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High frequency unsteady peristaltic flow of a Jeffrey fluid in uniform and tapered tube

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ABSTRACT

High frequency unsteady peristaltic flow of a Jeffrey fluid in uniform and non-uniform tube has been investigated. The governing equations has been solved numerically and investigations are made for two cases for uniform and tapered tubes. The velocity distribution and the pressure rise is obtained. The effect of Jeffrey parameter, Womersley parameter on the flow characteristics are discussed and are analyzed through graphs for both uniform and tapered tube. Furthermore, the results obtained for the flow characteristics reveal many interesting behaviors that warrant further study of the peristaltic transport models with physiological fluids.

Introduction

According to Newton's law of viscosity "the shear stress is directly and linearly proportional to velocity gradient. It is well known that Newtonian fluids posses a property called viscosity and obey a relation through Newtonian law between a stress and its strain. There are numerous fluids for which Newton's law of viscosity does not hold. These fluids are termed as non- Newtonian fluids. Many industrial and biological fluids are non-Newtonian in their flow characteristics and are referred as to as rheological fluids. Many materials such as slurries (china clay and coal in water, sewage sludge etc) and multiphase mixtures including oil-water emulsions and butter are non- Newtonian fluids. Further examples displaying a variety of non- Newtonian characteristics include biological fluids (blood, synovial fluid, salvia), pharmaceutical formulations, cosmetics and toiletries, paint, shampoo, synthetic lubricants and food stuffs (jams, jellies, soups). The non- Newtonian fluids are mainly classified into three categories. They are fluids for which the shear stress is dependent only upon the shear rate; the relation between shear stress and shear rate is time-dependent and the fluids which posses both elastic and viscous properties namely the viscoelastic fluids.

Peristalsis is a natural mechanism of pumping that is observed in the case of most physiological fluids. In the transport of some other fluids also peristaltic behavior is observed. This behavior is usually associated with a progressive wave of area contraction or expansion along the length of a fluid-filled distensible tube. This mechanism is also used in many biomedical appliances such as finger pumps, heart-lung machine and dialysis machine and also in industries for the transport of noxious fluid in nuclear industries, as well as roller pumps. For this reason in current years, studies of peristaltic transport have been receiving growing interest of scientific researchers. Most of the theoretical investigations have been carried out by assuming blood and other

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physiological fluids behave like a Newtonian fluid. Although, this approach provides satisfactory understanding of the peristaltic mechanism in the ureter, it fails to give a better understanding when the peristaltic mechanism involved in small blood vessels, intestine and in transport of spermatozoa in the cervical canal. It has been accepted that majority of the physiological fluids behave like a non-Newtonian fluids. At high shear rate the behavior of blood is almost Newtonian, while at low shear rate blood exhibits yield stress and non-Newtonian behavior.

Analytical solutions were obtained for peristaltic flows by assuming either small amplitude but arbitrary Reynolds number by Fung and Yih (1968), Yin and Fung (1969) using a laboratory frame of reference. Whereas Shapiro et al. (1969) have presented the solutions for arbitrary amplitude and negligible inertia under long wavelength approximation in a wave frame of reference. Much of the early literature of theoretical investigations, arranged according to the geometry, the types of fluids, the Reynolds number, wave amplitude, wavelength and the wave shape, along with an account of experimental studies on peristaltic transport have been reviewed by Srivastava and Srivastava (1984). In all study on peristaltic flow, much works are studied with no slip condition, but some few authors have studied the wall slip effect on the peristaltic transport of Newtonian and non- Newtonian fluids through channel/ tube. Elshehewey et al., (2006) studied the slip effects on the peristaltic flow of a non- Newtonian Maxwellian fluid. During the past few years, many authors have discussed the study of peristaltic flow of different non-Newtonian fluid models. The peristaltic analysis of MHD viscous fluid in a two-dimensional channel with variable viscosity under the effect of slip condition was investigated by Ali et al., (2008). Hayat et al., (2008) studied the influence of slip on the peristaltic motion of a third order fluid in an asymmetric channel. Ali et al., (2009) have analyzed the peristaltic motion of a non-Newtonian fluid in a channel having complaint boundaries. Peristaltic pumping of a micropolar fluid in an inclined tube is studied by Prasad et al., (2010). The effect of thickness of the porous material on the peristaltic pumping when the tube wall is provided with non-erodible porous lining was investigated by Hemadri Reddy et al., (2011). The peristaltic flow of a Prandtl fluid model in an asymmetric channel under long wavelength and low Reynolds number was investigated by Akbar et al., (2012).

Recently there has been an increasing interest in the flow of time-independent non-Newtonian fluids through tubes possessing a definite yield value because of their applications in polymer industries and bio-fluid dynamics. Due to the practical and fundamental association of non-Newtonian fluids to industrial applications several studies of these fluids in different geometries have been carried out. Unsteady flow with attenuation in a fluid filled elastic tube with a stenosis is studied by Ramachandra Rao (1983a). Li and Brasseur (1993) investigated the non-steady peristaltic transport in finite-length tubes. Eytan et al., (2001) studied the peristaltic flow in a tapered channel under lubrication approach using a fixed frame of reference. Hakeem et al., (2004) investigated the separation in flow through peristaltic motion of a Carreau fluid in uniform tube Mekheimer (2004) have studied the peristaltic flow of blood in a non-uniform channel under the effect of a magnetic field. Mandal (2005), studied the problem of non-Newtonian and non-linear blood flow through a stenosed artery, where the non-Newtonian rheology of the flowing blood is characterized by the generalized Power-law model. Series solution for the peristaltic flow of a tangent hyperbolic fluid in a uniform inclined tube is investigated by Nadeem and Akbar (2010). Recently, Sreenadh et al. (2012) have studied the unsteady flow of a Jeffrey fluid in an elastic tube with a stenosis.

