Available online at www.elixirpublishers.com (Elixir International Journal)

Advances in Pure Mathematics

Elixir Adv. Pure Math. 61 (2013) 17171-17173

ISSN: 2229-712X

# A texture approach to α-compactness in Ditop space

I. Arockia Rani and A.A.Nithya

Department of Mathematics, Nirmala College for Women, Coimbature.

**ARTICLE INFO**

**Article history:**
Received: 6 May 2013;
Received in revised form:
29 July 2013;
Accepted: 12 August 2013;

**Keywords**
Texture spaces,
Ditopology,
Ditopological Texture spaces,
Compactness,
α-compactness,
Dicover, Cover,
Cocover,
Pseudo α-open sets,
Pseudo α-closed sets.

**ABSTRACT**
The essence of this paper is to introduce the notion of pseudo α-open sets and pseudo α-closed sets. More results on compactness, co compactness, α- compactness and α-cocompactness in ditopological texture spaces are analysed. Many effective characterizations and properties of these concepts are also obtained.



**Introduction**

Textures were based on point-set concept for the study of fuzzy sets in 1998 by L.M.Brown[2]. Also textures offers a convenient setting for the investigation of complement-free concepts in general, so much of the recent work has proceeded independently of the fuzzy setting.

This concept is further extended by many researchers and generalized the sets and maps in texture setting. In this paper we present some results based on compactness, α-compactness, pseudo α open and closed sets. Many characterizations and properties of pseudo α-open sets and pseudo α-closed sets are discussed.

Definition.1.1 Let S be a set, a texturing T of S is a subset of P(S). If

(1) $(T,\subset)$ is a complete lattice containing S and φ, and the meet and join operations in $(T,\subset)$ are related with the intersection and union operations in $(P(S),\subset)$ by the equalities

$\wedge_{i\in I} A_i = \cap_{i\in I} A_i$, $A_i \in T$, $i \in I$, for all index sets I, while $\vee_{i\in I} A_i = \cup_{i\in I} A_i$, $A_i \in T$, $i \in I$, for all finite sets I.

(2) T is completely distributive.

(3) T separates the points of S. That is, given $s_1 \neq s_2$ in S we have $A \in T$ with $s_1 \in A$, $s_2 \notin A$, or $A \in T$ with $s_2 \in A$, $s_1 \notin A$.

If S is textured by T we call (S,T) a texture space or simply a texture.

For a texture (S; T), most properties are conveniently defined in terms of the p-sets

$P_s = \cap\{A \in T \setminus s \in A\}$ and the q-sets, $Q_s = \cup\{A \in T / s \notin A\}$: The following are some basic examples of textures.

Examples 1.2. Some examples of texture spaces,

(1) If X is a set and P(X) the powerset of X, then (X; P(X)) is the discrete texture on X.
For $x \in X$, $P_x = \{x\}$ and $Q_x = X \setminus\{x\}$.

(2) Setting I = [0; 1], T= {[0; r); [0; r]/r ∈ I } gives the unit interval texture (I; T). For $r \in I$, $P_r = [0; r]$ and $Q_r = [0; r)$.

(3) The texture (L;T) is defined by L = (0; 1], T = {(0; r] /r ∈ I }. For $r \in L$, $P_r = (0; r] = Q_r$ .

(4) T={φ, {a, b}, {b}, {b, c}, S} is a simple texturing of S = {a, b, c} clearly $P_a = \{a, b\}$, $P_b = \{b\}$ and $P_c = \{b, c\}$.

Since a texturing T need not be closed under the operation of taking the set complement, the notion of topology is replaced by that of dichotomous topology.

Definition.1.3 A ditopology is a pair (τ, κ) of subsets of T, where the set of open sets τ satisfies

1. S, φ ∈ τ ,
2. $G_1; G_2 \in \tau$ then $G_1 \cap G_2 \in \tau$ and
3. $G_i \in \tau$ , $i\in I$ then $\vee_i G_i \in \tau$ , and the set of closed sets κ satisfies
1. S, φ ∈ κ
2. $K_1; K_2 \in \kappa$ then $K_1 \cup K_2 \in \kappa$ and
3. $K_i \in \kappa$, $i \in I$ then $\cap K_i \in \kappa$. Hence a ditopology is essentially a 'topology" for which there is no priori relation between the open and closed sets.

