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Stochastic solution of a water-works pipeline network system

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ABSTRACT

This work is carried out to provide a corrective analysis of a typical water pipeline network with varied flow rates/discharges in order to obtain adequate pressure that must be maintained to cause a pipeline grid balance in a distribution network of a closed conduit. The network was taken to be analogous to a direct-electrical circuit analogy of current flow, which provides a simplified approach to complex distribution network problems. The fluid in the conduit is in a steady-state, hence the work adopts the application of steady-state energy equations of Darcy Weisbach, Hardy Cross and Hazen Williams in the analysis of head losses associated with flow through pipeline of varying sizes, bends, fittings, and surface roughness.

Introduction

For water to be distributed to either a large city or industrial plants from waterworks there is always a pipe network distribution system to convey and distribute the water; such a distribution system is composed of interconnected pipelines and fittings such as pumps, valves, etc. located at appropriate points in the network. The supply to the system is at a few points in the network known as supply points normally from elevated reservoirs and water is withdrawn from the network from several points as the demand points. The major problems associated in such distribution networks become the control of excessive head loss; corrective flow rate and adequate pressure that must be maintained to cause a pipeline grid balance in the distribution network. This work applied some mathematical models to analyze the behaviors of a steady- state fluid (water) flow under pressure in a network via long pipelines with different diameters pipes, pipe lengths and initial flow rates in series, parallel pipes using a direct-electrical circuit analogy. Such mathematical models include the Hardy Cross (HC) successive appropriation; Hazen-Williams (HW) and Darcy- Weisbach (DW) head loss equations.

Adequate pressure and water supply must be maintained in course of the design of a distribution system and the treated water must also be free from contamination during conveyance. Closed conduits have shown to be satisfactory in respect to contamination. There are basically a quite number of factors that do lead to losses of pressure or insufficient pressure along the drive and driven pipe distribution networks. Predominant among these losses include; (i) Frictional losses in pipe (ii) Reservoir to pipe connection and vice versa (iii) Sudden enlargements in pipes (iv) Gradual enlargements in pipes (v) Meters (vi) Sudden contractions in pipes. Among all losses aforementioned, review shows that frictional losses in pipe constitute the largest percentage [1]. Thus, for most practical purposes of analysis, the other losses are usually ignored.

The analysis of distribution network is usually a complex problem. Normally, the characteristics of the pipes in the network are known and the problem is one of determining the flow rates (Q) in the various parts of the network and the corresponding pressure at the nodal points especially for complex networks. A large number of mathematical models have been developed in addition to graphical approaches [2]. Most steady- State analyses have employed the Hardy Cross method, Newton-Rapson methods [3], Sparse Matrix methods [4] and the linear theory method [5], [6]. These methods may also be divided into categories, those which solve for unknown flow rates in pipes, thereafter referred to as Loop equations and those which solve for unknown heads (H) at the pipe junctions referred to as Node equations.

Hardy Cross (HC) method [7] becomes advantageous because both loop and node equations can be solved. This is however, applicable to Newton- Rhapson's methods which is a Hardy Cross extension in which both loop and nodal equations for the entire water distribution network system are solved at the same time. Both the linear theory and the sparse matrix methods solve linearized sets of the equations while the former solves loop equations; the latter solves the Nodal Equations.

Materials and applicable methods

Hydraulic Network and Electrical Circuit Analogies

The direct electrical circuit analogy method of pipeline network analysis depends on five analogies, the fifth resulting from the special characteristics of the non- linear resistors used. The first is that current (I) in an electric circuit behaves in a similar manner to flow of water (Q) in a pipe. The second is that the frictional head loss (H_f) resulting from the flow of water in a distribution pipe network is analogous to the voltage drop (V) caused by the flow of current in a resistor. The third analogy is expressed by the law that

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the total flow of water approaching any junction equals to the total flow leaving it (Fig. 1) where Qi = Q_2 + Qs. In an electrical circuit, the sum of the currents flowing towards any junction equals the sum of currents leaving it ($Ii=I_2+I_3$).