During the past few years, many authors have discussed the study of peristaltic flow of different non-Newtonian fluid models. Also, as the blood is frequently referred to as non-Newtonian fluid, amongst non-Newtonian fluid models the Jeffrey model is one of the simplest non-Newtonian model to account for reheological effects of viscoelastic fluids. Some of the works on Jeffrey model can be found in (Hayat et al., (2007), Hayat and Ali (2008), Hayat, et al., (2008), Nagendra et al., (2008)) and the references there in. Nadeem and Akbar (2010) studied the effects of temperature dependent viscosity on peristaltic flow of a Jeffrey-six constant fluid in a non-uniform vertical tube. Vajravelu et al., (2011) studied the influence of heat transfer on peristaltic transport of Jeffrey fluid in a vertical porous stratum. Peristsltic flow of a Jeffrey-six constant fluid in a non-uniform tube is investigated under the assumption of long wavelength and low Reynolds number by Akbar and Nadeem (2011). The flow of a two immiscible Jeffrey fluids in an inclined circular tube is investigated by Sreenadh et al., (2011) as the blood can be considered as two-fluid model based on its rheology. The effect of porous medium and magnetic field on peristaltic transport of a Jeffrey fluid in an asymmetric channel is studied by Mahmoud et al., (2011). Effects of induced magnetic field and slip condition on peristaltic flow of Jeffrey fluid in the presence of heat and mass transfer in a non-uniform channel is studied by Saleem et al., (2012). Jayarama Reddy et al., (2012) have studied the effect of variable viscosity on the peristaltic flow of Jeffrey fluid in a uniform tube.

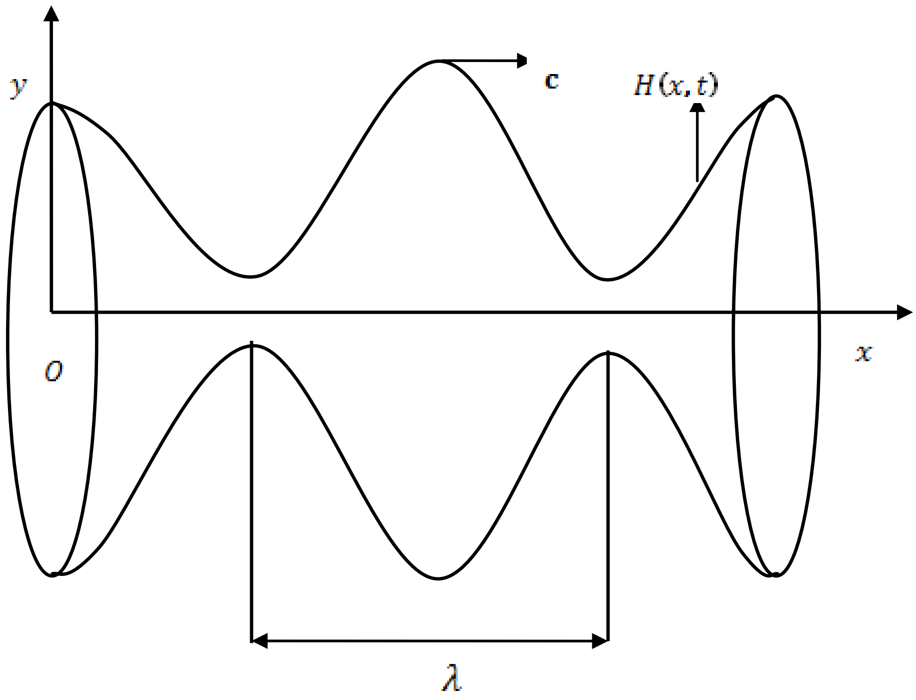
In this chapter, we investigate high frequency unsteady peristaltic flow of a Jeffrey fluid in a finite length uniform and non-uniform tube under lubrication theory approximation. Exact solutions are obtained for a prescribed time dependent pressure at the inlet and outlet when the Womersley parameter α is not negligible. The effect of Womersley parameter α, Jeffrey parameter λ_1, and frequency parameter ω_1 on the flow characteristics are discussed in detail and are analyzed through graphs. If the Jeffrey parameter $\lambda_1 \to 0$ our results agree well with the results with Ramachandra Rao (1983b).

2. MATHEMATICAL FORMULATION AND SOLUTION

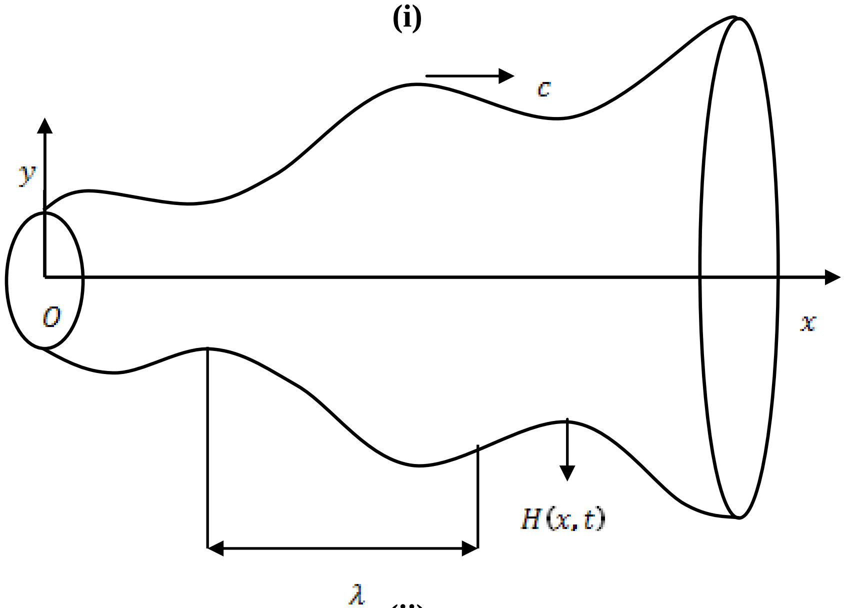
Consider the peristaltic transport of an incompressible Jeffrey fluid of viscosity μ in a circular cylindrical tube of finite length L. Cylindrical coordinate system (x, r, θ) is used to investigate the flow. The tube is flexible and a peristaltic wave propagates with constant velocity c and with frequency $\frac{c}{\lambda}$ on it in the axial direction, where λ is the wavelength of the peristaltic wave. The deformation of the tube wall is given by $H(x, c, \lambda, t)$.

