# Total time minimizing fuzzy transportation problem 

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#### Abstract

This paper develops a new technique for the time minimizing in fuzzy transportation problems using triangular fuzzy number. The procedure involves the fuzzy active transportation routes which are based on moving from basic fuzzy feasible solution to another till the last solutions arrived. A numerical example, illustrating the method is also included.


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## 1. Introduction

The basic difference between the cost minimizing transportation problem and the time minimizing transportation problem is that n the former the cost of transportation changes with variation in the quantity of the commodity and in the later the time involved remains unchanged irrespective of the quantities of the commodity in the occupied cells in the time minimization transportation problem. The time minimization transportation problem has been studied by Hammer [5]Garfinkel and Rao [3] and Szwarc[9]. Hammer [5] and Szwarc [11] used labeling to solve the problem by introducing a sufficiently large cost on certain routes. The transportation time is relevant in a variety of real transportation problems too. There are two types of problems regarding the transportation time [10];(i) minimization of the total time called minimization of first transportation time , and (ii) minimization of the transportation time of the longest active transportation route called minimization of second transportation time the problem of Barasov[1]. For (ii) the total numbers of units on transportation operation with longest time is minimized [5]. The total time problem is formulated and res olved in [6]. Lija Nikoli [8] proposed a new algorithm in total time transportation problem based on same criteria. This paper attempts to bring out the total time minimization in fuzzy transportation problem by using new techniques.
This paper is organized as follows: In section (2) some basic definition on fuzzy set theory is listed. In section (3) the new solution algorithm in the time minimizing fuzzy transportation problem is proposed. In section (4) Zero termination method algorithms to find out the optimal solution for the total fuzzy transportation on minimum cost is discussed. Section (5) is endowed with a new numerical example.
2.1 Definition: [6] A fuzzy number "a" represent with the three points as follow $a=[\underline{a}, a, \bar{a}]$.This representation is interpreted as membership function is
$\mu_{a}(x)=\left\{\begin{array}{lc}0 & \text { for } x<\underline{a} \\ \frac{x-\underline{a}}{a-\underline{a}} & \text { for } \underline{a} \leq x \leq a \\ \bar{a}-x & \text { for } a \leq x \leq \bar{a} \\ \frac{\bar{a}-a}{} & \text { for } x>\bar{a}\end{array}\right\}$
2.2 Fuzzy arithmetic operations [8] Operation on fuzzy numbers can be generalized from that of crisp interval as follows: $\forall a, b, \underline{a}, \underline{b}, \bar{a}, \bar{b} \in R$
Let $a=[\underline{a}, a, \bar{a}]$ and $b=[\underline{b}, b, \bar{b}]$
$a+b=\{\underline{a}+\underline{b}, a+b, \bar{a}+\bar{b}\}$
$a-b=\{\underline{a}-\bar{b}, a-b, \bar{a}-\underline{b}\}$
$\mathrm{a}_{\alpha}=\left[\underline{\mathrm{a}}^{\alpha}, \mathrm{a}^{\alpha}\right]=[(a-\underline{a}) \alpha+\underline{a},(a-\bar{a}) \alpha+\bar{a}] b_{\alpha}=\left[\underline{b}^{\alpha}, \bar{b}^{\alpha}\right]=[(b-\underline{b}) \alpha+\underline{b},(b-\bar{b}) \alpha+\bar{b}]$, Product of $\alpha$-cut

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$$
\begin{aligned}
& R_{\alpha}=a_{\alpha} \times b_{\alpha}\left[\underline{a}^{\alpha}, \bar{a}^{\alpha}\right] \times\left[\underline{b}^{\alpha} \bar{b}^{\alpha}\right]=\left[\underline{R}^{\alpha}, \bar{R}^{\alpha}\right] \\
& \text { Where } \\
& \underline{R}^{\alpha}=\min \left(\underline{a}^{\alpha} \underline{b}^{\alpha}, \underline{a}^{\alpha} \bar{b}^{\alpha}, \bar{a}^{\alpha} \underline{b}^{\alpha}, \bar{a}^{\alpha}-\alpha\right) \text { and } \\
& \bar{R}^{\alpha}=\max \left(\underline{a}^{\alpha} \underline{b}^{\alpha}, \underline{a}^{\alpha} \underline{b}^{\alpha},-_{\alpha}^{\alpha}, \underline{b}^{\alpha},-a^{\alpha} \bar{b}^{\alpha}\right)
\end{aligned}
$$