The fourth analogy is expressed by the law that the sum of the clockwise head losses around any loop in a pipe network equals the sum of the counter clockwise head losses around that loop ($H_2 + H_4 = H_3 + H_5$). Similarly, in an electrical circuit, the sum of the clockwise voltage drops bounding any loop equals the sum of the counter clockwise voltage drops around that loop ($V_2 + V_4 = V_3 + V_5$). The fifth analogy is a result of the development of a special non- linear resistor which is the electrical equivalent of a pipeline in a water distribution system. In a pipeline, the head loss varies nearly as the square of the flow, or $H=k_pQ1.85$ where k_p represents the pipe dimensions and constant (Hazen Williams formula). To obtain the direct solutions from an electrical circuit The voltage drop must vary nearly as the square of the current, or $V=kl^{185}$ where K is termed as the coefficient of the resistor and is analogous to k_p, the head loss constant of the pipeline. In this equation, V and I have a non- linear relationship. However, the voltage drop in an ordinary electrical circuit is directly proportional to the current or V=IR where the resistance, R is a constant. Thus, V and I have a linear relationship.

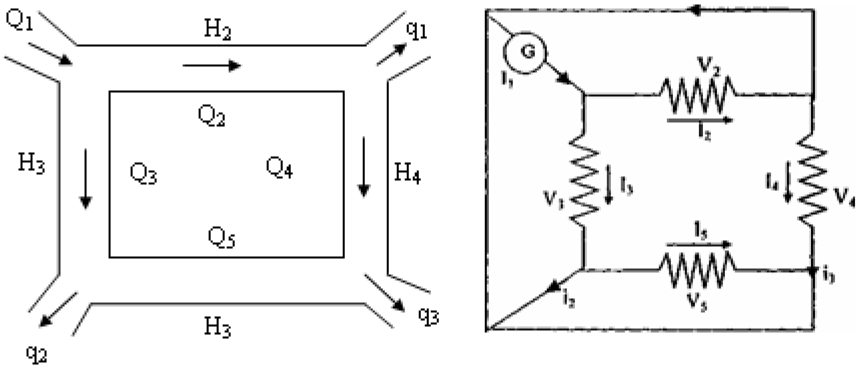

Fig 1. Hydraulic and Electrical Analogies

Theory of Pipeline Network Analysis

In a compound pipeline network, it is infrequently impossible to tell by inspection which way the flow travels, nevertheless the flow in any network however complicated, must satisfy the basic relations of continuity and energy as follows:

(1)The flow into each junction must be equal to the flowout of it.

(2) The flow in each pipe must satisfy the Darcy- Weisbach equation or equivalent exponential friction formula pipe friction laws) for flow in a single pipe ie the proper relation between head loss and discharge must be maintained for each pipe.

(3) The algebraic sum of the pressure drops around any closed circuit must be zero.

The first condition is the continuity equation while the third condition states that the pressure drop between the downstream and upstream must be constant or uniform. Pipe networks are generally too complicated to solve analytically, as was possible in simpler cases of parallel pipes, hence methods of successive approximations introduced by Hardy Cross method are utilized.

Applications of Frictional Head Losses

Head is energy per unit weight of fluid; it means the ratio of the product of force and length to weight of fluid. Energy equation represents elevation; pressure and velocity forms of energy. Energy losses for flow through ducts and pipes consist of major losses and minor losses. Major losses are losses owing to friction between the moving fluid and inside walls of the dusts/ pipes and are computed by applying either the Hazen -Williams equation or Darcy - Weisbach friction loss equation (which utilizes the moody friction factors, f).

Minor losses are due to pipe fittings such as elbows, bends, valves etc which may be included by using the equivalent length (L_{eq}) may be computed using the formula: U_g = KD/f. where f is the Darcy- Weisbach friction factor for the pipe containing the fitting and cannot be known with certainty until after the pipe network program is run. However, since f is needed to be known ahead of time, a reasonable value to use is f = 0.02 which is the default value. The minor loss coefficient, K, Values are from May (1999), although minor loss could also be calculated by applying $h_m = kV^2/2g$ where k = 0.85 depending on the type of fitting. The Darcy - Weisbach has an advantage over the Hazen-Williams loss equation. This is because the former is applicable in the frictional head loss analysis of any given fluid while the latter can be apply in the analysis of frictional head loss of water only within the temperature ranges of 40° F to 45° F or 4-25°C.

The Hazen Williams method is quite popular and much used in water works since its frictional coefficient, C does not depend on velocity or duct (pipe) diameter.

Industrial pipe frictional formula is usually empirical of the form [11];

$$\frac{H_f}{L} = \frac{RQ^n}{D^m} \qquad (1)$$

In which H_f/L is the head loss per unit length of the pipe, R is the resistance coefficient.