The inlet and outlet pressures are prescribed as

$$p(0,t) = p_0\, e^{i\omega t}, \qquad p(L,t) = p_L\, e^{i\omega t} \qquad (1)$$



(i)



(ii)

Fig 1: (i) Schematic diagram of the uniform tube

(ii) Schematic diagram of the tapered tube

The equations governing the motion are

$$\frac{\partial u}{\partial x} + \frac{1}{r}\frac{\partial (rv)}{\partial r} = 0, \qquad (2)$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r}\right) = -\frac{\partial p}{\partial x} + \frac{\mu}{1+\lambda_1}\left(\frac{\partial^2 u}{\partial x^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right), \qquad (3)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial r}\right) = -\frac{\partial p}{\partial r} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial r^2} - \frac{v}{r^2}\right), \qquad (4)$$

where u, v are the axial and radial velocity components, ρ is the fluid density and p is the pressure.

We introduce the following non-dimensional variables

$$\bar{x}=\frac{x}{\lambda}\ ,\quad \bar{r}=\frac{r}{a}\ ,\quad \bar{t}=\omega t\ ,\quad \bar{u}=\frac{u}{c}\ ,\quad \bar{v}=\frac{v}{c\delta}\ ,\quad \delta=\frac{a}{\lambda}\ ,\quad \bar{H}=\frac{H}{a}\ ,\quad \omega_1=\frac{c}{\lambda\omega}\ ,\bar{p}=\frac{a\delta}{\mu c}p \qquad (5)$$

where a is the inlet radius of the tube, into the governing equations (2)-(4), (dropping the bars) we get

$$\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial (rv)}{\partial r}=0\ , \qquad (6)$$

$$\alpha^2\frac{\partial u}{\partial t}+Re\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}\right)=-\frac{\partial p}{\partial x}+\frac{1}{1+\lambda_1}\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial r^2}\right), \qquad (7)$$

$$\alpha^2\delta^2\frac{\partial v}{\partial t}+Re\,\delta^2\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial r}\right)=-\frac{\partial p}{\partial r}+\delta^2\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{\partial^2 v}{\partial r^2}-\frac{v}{r^2}\right), \qquad (8)$$

where $\alpha^2=\frac{a^2\omega}{\nu}$ is the Womersley parameter, $\nu=\frac{\mu}{\rho}$ and $Re=\frac{ac}{\nu}\delta$ is the Reynolds number.

Here α^2 is more when comparing with the Reynolds number Re , therefore the ratio of peristaltic wave frequency to the frequency of pressure gradient oscillation ω_1 is less than one. Under the assumptions of long wavelength and negligible inertia (i.e. $\delta\ll 1$ and $Re\to 0$), the rate of momentum equations (7) and (8) reduce to

$$\alpha^2\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\frac{1}{1+\lambda_1}\left(+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial r^2}\right), \qquad (9)$$

$$\frac{\partial p}{\partial r}=0\ , \qquad (10)$$

The corresponding non-dimensional boundary conditions are

$$p(0,t)=p_0,\qquad p(L,t)=p_L \qquad (11)$$

$$u=0\ ,\qquad v=\frac{\partial H}{\partial t}+i\,S_t\,\xi_i \qquad \text{at} \qquad r=H \qquad (12)$$

$$\frac{\partial u}{\partial r}=0\ ,\qquad v=0 \qquad \text{at} \qquad r=0 \qquad (13)$$

where $S_t=\frac{\omega a}{c}$ (Strouhal number) , ξ_i is the radial displacement.

Substituting

$$\frac{\partial p}{\partial x}=A(x,t)e^{it} \quad \text{and} \quad u=u_1(x,r)e^{it} \qquad (14)$$

in (9), we obtain

$$\frac{\partial^2 u_1}{\partial r^2}+\frac{1}{r}\frac{\partial u_1}{\partial r}+(1+\lambda_1)\beta^2 u_1=(1+\lambda_1)A(x,t) \qquad (15)$$

where $\beta^2=i^3\alpha^2$

The solution of (15) which is regular on the axis $r=0$ is given by

$$u=\left(C_1 J_0\left(\sqrt{1+\lambda_1}\,\beta r\right)+\frac{A(x,t)}{\beta^2}\right)e^{it} \qquad (16)$$

By using the first condition of (12) in (16) we get

$$u=\frac{1}{\beta^2}\frac{\partial p}{\partial x}\left[1-\frac{J_0(\sqrt{1+\lambda_1}\,\beta r)}{J_0(\sqrt{1+\lambda_1}\,\beta H)}\right] \qquad (17)$$