For $A \in T$ we define the closure [A] or cl(A) and the interior ]A[ or int(A) under (τ, κ) by the equalities $[A] = \cap\{K \in \kappa / A \subset K\}$ and $]A[ = \cup\{G \in \tau / G \subset A\}$:

Definition.1.4An mapping $\sigma : T \to T$ is said to be complementation on (S,T) if $\kappa = \sigma(\tau)$, then (S, T , σ, τ, κ) is said to be a complemented ditopological texture space.

The ditopology (τ, τc) is clearly complemented for the complementation $\sigma_X : P(X) \to P(X)$ given by
$\sigma_X(Y) = X \setminus Y$.

**Tele:**
**E-mail addresses: aanithyajerry@gmail.com**

We denote by O(S; T; τ, κ), or when there can be no confusion by O(S), the set of open sets in S. Likewise, C(S; T; τ, κ) or C(S) will denote the set of closed sets.

Definition.1.5. A subset C of T × T is called a difamily on (S,T). Let C ={(Gα , Fα )/α ∈ A} be a family on (S,T). Then T is called a dicover of (S,T) if for all partitions A1 , A2 of A, we have ∩α∈A1 Fα ⊂ Vα∈A2 Gα

Definition.1.6 Let (τ, κ) be a ditopology on (S,T). Then a difamily C on (S, T, τ, κ). is called α-open(co-α-open) if dom(C)⊂ αO(S) (ran(C )⊂ αO(S).

Definition.1.7. Let (τ, κ) be a ditopology on (S,T). Then a difamily C on (S, T, τ, κ). is called α-closed(co-α closed) if dom(C)⊂ αC (S). (ran( C)⊂ αC (S)).

Definition.1.8. A ditopology (τ, κ) on (S; T) is called:
1. α-compact if every cover of S by α-open sets has a finite subcover.
2. α-cocompact if every cocover of φ by α-closed sets has a finite sub-cocover.

Definition.1.9 Let (τ ; κ) be a ditopology on the texture space (S; T).
1. (τ ; κ) will be called α-stable if every α-closed set F ∈ T / {S} is α-compact in S. That is, whenever Gj , j ∈ J, are α-open sets in (S; T; τ ; κ) satisfying F ⊂ ∪j∈J Gj , there exists a finite subset J' of J for which F ⊂ $\cup j \in J_0$ Gj .
2. (τ ; κ) will be called α-costable if every α-open set G /φ, is α cocompact in S. That is, whenever Fj , j ∈ J , are α-closed sets in (S; T; τ ; κ) satisfying ∩j∈J Fj ⊂ G, there exists a finite subset J' of J for which ∩j∈ J' Fj ⊂ G.

**α-compactness and α-cocompactness**

Theorem 2.1 For a ditopological texture (S, T ,τ, κ) is α − compact, α − cocompact, α stable and α-costable then every αclosed, co-αopen difamily with the finite exclusion property is bounded.

proof. To prove, here we use the method of contradiction. Consider a α-closed, co α-open difamily B = {(Fi , Gi )|i ∈ I } which satisfy finite exclusion property, which is not bounded in (S,T,τα, κα). Let F = ∩i∈I Fi , where each Fi is αC(S), so that F ∈ αC (S) and F ⊂ ∪i∈I Gi .

Then two cases arise

case(i)If F ≠S: Since F is closed, then by using α-stability there exists a finite subset JI of I with F ⊂ ∪j∈JI Gj .

case(ii) If F =S: Since F=S, which implies F is open, then by using α-compactness, there exist a finite subset JI of I with F ⊂ ∪j∈JI Gj .

Now let G = ∪j∈JI Gj , where each Gi is αO(S), so that G ∈ αO(S) and ∩i∈I Fi ⊂ G.

Then two cases arise:

case(i) G ≠ ø: Since G is open set we may use co α-stability so that we get ∩j∈J2 Fj ⊂ G for some finite subset J2 of I.

case(ii) G=ø: Then it is a closed set, so that we may use co α-compactness we get ∩j∈J2 Fj ⊂ G for some finite subset J2 of I . Using the above cases we get,

∩j∈J1 ∪J2 Fj ⊂ ∪j∈J1 ∪J2 Gj

This leads to a contradiction to the fact that B has the finite exclusion property.

Theorem 2.2 For a ditopological texture (S,T,τ, κ) if every α open, co-α closed dicover has a finite subcover then (S, T, τ, κ) is α − compact, α − cocompact, α stable and α-costable.