As earlier pinpointed the Hazen –Williams formula is well favoured and the expression for head is given by equation (1), from where we have,

$$H_t = \left(\frac{RL}{D^m}\right) \times Q^n \qquad (2)$$

The flow of water at ordinary temperature through pipes is of the form above with R given as; R = 4.727/C^n for U.S customary units and R = 10.675/C^n for S.I units. With n = 1.852; m = 4.8704 and C is dependent upon surface roughness. For each pipe of a system or network, there is a definite relationship between the head loss and discharge. This may be expressed as;

$$H_F = KQ^z \qquad (3)$$

Tables 1: Hazen Williams' Major Loss Coefficients (C has no units)

Materials	C	Materials	C
Extremely smooth, str pipe, asbestos cement	140	Copper	130-140
		Galvanized iron	120
Brass	130-140	Glass	140
Brick sewer	100	Lead	13—140
		Plastic	140-150
CAST – IRON			
New, Unlined	130	**STEEL**	
10yrs old	100-113	Coal-tar enamel lined	145-150
20yrs old	89-100	New unlined	140-150
30yrs old	75-90	Riveted	100-110
40yrs old	64-83	New welded	120
Concrete/concrete-lined:		Tin	130
Steel forms	140	Vitrify, clay (good condition)	100-140
Wooden forms	120	Wood stave (Avg condition)	120
Centrifugally spun	135		
Very smooth pipes, concrete	140		

Table 2: Minor Loss Coefficients (K has no units)

The following minor loss coefficient table was assembled from data shown in [8]; [9]; and [10].

Fittings	K	Fittings	K
Valves		Elbows:	
Globe, fully open	10	Regular 90^0, flanged	0.3
Angle, fully open	2	Regular 90^0, threaded	1.5
Gate, fully open	0.15	Long radius 90^0, flanged	0.2
Gate, ¼ closed	0.26	Long radius 90^0, threaded	0.7
Gate, ½ closed	2.1	Long radius 45^0, threaded	0.2
Gate, ¾ closed	17	Regular 45^0, threaded	0.4
Swing check, forward flow	2		
Swing, check, backward flow	Infinity		
		TEES	
180 RETURN BENDS		Line flow, flanged	0.2
Flanged	0.2	Line flow, threaded	0.9
Threaded	1.5	Branch flow, flanged	1.0
		Branch flow, threaded	2.0
PIPE ENTRANCE (RESERVOIR TO PIPE)		**PIPE EXIT (PIPE TO RESERVOIR)**	
Square connection	0.5		
Rounded connection	0.2	Square connection	1.0
Re-entrant (pipe juts into tank)	1.0	Rounded connection	1.0
		Re-entrant	1.0

Where exponent (z) is dependent upon the particular pipe friction formula utilized. In actual fact the exponent z is not strictly constant for the system unless the pipes are operating in the rough turbulence zone which is unlikely. However, since the range of velocity is not large and in view of unavoidable limitations in the accuracy of the basic data, it is generally reasonable to assume that z is constant throughout. Hence, equation (2) becomes;

$$H_t = \frac{4.727L}{C^{1.852}\,D^{4.8704}}\,Q^{1.852}\ for\ US\ Units \qquad (4)$$

Similarly, $$H_t = \frac{10.675L}{C^{1.852}\,D^{4.8704}}\,Q^{1.852}\ for\ SI\ Units \qquad (5)$$

Now, equating RHS of (2) and (3) will subsequently yields,

$$\left(\frac{RL}{D^m}\right) Q^n = KQ^2 \quad \textbf{(6)}$$

Where $k = \frac{RL}{D^m}$ and n = z = 1.852 for both cases of the units, thus;

$$K = \frac{4.727L}{C^{1.852} D^{4.8704}} Q^{1.852} \; for\ US\ Units \quad (7)$$

Also,

$$K = \frac{10.675L}{C^{1.852} D^{4.8704}} Q^{1.852} \; for\ SI\ Units \quad (8)$$