Using (17) and second condition of (13) in the equation of continuity, we get

$$v=\frac{1}{\beta^2}\frac{\partial^2 p}{\partial x^2}\left[\frac{J_1(\sqrt{1+\lambda_1}\,\beta r)}{(\sqrt{1+\lambda_1}\,\beta)J_0(\sqrt{1+\lambda_1}\,\beta H)}-\frac{r}{2}\right]+\frac{1}{\beta^2}\frac{\partial p}{\partial x}\frac{\partial H}{\partial x}\frac{J_1(\sqrt{1+\lambda_1}\,\beta r)J_1(\sqrt{1+\lambda_1}\,\beta H)}{J_0^{\,2}(\sqrt{1+\lambda_1}\,\beta H)} \qquad (18)$$

Using the second condition of (12) in the equation (18), we have

$$\frac{\partial H}{\partial t}+i\,S_t\,\xi_i=\frac{\partial p}{\partial x}\frac{\partial H}{\partial x}\frac{J_1^{\,2}\left(\sqrt{1+\lambda_1}\,\beta H\right)}{\beta^2 J_0^{\,2}\left(\sqrt{1+\lambda_1}\,\beta H\right)}+\frac{1}{\beta^2}\frac{\partial^2 p}{\partial x^2}\left[\frac{J_1\left(\sqrt{1+\lambda_1}\,\beta H\right)}{\left(\sqrt{1+\lambda_1}\,\beta\right)J_0\left(\sqrt{1+\lambda_1}\,\beta H\right)}-\frac{H}{2}\right] \qquad (19)$$

Integrating (19) once with respect to x ,we get

$$\frac{\partial p}{\partial x}=\frac{2\left(\beta^2\sqrt{1+\lambda_1}\right)J_0\left(\sqrt{1+\lambda_1}\,\beta H\right)}{2HJ_1\left(\sqrt{1+\lambda_1}\,\beta H\right)-H^2\left(\sqrt{1+\lambda_1}\,\beta\right)J_0\left(\sqrt{1+\lambda_1}\,\beta H\right)}\left[f(t)+\int_0^x H(s,t)\left(\frac{\partial H(s,t)}{\partial t}+i\,S_t\,\xi_i\right)ds\right] \quad (20)$$

Integrating (20) once again with respect to x ,we obtain

$$p(x,t)=p_0 e^{it}+\int_0^x \frac{\partial p(s,t)}{\partial s}ds \quad (21)$$

$f(t)$ is determined by evaluating (20) at $x=L$,

(22)

where $\Delta p(t)=(p_L-p_0)e^{it}=\Delta p\, e^{it}$ and $\Delta p=p_L-p_0$

The dimension-less volume flow rate is given by

$$Q(x,t)=2\int_0^H u\,r\,dr$$

$$=\frac{1}{\beta^2}\frac{\partial p}{\partial x}\left[H^2-\frac{2\,H\,J_1\left(\sqrt{1+\lambda_1}\,\beta H\right)}{\left(\sqrt{1+\lambda_1}\,\beta\right)J_0\left(\sqrt{1+\lambda_1}\,\beta H\right)}\right] \quad (23)$$

Time averaged volume flow rate over one period $T\left(=\frac{1}{\omega_1}\right)$ of the peristaltic wave is given by

$$\bar{Q}=\frac{1}{T}\int_0^T Q(x,t)\,dt \quad (24)$$

The relation between $\bar{Q}$ and Δp is obtained by substituting (20) and (22) into (24):

$$\bar{Q}=\bar{Q}_0\left[1-\frac{\Delta p}{\Delta p_0}\right] \quad (25)$$

where

$$\bar{Q}_0=\frac{2}{T}\int_0^T \frac{\int_0^L K(s_1,t)\left[\int_x^{s_1}\int_0^x H(s,t)\left(\frac{\partial H(s,t)}{\partial t}\right)ds\right]ds_1}{\int_0^L K(s_1,t)ds_1}\ dt \quad (26)$$

$$\Delta p_0=\frac{\bar{Q}_0}{2}\left[\frac{1}{T}\int_0^T \frac{e^{it}}{\int_0^L K(s_1,t)ds_1}\,dt\right]^{-1} \quad (27)$$

and

$$K(s_1,t)=\frac{2(\beta^2\sqrt{1+\lambda_1})\,J_0(\sqrt{1+\lambda_1}\,\beta H(s_1,t))}{2HJ_1(\sqrt{1+\lambda_1}\,\beta H(s_1,t))-H^2(\sqrt{1+\lambda_1}\,\beta)J_0(\sqrt{1+\lambda_1}\,\beta H(s_1,t))} \quad (28)$$

3. RESULTS AND DISCUSSIONS

The objective of this analysis is to study the flow characteristics of a Jeffrey fluid in an uniform tube and in a tapered tube when it is pumped by a peristaltic wave.

Peristaltic flow in an uniform tube

In this section the wall deformation in dimension-less form is taken as

$$H(x,t)=1+\phi\cos 2\pi(x-\omega_1 t) \quad (29)$$

To study the behavior of axial velocity u, numerical calculations for several values of Womersley parameter α, frequency ratio ω_1, Jeffrey parameter λ_1 and amplitude ratio ϕ are carried out. Fig (2) shows that as the Womersley parameter α increases the velocity decreases. From the Fig (3), we observe that the velocity decreases as the frequency ratio ω_1 increases. The effect of Jeffrey parameter λ_1 on the velocity distribution can be seen through Fig (4). It reveals that the velocity increases with increase in λ_1. Fig (5) shows that increase in amplitude ratio ϕ results in decrease in the velocity distribution.

