}'$ - (X – A) is a finite sub cover of $\mathcal{C}$.

Hence A is nearly compact .

Theorem 2.4 :A nearly compact subset of an almost Hausdorff space is regularly closed .

Proof : Suppose that X is an almost Hausdorff space .

Let A be a nearly compact subset of X .

Take x $\in$ X – A .

Then there exist disjoint open sets U_x and V_x containing x and A respectively .

We have x $\in U_x \subset X - U_x \subset X - A$.

Therefore , X – A is regularly open .

Thus , A is regularly closed .

Theorem 2.5 : Suppose that (X , τ) is nearly compact . Let f : (X , τ) $\to$ (Y , σ) be a δ - continuous surjection . Then (Y , σ) is nearly compact .

Proof : Let $\mathcal{C} = \{G_\alpha :\ \alpha \in \Lambda\}$ be a regularly open cover of Y .

$\Rightarrow$ Y = $\cup\{G_\alpha :\ \alpha \in \Lambda\}$.

$\Rightarrow$ X = $\cup\{f^{-1}(G_\alpha) :\ \alpha \in \Lambda\}$.

$\Rightarrow$ The collection $\mathcal{D} = \{f^{-1}(G_\alpha) :\ \alpha \in \Lambda\}$ is a regularly open cover of X .

But X is nearly compact .

Hence $\mathcal{D}$ has a finite sub cover $\mathcal{D}' = \{f^{-1}(G_{\alpha_i}) :\ i = 1, 2, \dots, n\}$ for X .

$\Rightarrow$ X = $\cup\{f^{-1}(G_{\alpha_i}) :\ i = 1, 2, \dots, n\}$

$\Rightarrow$ Y = $f\{f^{-1}(G_{\alpha_1}) \cup f^{-1}(G_{\alpha_2}) \cup \cdots \cup f^{-1}(G_{\alpha_n})\}$

= f $f^{-1}(G_{\alpha_1}) \cup \dots \dots \dots \cup$ f $f^{-1}(G_{\alpha_n})$

= $G_1(\alpha_1 1)$ (... ... ... $(G_1(\alpha_1 n)$.

$\Rightarrow\{G_{\alpha_1}$ (... ... ... $(G_{\alpha_n}\}$ is a finite regularly open sub cover of $\mathcal{C}$ for Y

$\Rightarrow$ Y is nearly compact .

Corollary 2.1 : Nearly compactness is a topological property .

Theorem 2.6 : Let X and Y be nonempty topological spaces . The product space $X \times Y$ is nearly compact $\Leftrightarrow$ X and Y are nearly compact .

Proof :

Step 1 : Suppose that $X \times Y$ is nearly compact .

The projection $p_1 : (X \times Y) \to X$ and $p_2 : (X \times Y) \to Y$ are δ - continuous .

Hence $X' = p_1 (X \times Y)$ and $Y' = p_2 (X \times Y)$ are nearly compact .

Step 2 : Suppose that X and Y are nearly compact .

Let $\mathcal{C}$ be a cover consisting of basic regularly open sets of the form $U \times V$, where U is a regularly open set in X and V is a regularly open set in Y .

Let $x \in X$.

Then for each $y \in Y$, there exists a set $U_y \times V_y$ in $\mathcal{C}$ containing (x , y) .

The collection $\{ V_y : y \in Y \}$ is a regularly open cover of Y .

But Y is nearly compact .

Consequently , this collection has a finite sub cover $\{V_{y_1}, \dots \dots, V_{y_n}\}$.

Let $\mathbf{U}_x = U_{y_1} \cap \dots \dots \dots \cap U_{y_n}$.

Then $\{ x \} \times Y = \mathbf{U}_x \times Y$

$$= \mathbf{U}_x \times \{V_{y_1} \cup \dots \dots \cup V_{y_n}\}$$

$$= (\mathbf{U}_x (V_{y_1}) \cup (\mathbf{U}_x (V_{y_2}) \cup \dots \cup (\mathbf{U}_x (V_{y_n}) \subset (U_{y_1} (V_{y_1}) \cup \dots \cup (U_{y_n} (V_{y_n})$$

Hence for each $x \in X$, there is a set $\{ x \} \times Y \subset \mathbf{U}_1 x (Y$ and such that $\mathbf{U}_1 x (Y$ is contained in the union of a finite member of sets in $\mathcal{C}$.