(1)⟶(2) To prove α-compactness: Let Gi ∈ αO(S), i ∈ I , with S = Vi∈I Gi . For i ∈ I let Fi = ø. Then C={(Gi , Fi )|i ∈ I } is an α open, co-α closed dicover, so has a finite sub-dicover

{(Gj , Fj )| j ∈ J }. For the partition J1 = ø, J2 = J of I, S= ∩j∈J1 Fj ⊂ ∪j∈J2 Gj ,

hence S=∪j∈J Gj and (S, T, τ, κ) is α compact. co-α compactness is proved similarly.

To prove α-stability take F ∈ αC (S), F = S and Gi ∈ αO(S), i ∈ I with F ⊂ Vi∈I Gi . Define C={(S, F )} ∪ {(Gi , ø)|i ∈ I }. It is clear C is an α open, co-α closed dicover, and hence has a finite sub-dicover C∞. If C∞={(Gj , ø)|j ∈ J }, J finite, then the fact that C∞ is a dicover implies ∪j∈J Gj = S, hence F ⊂ ∪j∈J Gj . On the otherhand, if (S,F) ∈ C∞, then we again obtain F ⊂ ∪j∈J Gj as required. co-α stability can be proved similarly.

**Pseudo α-open sets and Pseudo α-closed sets**

Definition 3.1 Let (S, T , τ, κ) be a ditopological texture space and A ∈ T.

(1) αQ(A) = ∩{αint(Qs )|Ps ⊄ A} if αint A = αQ(A), then A is called as pseudo α-open set.

(2) αP(A) = V{αcl(Ps )|A ⊄ Qs } if αcl A = αP(A), then A is called as pseudo α-closed set.

Lemma 3.2 Let (S, T , τ, κ) be a ditopological texture space and A ∈ T. Then

(1) αint(A) ⊂ αQ(A) ⊂ A.

(2) A ⊂ αP(A) ⊂ αcl(A).

Proof. Suppose that αint(A) ⊄ αQ(A). Then there exists s ∈ S with s ∈ αint(A) and s ∉ αQ(A).(i.e) s ∉ ∩{αint(Qs )|Ps ⊄ A}, which implies there exists atleast one αint(Qs ) such that s ∉ αint(Qs ) with Ps ⊄ A (i.e) αint(A) ⊄ αint(Qs ). Since Ps ⊄ A, which implies s ∉ A then A ⊂ Qs using the property of αinterior we get αint(A) ⊂ αint(Qs ), which is a contradiction. Hence αint(A) ⊂ αQ(A).

Secondly, to prove αQ(A)⊂ A. αQ(A) = ∩{αint(Qs )|Ps ⊄ A} ⊂ ∩{Qs |Ps ⊄ A} = A. Hence(1) is proved.

(2) A = V{Ps |A ⊄ Qs } ⊂ V{αcl(Ps )|A ⊄ Qs }=αP(A). Next to prove the second inclusion, suppose that αP(A) ⊄ αcl(A). Then there exist s ∈ S with αcl(Ps ) ⊄ αcl(A) and A ⊄ Qs . From A⊄ Qs we have Ps ⊂ A and so αcl(Ps ) ⊂ αcl(A), which is a contradiction. Hence αP(A) ⊂ αcl(A).

Corollary 3.3 Let (S, T , τ κ) be a ditopological texture space. (1) Every set A ∈ αO(S) is pseudo α-open.

(2) Every set A ∈ αC (S) is pseudo α-closed.

Proof. (1) If A ∈ α O(S) then A = αint(A) so by Lemma 5.3.2(1) we have αint(A) ⊂ αQ(A) ⊂ A, hence αQ(A)=A which means that A is pseudo α-open set.

(2)If A ∈ αC (S) then A=αcl(A) so by Lemma 5.3.2(2) we have A ⊂ αP (A) ⊂ αcl(A), hence αP(A)=A which means that A is pseudo α-closed set.

Theorem 3.4 Let (S, T , τ, κ) be a ditopological texture space.

(1) Suppose that the given space is α compact, α stable and α-R0 . Then every pseudo α closed set A ∈ T is α-compact.

(2) Suppose that the space is α-cocompact, α-costable and co-αR0. Then every pseudo α open set A ∈ T is α-cocompact.