The variation of pressure rise verses average flow rate is analyzed graphically and are plotted in Figs (6 - 9) for several values of Womersley parameter α, frequency ratio ω_1, Jeffrey parameter λ_1 and amplitude ratio ϕ. From Fig (6) we can observe the variation of Δp with $\bar{Q}$ for the variation of Womersley parameter α. It is interesting to note that all the curves are intersecting in the free pumping region $(\Delta p > 0)$ at $\bar{Q} = 1.1$. For $0 \le \bar{Q} \le 1.1$ we observe that Δp increases with increase in α, and in the rest of the region i.e. $1.2 \le \bar{Q} \le 2$ Δp decreases with increase in α. The variation of Δp with $\bar{Q}$ for different values of ω_1 is shown in Fig (7). It is observed that the pumping increases with increase in ω_1. Fig (8) shows the variation of Δp with $\bar{Q}$ for variation in Jeffrey parameter λ_1. We can observe that the pumping region decreases with increase in λ_1 for $0 \le \bar{Q} \le 0.62$. The effect of Jeffrey parameter has no significant change in the pumping phenomenon in the region $0.62 \le \bar{Q} \le 0.7$ and pumping decreases as λ_1 increases in the rest of the region i.e. $0.7 \le \bar{Q} \le 1$. Fig (9) reveals that Δp decreases with increase in amplitude ratio ϕ.

Peristaltic flow in a tapered tube

In this case the wall deformation in dimension-less form is taken as

$$H(x,t) = 1 + \frac{k\lambda x}{a} + \phi \sin 2\pi(x - t) \tag{30}$$

where k is a constant whose magnitude depends on the length of the channel, exit and inlet dimensions.

In order to see the quantitative effects of the various emerging parameters on velocity, numerical calculations for several values of Womersley parameter α, amplitude ratio ϕ, Jeffrey parameter λ_1 are carried out and are analyzed through graphs (10-13).The effect of Womersley parameter α and amplitude ratio ϕ on velocity is shown in Figs (10) and (11). It is observed that as α and ϕ increases velocity decreases. Fig (12) reveals that velocity increases with increase in the value of Jeffrey parameter λ_1.

Graphs are plotted and analyzed to study the effects of emerging parameters i.e. Womersley parameter α, amplitude ratio ϕ, Jeffrey parameter λ_1 and frequency ratio ω_1 on pumping characteristics and is shown in Figs (13-16). Fig (13) illustrate the variation of Δp with $\bar{Q}$ for variation in Womersley parameter α. It is observed that, as α increases Δp increases in the region $0 \le \bar{Q} < 0.6$ and also it is observed that as α increase Δp is decreasing in the rest of the region i.e. $0.6 < \bar{Q} \le 1$. The variation of pressure rise Δp with the time averaged flux $\bar{Q}$ for different values of amplitude ratio is presented in Fig (14). It is noticed that pumping decreases as amplitude ratio ϕ increases. The effect of Jeffrey parameter λ_1 on Δp is studied in Fig (15). We observe that increase in λ_1 results in increase in Δp in the region $0 \le \bar{Q} < 0.4$ and also it is interested to note that λ_1 has no significant effect on the pumping in the region $0.4 \le \bar{Q} < 0.5$. The variation of Δp with $\bar{Q}$ for different values of frequency ratio ω_1 is shown in Fig (16). It is observed that Δp increases as ω_1 increases.

4. CONCLUSIONS

High frequency unsteady peristaltic transport of a Jeffrey fluid in finite length uniform and tapered tube is studied under lubrication theory approach using a fixed frame of reference. Here the effects of Womersley, Jeffrey parameters are analyzed through graphs and we find some interesting observations as given below:

1. velocity increases with increase in Womersley parameter and amplitude ratio for both uniform and tapered tube.
2. we observe that velocity increases with increase in Jeffrey parameter for both the cases.
3. pumping increases with increase in frequency ratio for uniform tube and for tapered tube.
4. as amplitude ratio increases we see that pumping decreases for both uniform and tapered tube.

Velocity for Uniform tube

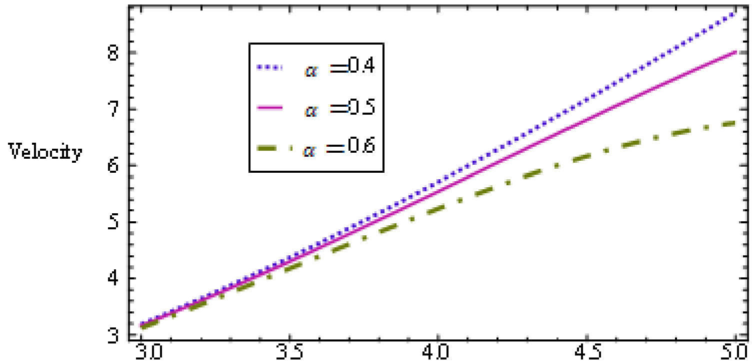


Fig 2: The velocity profiles for different α with
$\phi = 0.6, \quad \lambda_1 = 0.5, \quad P = 1, \quad t = 0.2, \quad \omega_1 = 0.3$

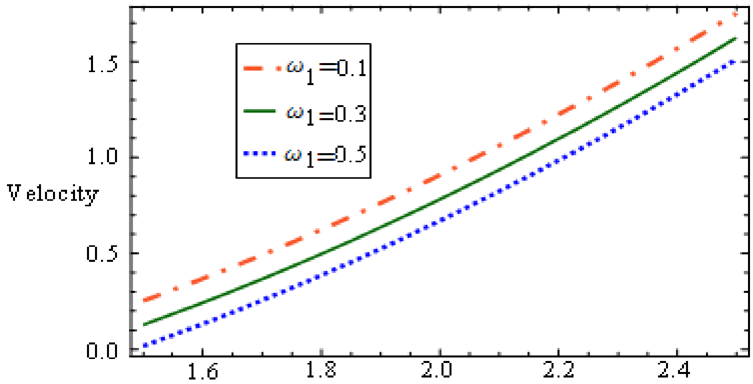


Fig 3: The velocity profiles for different ω_1
with $\phi = 0.6, \quad \lambda_1 = 0.5, \quad P = 1, \quad t = 0.2, \quad \alpha = 0.4$