2.3 Measure of a fuzzy triangular number [2] If $A=[\underline{a}, a, \bar{a}]$ is a Triangular Fuzzy Number, then the measure of A denoted by $M^{T R I}(A)^{\text {is defined as }} M^{T R I}(A)=\frac{1}{4}(2 a+\underline{a}+\bar{a})$. It easily obtained that $A \leq B$ iff $M(A) \leq M(B)$
2.4 Fuzzy Transportation problem [6] Consider transportation with m sources and n destinations. Let $a_{i}$ be the fuzzy availability at source i and $b_{j}$ be the fuzzy requirements at destination j . Let $c_{i j}$ be the fuzzy unit transportation cost from source i to destination j . Let $x_{i j}$ denote the number of fuzzy units to be transported from source ito destination j . Then the problem is to determine a feasible way of transporting the available quantity at each source to satisfy the demand at each destination, so that the total transp ortation cost is minimum. The mathematical formulation of the fuzzy transportation problem whose parameters are fuzzy numbers under the given constraint is;

$$
\begin{aligned}
& \min z=\sum_{i} \sum_{j} c_{i j} x_{i j} \\
& \text { s.t } \sum_{j=1}^{n} x_{i j}=a_{i} \quad i=1,2 . . m \\
& \sum_{i=1}^{m} x_{i j}=b_{j} \quad j=1,2 . . n \\
& \sum_{i=1}^{m} a_{i}=\sum_{j=1}^{n} b_{j} \quad \text { and } \quad x_{i j} \geq 0
\end{aligned}
$$

where the fuzzy parameters are fuzzy numbers

## 3 Formulation of the time minimizing fuzzy transportation Problem

In a time minimizing fuzzy transportation problem, the time of fuzzy transporting goods from m origins to n destinations is minimized, satisfying certain conditions in respect of availabilities at sources and requirements at the destinations.
Thus, a time minimizing fuzzy transportation problem is:
$Z=\left[\operatorname{maxt}_{\mathrm{ij}} / x_{i j}>0\right]$
subject to $\sum_{j=1}^{n} x_{i j}=a_{i}, i=1,2, \ldots m$

$$
\sum_{i=1}^{m} x_{i j}=b_{j}, j=1,2, \ldots n, \quad x_{i j} \geq 0
$$

Here $t_{i j}=\left[t_{i j}, t_{i j}, \overline{t_{i j}}\right]^{\text {is }}$ the time of transporting goods from the $\mathrm{i}^{\text {th }}$ origin, where availability is

$$
a_{i}=\left[\underline{a_{i}}, a_{i}, \bar{a}_{i}\right] \text { to }
$$ the $j^{\text {th }}$ destination, where the requirement is $b_{i}=\left[\underline{b_{i}}, b_{i}, \overline{b_{i}}\right]$. For any given feasible solution, $X_{i j}=\left[x_{i j}, x_{i j}, \overline{x_{i j}}\right]$ satisfying (1), the time of transportation is the maximum of $t_{i j}=\left[t_{\underline{i j}}, t_{i j}, \overline{t_{i j}}\right]$ among the cells in which there are positive allocations, i.e., corresponding to the solution X , the time if Fuzzy transportation is $\mathrm{Z}=\left[\max t_{i j} / x_{i j}>0\right]$, the total fuzzy transportation time, often is observed the " fuzzy transportation efficiency" and minimized is;

$$
\begin{equation*}
F(x)=\sum_{i \in I} \sum_{j \in J} t_{i j} x_{i j} \tag{2}
\end{equation*}
$$

But, we prefer that in many real life problems is mostly natural to focus to minimization only the time of active fuzzy transportation routes ( $\mathrm{i}, \mathrm{j}$ ), as next objective is
$T(x)=\sum_{i \in I} \sum_{j \in J} t_{i j} h_{i j}$, where $h_{i j}=\left[\underline{h_{i j}}, h_{i j}, \overline{h_{i j}}\right]$
$h_{i j}$ is the auxiliary function show active and non active fuzzy transportation route is;
$h_{i j}=\left\{\begin{array}{lll}1 & \text { if } & x_{i j}>0 \\ 0 & \text { if } & x_{i j}=0\end{array}\right\}$

If the multiple fuzzy optimal solutions exist with $T^{*}$ as minimal value of (3), it is recommended to optimize an another criteria retain $T^{*}$, like the fuzzy transportation efficiency (2), the time of the longest fuzzy active transportation operation, the number of units on fuzzy transportation operation with fuzzy longest time, the total fuzzy transportation cost etc.