Proof.(1) Let A ∈ T be pseudo α-closed. We must prove that A is α compact. Let Gi ∈ αO(S), i ∈ I satisfy the condition A⊂ Vi∈I Gi. First we prove αP(A) ⊂Vi∈I

Gi. Assume if $\alpha P(A) \not\subset \vee Gi$ . By the definition of $\alpha P(A)$ there exists $s \in S$ with $A \not\subset Qs$ and $\alpha cl(Ps) \not\subset Gi$ , for each i. Now $A \not\subset Qs \longrightarrow \vee Gi \not\subset Qs$ (i.e) $Ps \subset \vee Gi$ which implies $\alpha cl(Ps) \subset Gi$ since (τ, κ) is αR0, which is a contradiction, hence $\alpha P(A) \subset \vee Gi$ . Now given that A is pseudo α closed, therefore we have $\alpha P(A) = \alpha cl(A)$ and so $\alpha cl(A) \subset \vee Gi$

Then two cases arise

case(1) :If $\alpha cl(A) = S$. As S is α compact, we have $\alpha cl(A)$ is αcompact. Then by definition there exists i1, i2......in such that $\alpha cl(A) \subset Gi1 \cup Gi2 \cup ..... \cup Gin$ . Since $A \subset \alpha cl(A)$ we have $A \subset \cup Gi$ , i = 1, 2, ...., n which shows that A is α-compact.

case(2) $\alpha cl(A) \neq S$. As S is αstable, we have $\alpha cl(A)$ is α compact. (2) is dual of (1).

**Reference**

[1] M.E.Abd El monsef, E.F Lashien and A.A Nasef on I-open sets and Icontinuous func- tions, Kyungpook Math., 32 (1992) 21-30.
[2] L. M. Brown, M. Diker, Ditopological texture spaces and intuitionistic sets, Fuzzy sets and systems 98, (1998), 217-224.
[3] L. M. Brown, R. Erturk, Fuzzy Sets as Texture Spaces, I. Representation Theorems, Fuzzy Sets and Systems 110 (2) (2000), 227-236.
[4] L. M. Brown, R. Erturk, Fuzzy sets as texture spaces, II. Subtextures and quotient textures, Fuzzy Sets and Systems 110 (2) (2000), 237-245.
[5] L. M. Brown, R. Erturk, and S. Dost, Ditopological texture spaces and fuzzy topology, I. Basic Concepts, Fuzzy Sets and Systems 147 (2) (2004), 171-199. 3
[6] L. M. Brown, R. Erturk, and S. Dost, Ditopological texture spaces and fuzzy topology, II. Topological Considerations, Fuzzy Sets and Systems 147 (2) (2004), 201-231.
[7] L. M. Brown, R. Erturk, and S. Dost, Ditopological texture spaces and fuzzy topology, III. Separation Axioms, Fuzzy Sets and Systems 157 (14) (2006), 1886-1912.
[8] M. Demirci, Textures and C-spaces, Fuzzy Sets and Systems 158 (11) (2007), 1237-1245.
[9] S. Dost, L. M. Brown, and R. Erturk, β-open and β-closed sets in ditopological setting, submitted.
[10] S. Dost, Semi-open and semi-closed sets in ditopological texture space, submitted. [11] S. Dost, C-sets and C-bicontinuity in ditopological texture space, preprint.
[12] J.Dontchev, On pre -I-open sets and a decomposition of I-continuity, Banyan Math.J.,2(1996).
[13] J.Dontchev, M.Ganster and D.Rose, Ideal resolvability, Topology Appl., 93 (1) (1999),1-16.
[14] M. M. Gohar, Compactness in ditopological texture spaces,(PhD Thesis, Hacettepe Uni- versity, 2002).
[15] T.R.Hamlett and D.S.Jankovic, Compatible extensions of ideals, Boll.Un.Mat. Ital., 7 (1992), 453-465.
[16] S. Jafari, Viswanathan K., Rajamani, M., Krishnaprakash, S. On decomposition of fuzzy A-continuity, The J. Nonlinear Sci. Appl. (4) 2008 236-240.
[17] O. Njastad, On some classes of nearly open sets, Pacic J. Math. 15, (1965), 961-970
[18] M. K. Singal, N. Rajvanshi, Fuzzy alpha-sets and alpha-continuous maps, Fuzzy Sets and Systems 48 (1992), 383-390.