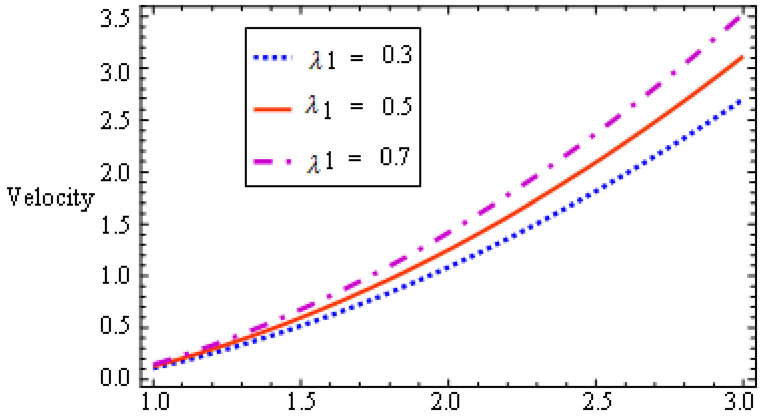


Fig 4: The velocity profiles for different λ_1
with $\phi = 0.7,\quad \omega_1 = 0.5,\quad P = 1,\quad t = 0.1,\quad \alpha = 0.2$

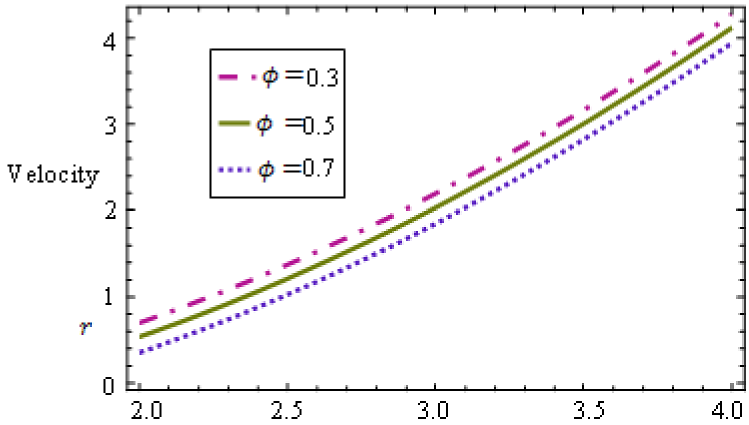


Fig 5: The velocity profiles for different ϕ
with $\omega_1 = 0.8,\quad P = 1,\quad t = 0.2,\quad \alpha = 0.4,\quad \lambda_1 = 0.2$

Pressure gradient for uniform tube

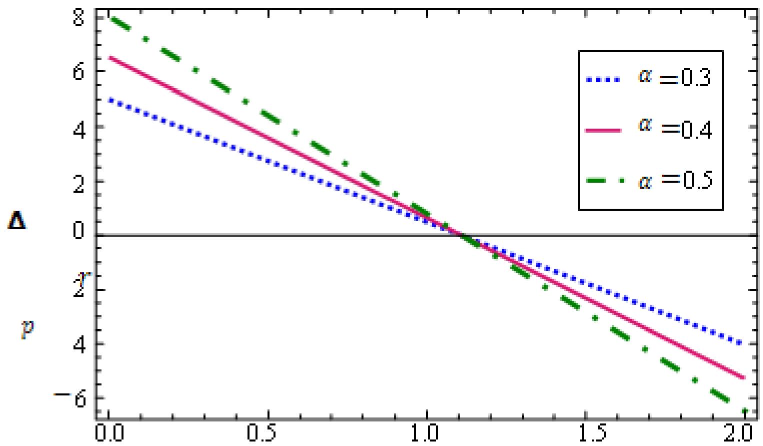


Fig 6: The variation of Δp with $\bar{Q}$ for different values of α with
$\lambda_1 = 0.6,\quad \omega_1 = 0.8,\quad \phi = 0.5$

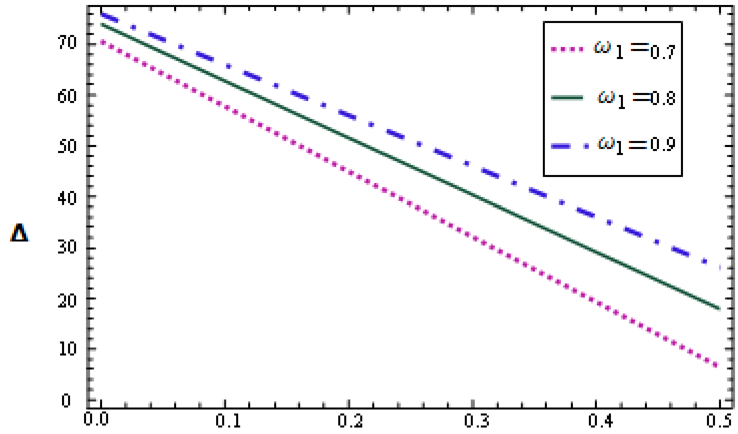


Fig 7: The variation of Δp with $\overline{Q}$ for different values of ω_1 with $\lambda_1 = 0.5$, $\alpha = 0.5$, $\phi = 0.7$