### 3.1 Proposed New Algorithm

In this paper we proposed a new algorithm regarding the time minimization in fuzzy transportation problem which is based on the following criteria:
Let $x^{(k)}$ and $x^{(k+1)}$ are two basic neighboring fuzzy feasible solutions, where $x_{i j}^{(k)}$ is entering basis variable and $x_{i s}^{(k)}$ is leaving fuzzy basis variable for $x^{(k)}$ :

$$
\begin{aligned}
& x^{(k)} \text { Contain: } x_{i j}^{(k)}=0 \text { and } x_{i s}^{(k)>0,} \\
& x^{(k+1)} \text { Contain: } x_{i j}^{(k+1)}>0 \text { and } x_{i s}^{(k+1)}=0
\end{aligned}
$$

Therefore
$x_{i j}^{(k+1)}=x_{i s}^{(k)}$
$x^{(k)}, x^{(k+1)}, x_{i j}^{(k)}$ and $x_{i s}^{(k)}$ be the triangular fuzzy number,
In moving from $x^{(k)}$ to $x^{(k+1)}$ the total time $\mathrm{T}(\mathrm{x})$ given as (3) will be changed with the following values:
$q_{i j}^{(k)}=t_{i j}-t_{i s}$
The characteristic $q_{i j}=\left[\underline{q_{i j}}, q_{i j}, \overline{q_{i l}}\right]$ is the change of the fuzzy transportation time in problem (3).

Clearly, the total time $T^{(k+1)}$ is determined by values $q_{i j}^{(k) \text { as following: }}$

$$
T^{(k+1)}=\left\{\begin{array}{lll}
>T^{(k)} & \text { if } & q_{i j}^{(k)}>0  \tag{7}\\
\leq T^{(k)} & \text { if } & q_{i j}^{(k)} \leq 0
\end{array}\right\}
$$

Let $T^{*}$ is minimum value of $\mathrm{T}(\mathrm{x}), x^{*}$ is the fuzzy optimal solution of (3) and $X_{T}$ is a set of multiple fuzzy optimal solution of (3);

$$
\begin{gather*}
T^{*}=\min \left\{T(x)=\sum_{i \in I} \sum_{j \in J} t_{i j} h_{i j}\right\}  \tag{8}\\
x_{T}^{*}=\left\{x / T^{*}=\min \left\{T(x)=\sum_{i \in I} \sum_{j \in J} t_{i j} h_{i j}\right\}\right\}  \tag{9}\\
X_{T}^{*}=\left\{x_{T}^{*}\right\} \tag{10}
\end{gather*}
$$

Above discussion makes possible to develop the method of solution for fuzzy transportation problem (3). If these problems have multiple fuzzy optimal solutions (10), clearly it may be required to optimize next criterion.
Minimize the some of the following criterions:
The fuzzy transportation efficiency from (2) is

$$
\begin{equation*}
\min \left(T(x)==T^{*}\right)=\left\{F(x)=\sum_{i} \sum_{j} t_{i j} x_{i j}\right\} \tag{11}
\end{equation*}
$$

Minimize the time of the longest active transportation operation

$$
\begin{equation*}
\min \left(T(x)==T^{*}\right)=\left\{t(x)=\max t_{i j}, \quad x_{i j}>0\right\} \tag{12}
\end{equation*}
$$

Minimize the number of units on fuzzy transportation operation with longest fuzzy time

$$
\begin{equation*}
\min \left(T(x)==T^{*}\right)=\left\{Q(x)=\sum_{t_{i j}=t(x)} x_{i j}\right\} \tag{13}
\end{equation*}
$$

Minimize the total fuzzy transportation cost

$$
\min \left(T(x)==T^{*}\right)=\left\{F(x)=\sum_{i} \sum_{j} c_{i j} x_{i j}\right\}
$$

Where $c_{i j}=$ the units transportation costs.