One can also develop a special purpose formula for a particular application by using Darcy-Weisbach Equation and employing the Colebrook's equation or the much simpler Moody's equation in calculating the friction coefficient or alternatively by using experimental data if available. The Darcy Weisbach head loss equation is given by,

$$H_f = \frac{fLV^2}{D2g} \quad (9)$$

But

$$Q = AV \; and \; A = \frac{\pi D^2}{4}$$

Therefore equation (9) becomes [8]; [9],

$$H_f = \frac{8fLQ^2}{\pi^2 D^5 g} \quad (10)$$

Here f is not a constant but is referred to as Moody's friction factors which is determined by the nature of flow [12]: If laminar flow ($Re < 4000 \; and \; any \; \frac{e}{D}$) then $f = \frac{64}{Re}$; If turbulent flow $\left(4000 \le Re \le 10^8 \; and \; 0 \le \frac{e}{D} < 0.05\right)$, then Colebrook equation [13],

$$\frac{1}{\sqrt{f}} = -2.0 \log_{10}\left(\frac{e/D}{3.7} + \frac{2.51}{RE\sqrt{f}}\right) \quad (11)$$

If fully turbulent flow ($Re > 10^8 \; and \; 0 \le \frac{e}{D} < 0.05$),

$$f = \left[1.14 - 0.869 ln\left(\frac{e}{D}\right)\right]^{-2} \quad (12)$$

Where Re is Reynold's number and e is Roughness of pipe in meters.

Theory of Hardy Cross Method

The pipe network calculation uses the steady-state energy equation, Darcy –Weisbach and Hazen-Williams friction losses and Hardy Cross method to determine the flow rate in each pipe, losses in each pipe and node pressure. Hardy Cross (HC) analysis method is otherwise known as the single-path –Adjustment method [7]; [14]. It is a method adopt for analysis of flow are assured for each pipe so that continuity is satisfied at every junction.

A correction to the flow in each circuit is then computed in turn and applied to bring the circuits into a closer balance. Pipe networks are usually complicated to solve analytically as well as possible in simpler cases of parallel pipes hence method of successive approximations introduced by Hardy Cross using head balance can be employed. The flow rates (Q) in each pipe is adjusted iteratively until all equations are satisfied. The method is based on some primary physical laws such as.

i. The sum of pipe flows into and out of a node equals the flow entering or leaving the system via the node.
ii. Continuity Equations
iii. Steady-State energy equations/conservation of energy
iv. Hydraulic heads (Elevation head + pressure head, Z + P/S) which is a single valued. This means that the hydraulic head at node is the same whether it is computed from upstream or downstream directions.

Pipe flows are adjusted iteratively using the following equation;

$$\Delta Q_i = \pm \sum_{i=1}^{in\ Loop\ Pipes} H_i \Bigg/ \left[n \sum_{i=1}^{in\ Loop\ Pipes} \left(\frac{H_i}{Q_i}\right) \right] \quad (13)$$

Until the change in flow in each pipe is less than convergence criteria; n = 2.0 for Darcy Weisbach losses or 1.85 for Hazen Williams losses.