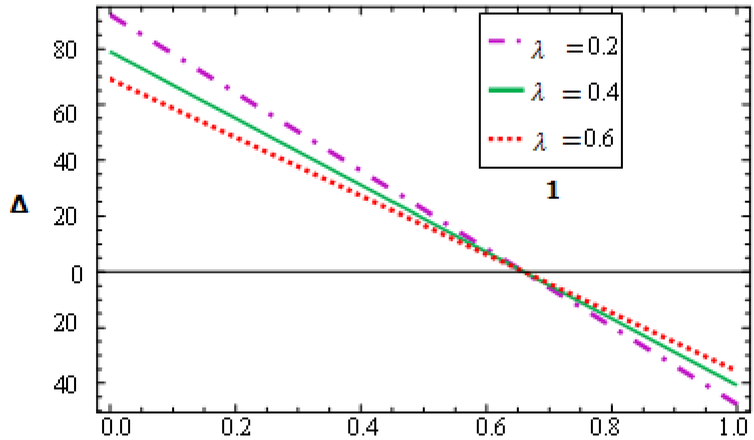


Fig 8: The variation of Δp with $\overline{Q}$ for different values of λ_1 with $\alpha = 0.2$, $\omega_1 = 0.8$, $\phi = 0.7$

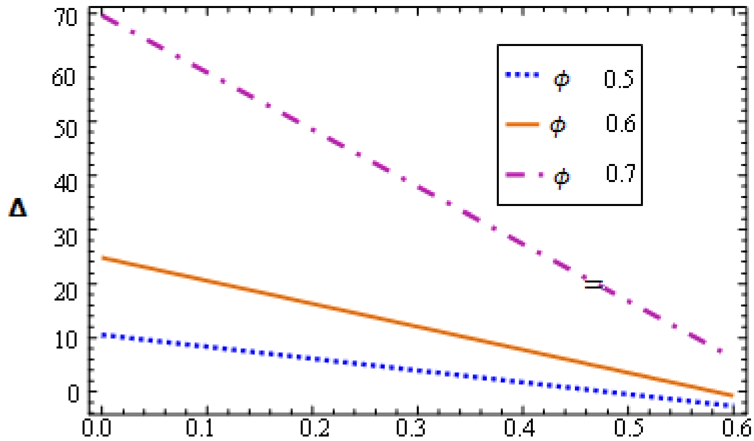


Fig 9: The variation of Δp with $\overline{Q}$ for different values of ϕ with $\alpha = 0.6$, $\omega_1 = 0.8$, $\lambda_1 = 0.6$

Velocity for Tapered tube

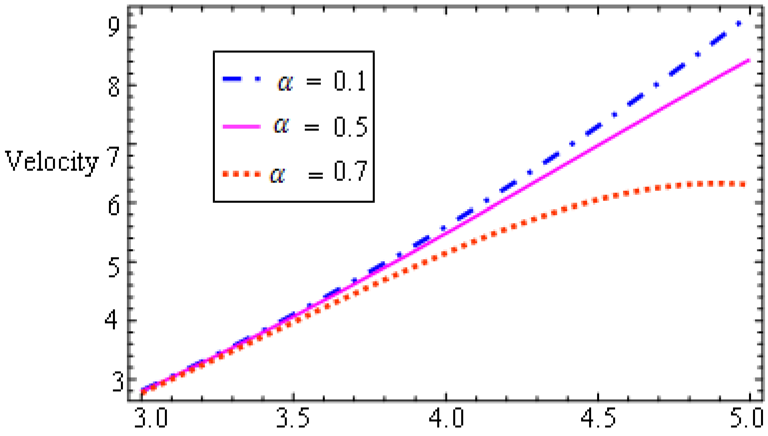


Fig 10: The velocity profiles for different α **with** $\phi = 0.6,\quad P = 1,\quad t = 0.2,\quad \lambda_1 = 0.6$

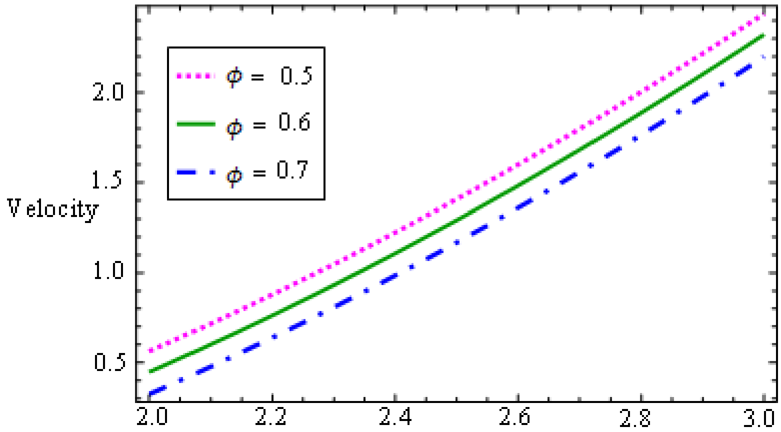


Fig 11: The velocity profiles for different ϕ **with** $\alpha = 0.2,\quad P = 1,\quad t = 0.1,\quad \lambda_1 = 0.5$