### 3.2 The Proposed Solutions Algorithm

Step 1: Find the basic fuzzy feasible solution $\mathrm{x}^{(1)}$ by zero termination method. Let number of iteration $\mathrm{k}=1$
Step 2: Determine the fuzzy indicators $h_{i j}^{(k)}$ of active fuzzy transportation routes $x_{i j}^{(k)}>0$, and the total fuzzy time $\mathrm{T}^{(\mathrm{k})}=\mathrm{T}\left(\mathrm{x}^{(\mathrm{k})}\right)$

$$
\begin{align*}
& h_{i j}^{(k)}=\left\{\begin{array}{ll}
1 & \text { if } x_{i j}^{(k)}>0 \\
0 & \text { if } x_{i j}^{(k)} \leq 0
\end{array}\right\}  \tag{15}\\
& T^{(k)}=\sum_{i} \sum_{j} t_{i j} h_{i j}^{(k)} \tag{16}
\end{align*}
$$

Step 3: Obtain the characteristics $q_{i j}^{(k)}$ for all nonbasic variables $x_{i j}^{(k)}=0^{\text {using (5).Use the changing path of the fuzzy basic }}$ solution and corresponding leaving fuzzy basic variables,
Step 4: Check the optimality of the fuzzy total time (3), using (7).If all $q_{i j}^{(k)}>0$, the optimal solution $x^{*}$ is found .stop otherwise, go to step (5)
Step 5: Obtain next fuzzy basic solution, using fuzzy entering variable $x_{i j}$ with minimum $q_{i j}^{(k)}$, regarding $q_{i j}^{(k)}<0^{\text {set } \mathrm{k}=\mathrm{k}+1}$ and go to step 2.
If the fuzzy optimal solution $x^{*}$ in last step 2 has $q_{i j}^{(k)}<0$ for nonbasic variables $x_{i j}^{(k)}=0$, there is no unique fuzzy optimal solution and exist a set of multiple fuzzy optimal solution $x_{T}^{*}$. Each of these variables gives an alternative an fuzzy optimal solution for (3), go to fuzzy optimize other criteria.
Step 5: Choose one of the fuzzy criteria from (11) to (14) and calculate the value increase $\Delta_{i j}^{(k)}$ for each nonbasic variable $x_{i j}^{(k)} \leq 0$

Step 6; If there are negative increase, $\Delta_{i j}^{(k)}<0$, for regarded criterion, chose minimum of them and minimize this criterion inset of multiple optimal solution for (3).
Step 7: Repeat step 2 with each of negative increase for regarded criterion and choose solution with minimum criterion value

## 4 Zero Termination Method.

The Initial fuzzy feasible solution is obtained using zero termination method. Its procedure is summarized below
Step 1: Construct the transportation table
Step 2: Select the very smallest unit transportation cost value for each row and subtract each entries of the transportation table from the corresponding row minimum. After that using the similar way subtract each column entries of the transportation table from the corresponding column minimum
Step 3: In the reduced cost matrix, there will be at least one zero in each row and column, then find the termination value of all the Zero cells in the reduced cost matrix by the following simplification: The termination cost $T=$ Sum of the costs of all the cells adjacent to zero cell is divided by the number of non- zero cells added.
Step 4: Choose the cell having maximum termination cost ( T ), if it has one maximum value, then first maximum possible allocation (demand) is made to the cell. If it has one (or) more equal values then select the cell having minimum cost of transportation and allocation is made.
Step 5: After the above step, the exhausted demands (column) (or) supplies (row) to be trimmed. The resultant matrix must possess at least one zero is each row and column, else repeat step (2)
Step 6: Repeat step (3) to step (5) until the fuzzy feasible solution is obtained.

## 5 Example

Let us consider the following fuzzy transportation problem with $m=4$ sources and $n=5$ destinations, each row corresponds to a supply point and each column to a demand point.

Table 1 Fuzzy Transportation Problem

|  |  |  |  |  | Source |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | (-2 0 2) | (0 1 2) | (-202) | (-2 0 2) | [ $\left.\begin{array}{llll}0 & 1 & 2\end{array}\right]$ |
|  | ( 4812 ) | (47 10) | (246) | (135) | [246] |
|  | (246) | (468) | (468) | (4710) | [4 6 8] |
| Demand | [135] | [02 4] | [135] | [13 5] |  |

In each cell ( i j ) top left corner represents the time $t_{i j}$ required for fuzzy transporting $x_{i j}$ fuzzy units from source $a_{i}$ to $b_{j}$ destinations. The fuzzy basic variable $x_{i j}$ are presented in the middle of corresponding cells and the increase $q_{i j}$ of time in bottom right corner of each cell
(ij) with non-basic variable by using Step ( 1 to 4 ) may be determined using Zero termination method in given FTP,