Iterative analysis of a hw water works pipeline network

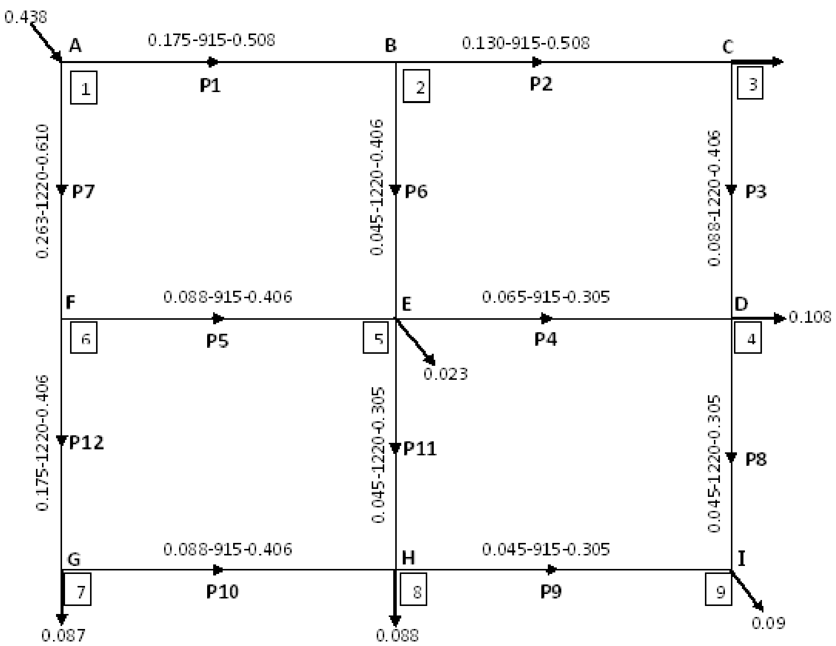

Fig. 2: A Typical Water Pipeline Network

Table 3: First Iteration of Loops

Lines	L(m)	D(m)	$Q_0 (m^3/s)$	K	KQ_0^n	KnQ_0^{n-1}	ΔQ
AB	915	0.508	0.175	37.300	1.479	15.647	0.0136
BE	1220	0.406	0.045	148.156	0.475	19.539	0.0136 – 0.0055 = 0.0081
EF	915	0.406	-0.088	111.119	-1.213	25.950	0.0136 – 0.02055 = 0.00695
FA	1220	0.610	-0.263	20.399	-1.719	12.107	0.0136
					-0.998	**73.243**	
BC	915	0.508	0.130	37.300	0.853	12.146	0.0055
CD	1220	0.406	0.088	148.156	1.644	34.599	0.0055
DE	915	0.305	-0.065	447.519	-2.834	80.734	0.0055-(-0.0047) = 0.0102
EB	1220	0.406	-0.045	148.156	-0.475	19.539	0.055-0.0136=-0.0081
					-0.812	**147.018**	
FE	915	0.406	0.088	111.110	1.233	25.950	0.02055-0.0136=0.00695
EH	1220	0.305	0.045	596.692	1.912	78.692	0.02055-(-0.0047)=0.02525
HG	915	0.406	-0.088	111.119	-1.233	25.950	0.02055
GF	1220	0.406	-0.175	148.156	-5.873	62.148	0.02055
					-3.961	**192.740**	
ED	915	0.305	0.065	447.519	2.834	80.734	-0.0047-(0.0055)=-0.0102
DI	1220	0.305	0.045	569.692	1.912	78.692	-0.0047
IH	915	0.305	-0.045	447.519	-1.434	59.019	-00047
HE	1220	0.305	-0.045	596.692	-1.912	78.692	-0.0047-(0.02055)=-0.02525
					1.400	**297.137**	

Table 4: Second Iteration of Loops

Lines	L(m)	D(m)	$Q_1(m^3/s)$	K	KQ_1^n	$N\Sigma KQ_1^{n-1}$	ΔQ
AB	915	0.508	0.1886	37.300	1.698	16.677	0.0081
BE	1220	0.406	0.0531	148.156	0.645	22.498	0.0081-0.00035 = 0.00775
EF	915	0.406	-0.0950	111.119	-1.421	27.698	0.0081-(-0.0017) = 0.0098
FA	1220	0.610	-0.2494	20.399	-1.558	11.572	0.0081
					-0.636	**78.445**	
BC	915	0.508	0.1355	37.300	0.921	12.583	-0.00035
CD	1220	0.406	0.0935	148.156	1.839	36.433	-0.0035
DE	915	0.305	-0.0548	447.519	-2066	69.806	-0.00035-(0.0077) = -0.000805
EB	1220	0.406	-0.0531	148.156	-0.645	22.498	-000035-(0.0081) = -0.00845
					0.049	**141.320**	
FE	915	0.406	0.0950	111.119	1.421	27.698	-0.0017-(0.0081) = -0.0098
EH	1220	0.305	0.0703	596.692	4.368	115.079	-0,0017-(0.0077) = -0.0094
HG	915	0.406	-0.0675	111.119	-0.754	20.701	-0.0017
GF	1220	0.406	-0.1545	148.156	-4.662	55.889	-0.0017
					0.373	**219.367**	
ED	915	0.305	0.065	447.519	2.834	80.734	-0.0047-(0.0055)=-0.0102
DI	1220	0.305	0.045	569.692	1.912	78.692	-0.0047
IH	915	0.305	-0.045	447.519	-1.434	59.019	-00047
HE	1220	0.305	-0.045	596.692	-1.912	78.692	-0.0047-(0.02055)=-0.02525
					-2.368	**320.75**	