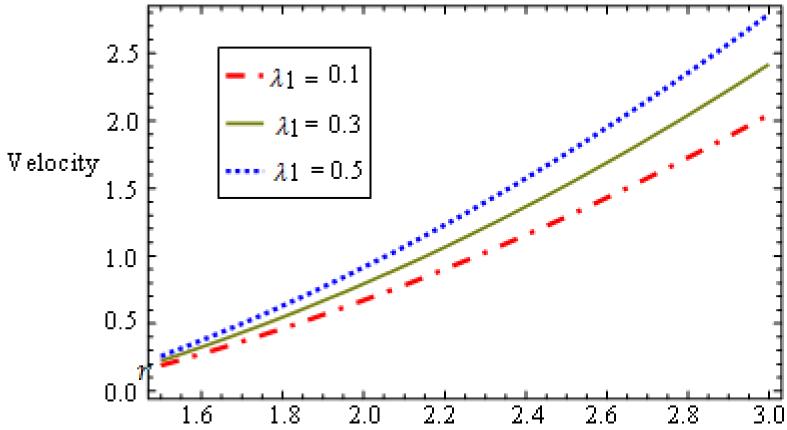


Fig 12: The velocity profiles for different λ_1 **with** $\phi = 0.6,\quad P = 1,\quad t = 0.4,\quad \alpha = 0.2,$

Pressure gradient for Tapered tube

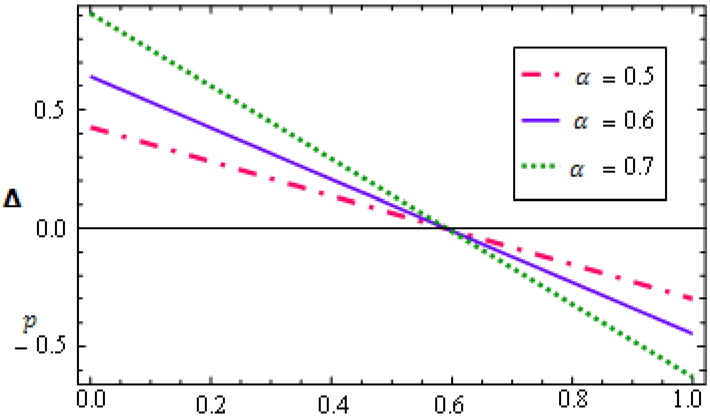


Fig 13: The variation of Δp with $\bar{Q}$ for different values of α with
$\phi = 0.5,\quad \omega_1 = 0.8,\quad a = 0.5,\quad \lambda_1 = 0.6$

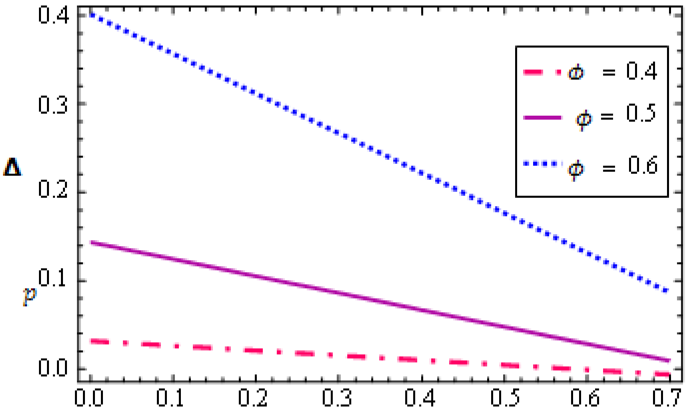


Fig 14: The variation of Δp with $\bar{Q}$ for different values of ϕ with
$\alpha = 0.2,\quad \omega_1 = 0.8,\quad a = 0.5,\quad \lambda_1 = 0.2$

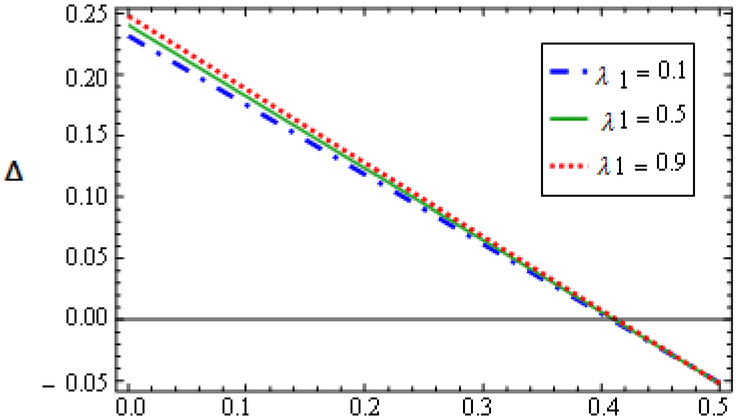


Fig 15: The variation of Δp with $\bar{Q}$ for different values of λ_1 with
$\alpha = 0.8,\quad \omega_1 = 0.8,\quad a = 0.8,\quad \phi = 0.4$

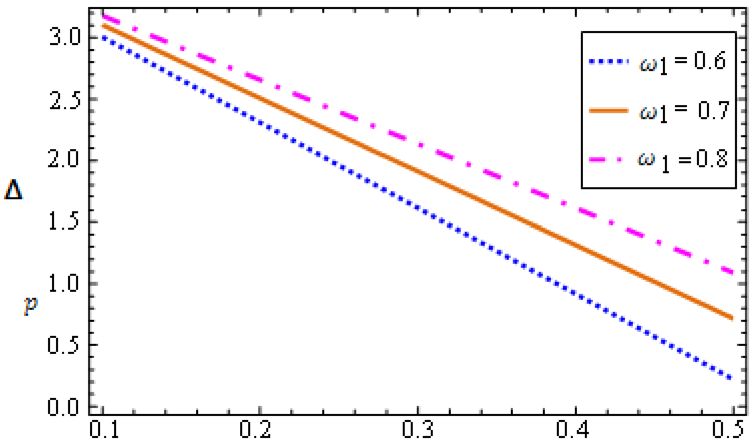


Fig 16: The variation of Δp with $\bar{Q}$ for different values of ω_1 with $\alpha = 0.8$, $\lambda_1 = 0.5$, $\phi = 0.6$, $a = 0.1$

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