Table 2: The Fuzzy optimal solution of ( $\mathbf{X}^{\mathbf{1}}$ )

|  |  |  |  |  | Source |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & (-202) \\ & *\left[\begin{array}{lll} -4 & -1 & 2 \end{array}\right] \end{aligned}$ | $\begin{gathered} \left(\begin{array}{lll} 0 & 1 & 2 \end{array}\right) \\ {\left[\begin{array}{llll} 0 & 1 & 2 \end{array}\right]} \end{gathered}$ | $\begin{aligned} & \left(\begin{array}{lll} -2 & 0 & 2 \end{array}\right) \\ & \quad *\left[\begin{array}{lll} 2 & 6 & 10 \end{array}\right] \end{aligned}$ | $\begin{aligned} & \left(\begin{array}{lll} -2 & 0 & 2 \end{array}\right) \\ & *\left[\begin{array}{llll} -3 & -1 & 2 \end{array}\right] \end{aligned}$ | [ $\left.\begin{array}{llll}0 & 1 & 2\end{array}\right]$ |
|  | $\begin{aligned} & \hline(4812) \\ & \quad *[-2410] \\ & \hline \end{aligned}$ | $\left.\begin{array}{l} \hline\left(\begin{array}{lll} 4 & 7 & 10 \end{array}\right) \\ * \\ * \end{array} \mathbf{- 2} 388\right]$ | $\begin{aligned} & \hline\left(\begin{array}{lll} 2 & 4 & 6 \end{array}\right. \\ & {\left[\begin{array}{llll} -3 & 1 & 5 \end{array}\right]} \\ & \hline \end{aligned}$ | $\begin{array}{lll} \left(\begin{array}{lll} 1 & 3 & 5 \end{array}\right. \\ {\left[\begin{array}{lll} 1 & 3 & 5 \end{array}\right.} \\ \hline \end{array}$ | [2 4 6] |
|  | $\begin{aligned} & (246) \\ & {[135]} \\ & \hline \end{aligned}$ | $\begin{aligned} & \left(\begin{array}{lll} 4 & 6 & 8 \end{array}\right) \\ & {\left[\begin{array}{llll} -2 & 1 & 4 \end{array}\right]} \end{aligned}$ | $\begin{aligned} & \left(\begin{array}{lll} 4 & 6 & 8 \end{array}\right) \\ & {\left[\begin{array}{lll} -4 & 2 & 8 \end{array}\right]} \end{aligned}$ | $\begin{gathered} (4710) \\ \quad *[-149] \end{gathered}$ | [4 6 8] |
| Demand | $\left[\begin{array}{lll}1 & 3\end{array}\right]$ | [02 4] | [135] | [135] |  |

$F^{*}=F^{1}=(012)(012)+(246)(-315)+(135)(135)+\left(\begin{array}{ll}2 & 4\end{array}\right)\left(\begin{array}{ll}1 & 3\end{array}\right)+(468)(-214)$

$$
\left.\begin{array}{rl} 
& +\left(\begin{array}{lll}
4 & 6 & 8
\end{array}\right)(-4 \\
- & 2
\end{array}\right)
$$

Table 2 shows only indicators $h_{i j}$ with value $=1\left[h_{12}=h_{23}=h_{24}=h_{31}=h_{32}=h_{33}=1\right]$
The corresponding fuzzy total time $\mathrm{T}^{(1)}=[13,24,35]=\mathrm{T}^{(1)}$
The longest active fuzzy transportation operation $t^{(1)}=\left[\begin{array}{lll}0 & 1 & 2\end{array}\right]$
The Number of fuzzy transported units $\mathrm{Q}^{(1)}=\left[\begin{array}{lll}-2 & 1 & 4\end{array}\right]$
To calculate the increase of $q_{i j}$ in table (2) which are verify the unique fuzzy optimal solution.