But this collection $\{ \mathbf{U}_x : x \in X \}$ covers X .

Since X is nearly compact , y this collection had a finite sub cover $\{U_{x_1}, \dots \dots, U_{x_m}\}$ then

$$X \times Y = \left(\bigcup_{i=1}^{m} U_{x_i}\right) \times Y \quad .$$

$$= \left(\bigcup_{i=1}^{m} U_{x_i} \times Y\right) \quad .$$

Since for each i with $1 \leq i \leq m$, $U_{x_i} \times Y$ is contained in the union of a finite number of sets in $\mathcal{C}$. It follows that $X \times Y$ is equal to union of a finite number of sets in .

Hence $X \times Y$ is nearly compact .

3.SC - Compact Spaces

In this section we discuss the SC compactness of subsets .

Definition 3.1 [3] -A topological space (X , τ) is said to be SC - Compact if for each closed subset A of X and τ- semi open over u of A $\exists$ a finite subfamily of elements of u , say $V_1 , V_2 , \dots , V_n$ such that

$A \subset \bigcup_{i=1}^{n} cl\, V_i$.

Theorem 3.1 : Every SC - Compact subset of a Hausdorff space is closed .

Proof : Suppose that A is an SC - Compact subset of a Hausdorff space X .

Since X is Hausdorff , A is Hausdorff .

Thus , A is Hausdorff SC - Compact subset of X .

For each $a \in A$ and $x \in X - A$ there exists disjoint open neighborhoods $\mathbf{U}_a$ of a and $\mathbf{V}_a$ of x respectively .

The collection $\{\mathbf{U}_a : a \in A\}$ is an open cover of A .

But A is SC - Compact .

Consequently there exists a finite sub over $\{U_{a_1}, \dots \dots, U_{a_n}\}$ for A .

Put U = $\{U_{a_1} \cup \cdots \dots \cup U_{a_n}\}$.

Then $A \subset U$ and U is open .

Take the corresponding sets $V_{a_1}, \dots \dots, V_{a_n}$ and put V = $V_{a_1} \cap \dots \dots \dots \cap V_{a_n}$.

Then V is an open subset and $x \in V$.

Also $U_{a_i} (V_{a_i} = \phi$ for i = 1 , 2 , … , n .

Accordingly , $U \cap V = \phi$.

$\Rightarrow x \in U \subset X - V \subset X - A$.

$\Rightarrow$ X – A is an open set .

$\Rightarrow$ A s a closed set in X .

4.Anti Compact spaces

In this section we prove some results for anti compact spaces .

Definition 4.1:(X , τ) is said to be anti compact if every compact subset K of X is finite .**Theorem 4.1 :**Let f : $X \rightarrow Y$ be a homeomorphism . If X is anti compact then Y is anti compact . Consequently , anti compactness is a topological property .

Proof : Suppose that X is anti compact .

Let K be any compact subset of Y .

$\Rightarrow f^{-1}(K)$ is compact in X because f^{-1} is continuous .

$\Rightarrow f^{-1}(K)$ is a finite set , because f is anti compact .

$\Rightarrow f(f^{-1}(K))$ is a finite set .

$\Rightarrow$ K is a finite set in Y .

$\Rightarrow$ Y is anti compact .

Theorem 4.2 : Every closed subset F of an anti compact space X is anti compact .

Proof : Suppose that X is anti compact .

Let F be a closed subspace of X .

Let K be a compact subset of F .

$\Rightarrow$ K is a compact subset of F .

$\Rightarrow$ K is finite .

$\Rightarrow$ F is anti compact .

Definition 4.3 : X is relatively anti compact if for every compact set K in X its closure $\overline{K}$ is finite **Theorem 4.3 :** If X is anti compact and Hausdorff , X is relatively anti compact .

Proof : Suppose that X be anti compact and Hausdorff .

Let K is compact subset of X .

But X is Hausdorff .

$\Rightarrow$ K is closed

$\Rightarrow$ K = $\overline{K}$ … … (1)

Also K is a finite set , because X is anti compact … … (2)

$\Rightarrow \overline{K}$ is finite [by (1) & (2)]

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