Table 5: Third Iteration of Loops

Lines	L(m)	D(m)	$Q_2(m^3/s)$	K	KQ_2^n	$N\Sigma KQ_2^{n-1}$	ΔQ	Q_3
AB	915	0.508	0.1967	37.300	1.836	17.285	0.0005	0.1962
BE	1220	0.406	0.0609	148.156	0.331	25.284	0.0005-0.005 = -0.0055	0.0554
EF	915	0.406	-0.0852	111.119	-1.161	25.244	0.0005-0.0043= 0.0048	-0.090
FA	1220	0.610	-0.2413	20.399	-1.466	11.509	0.0005	-0.2418
					0.04	**79.322**		
BC	915	0.508	0.1355	37.300	0.971	12.559	-0.0050	0.1402
CD	1220	0.406	0.0932	148.156	1.828	36.333	-0.0050	0/0982
DE	915	0.305	-0.0629	111.119	-2666	78.506	-0.005-(0.0007) = -0.0057	-0.0572
EB	1220	0.406	-0.0616	20.399	-0.849	25.532	0.005-(0.0005) = -0.0055	-0.0561
					-0.077	**152.932**		
FE	915	0.406	0.0852	111.119	1.161	25.244	-0.0043-(0.0005) = -0.0048	0.090
EH	1220	0.305	0.0609	596.692	3.349	101.832	-0.0043-(0.0007) = -0.005	0.0659
HG	915	0.406	-0.0692	111.119	-0.624	21.445	-0.0043	-0.0649
GF	1220	0.406	-0.1562	148.156	-4.758	56.413	-0.0043	-0.1519
					0.872	**204.634**		
ED	915	0.305	0.0629	447.519	2.666	78.506	-0.0007-(0.005) = -0.0057	0.0572
DI	1220	0.305	0.048	569.692	2.155	83.140	-0.0007	0.0473
IH	915	0.305	-0.042	447.519	-1.262	55.649	-00007	-0.0427
HE	1220	0.305	-0.0609	596.692	-3.349	101.832	-0.0007-(0.0043)=-0.005	-0.0659
					-0.21	**319.127**		

Results

The result of the computed flows after 3-corrections (3 iterative Analysis) are as follows (all in m^3/s); P1– AB = 0.1962; P2– BC = 0.1402; P3– CD = 0.0982; P4– ED = 0.0572; P5– FE = 0.090; P6– BE = 0.0554; P7– AF = 0.2418; P8– DI = 0.0427; P9– HI = 0.0427; P10– GH = 0.0649; P11– EH = 0.0659; P12– FG = 0.1519

The elevation of the hydraulic grade line at A is 61.00 + 45.72 = 106.72m. The head loss to I can be computed by any route from A to I, adding the losses in the usual manner (direction of flow). Applying ABCDI, we will obtain;

$$H_f = H_{AB} + H_{BC} + H_{CD} + H_{DI} \qquad (14)$$

Thus; H_f (AI) = 1.836 + 0.917 + 1.828 + 2.155 = 6.736m. Similarly applying AFGHI, we will obtain;

$H_{c\,(AI)} = H_{AF} + H_{FG} + H_{GH} + H_{HI}$ = 1.466 + 4.758 + 0.624 + 1.262 = 8.11m

Taking the mean value of $H_{f\,(AI)}$ = (6.736+8.11)/2 = 7.42m

The elevation of hydraulic grade line at I is then 106.72 – 7.42 = 99.3m

Then the pressure head at 1 = 99.3 -30.5m – 68.8m

The method of analysis of network can be summarized as follows: Starting with an initial flow distribution which satisfies continuity at the junctions, a corrective flow rate is computed, consecutively for each loop, which tends to satisfy the energy equation.

This process is repeated using the latest available solution as the starting point for next iteration, until the average corrective flow rate in pipes is below a prescribed tolerance level. The correction of the flow rate in pipes in one loop disturbs the energy relationships in all adjacent loops to which they are common. This explains the slow convergence of the method. The main handicap as earlier stated of the HC method is the assumption of initial flow for the pipes. This procedure could be very tedious in the case of large n etworks. The rate of convergence was dependent on the initial guess of the flow distribution and the number of loops in the network. Also the computer outputs show that computed results were independent on the initial flow distribution prescribed at the start of the iteration process. Inspite of this, the energy equation around the loop, appeared satisfied in each case.

Conclusion

The direct electrical - analogy method of pipeline network analysis is a convenient, rapid and accurate means of studying the performance of pipeline grids. It provides immediate visual indication of the section of a network where head losses are excessive, since the only values of flow rates or head losses which are necessary to be measured are those of direct interest, no time is wasted in recording unimportant information. No guesses, trial values of flow rates or head losses or successive approximations are required in reaching a solution based on the Hazen-Williams or Darcy-Weisbach formula. Changes to represent alternate plans of construction or different separating conditions of the sources or loads can easily be made, and their effects can be measure rapidly. Hence, the method is useful aid in designing networks for maximum economy, as well as in analyzing their performance in detail.

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