| Non - Basic Variables <br> $x_{i j}^{(1)}=0$ | Indicators $d_{i j}^{(1)}$ for $\mathrm{F}(\mathrm{x})$ | Indicators $q_{i j}^{(1)}$ for $\mathrm{T}(\mathrm{x})$ |
| :--- | :--- | :--- |
| $x_{11}^{(1)}=0$ | $d_{11}^{(1)}=[-6,1,8]>0$ | $q_{11}^{(1)}=[-4,-1,2]<0$ |
| $x_{13}^{(1)}=0$ | $d_{13}^{(1)}=[-10,-2,6]<0$ | $q_{13}^{(1)}=[2,6,10]>0$ |
| $x_{14}^{(1)}=0$ | $d_{14}^{(1)}=[-10,0,11]=0$ | $q_{14}^{(1)}=[-3,-1,2]<0$ |
| $x_{21}^{(1)}=0$ | $d_{21}^{(1)}=[-4,6,10]>0$ | $q_{21}^{(1)}=[-2,4,10]>0$ |
| $x_{22}^{(1)}=0$ | $d_{22}^{(1)}=[-2,3,8]>0$ | $q_{22}^{(1)}=[-2,3,8]>0$ |
| $x_{34}^{(1)}=0$ | $d_{34}^{(1)}=[-7,2,11]>0$ | $q_{34}^{(1)}=[-1,4,9]>0$ |

lon changing the path $l_{11}^{(1)}=\{(1,1)(1,2)(3,2)(3,1)\}$ for $x_{i j}^{(1)}=0$, there is
$x_{11}^{(2)}=\min \{(0,1,2)(2,4,6)\}=(0,1,2)=x_{12}^{(1)} \quad$, and $\quad x^{(1)} \quad$ is no fuzzy optimal solution for $T(x)$ because ${ }^{\text {of }} q_{11}^{(1)}=(-4,-1,2)<0$, next go to the Step (1 to 4 )

Table (3) Time minimizing fuzzy optimal solution stage 2 is: ( $\mathbf{X}^{(2)}$ )

|  |  |  |  |  | Source |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | (-2, 0,2 | (0, 1, 2) | (-2, 0,2$)$ | (-2, 0, 2) | [ $\left.\begin{array}{llll}0 & 1 & 2\end{array}\right]$ |
|  | [0, 1 , 2] | *[-2 144$]$ | *[000] | *[-3-12] |  |
|  | $(4,8,12)$ | $(4,7,10)$ | (2, 4, 6) | $(1,3,5)$ | [2 4 6] |
|  | *[-2 4 10] | *[-2 3 8] | $[-3,1,5]$ | [1, 3,5] |  |
|  | $(2,4,6)$ | $(4,6,8)$ | $(4,6,8)$ | $(4,7,10)$ | [4 6 8] |
|  | [-1,2,5] | $\left[\begin{array}{lll}-2 & 2 & 6\end{array}\right]$ | $[-4,2,6]$ | *[-1,4,9] |  |
| Demand | [135] | [024] | [135] | [135] |  |

$x_{11}$ be the entering fuzzy variable and $x_{12}$ is leaving the fuzzy variable, the entering fuzzy variable $x_{11}$ with solution of $x^{(2)}$ decrease $\mathrm{T}^{(1)}$ to $\mathrm{T}^{(2)}$, therefore $\mathrm{T}^{(2)}=\mathrm{T}^{(1)}+t_{11}^{(1)}=23-1=22$ [table (i)]
$\mathrm{F}^{2}=[-31,45,185]=61, \mathrm{t}^{(2)}=[4,6,8], \mathrm{Q}^{(2)}=[-2,2,6]$
To calculate the increase of $q_{i j}$ in table (3) which are verify the unique optimal solution.

| Non - Basic Variables <br> $x_{i j}^{(2)}=0$ | Indicators $d_{i j}^{(1)}$ for $\mathrm{F}(\mathrm{x})$ | Indicators $q_{i j}^{(1)}$ for $\mathrm{T}(\mathrm{x})$ |
| :--- | :--- | :--- |
| $x_{12}^{(2)}=0$ | $d_{12}^{(2)}=[-8,1,6]>0$ | $q_{12}^{(2)}=[-2,1,4]>0$ |
| $x_{13}^{(2)}=0$ | $d_{13}^{(2)}=[-10,-2,6]<0$ | $q_{13}^{(2)}=[0,0,0]=0$ |
| $x_{14}^{(2)}=0$ | $d_{14}^{(2)}=[-10,0,11]=0$ | $q_{14}^{(2)}=[-3,-1,2]<0$ |
| $x_{21}^{(2)}=0$ | $d_{21}^{(2)}=[-4,6,10]>0$ | $q_{21}^{(2)}=[-2,4,10]>0$ |


| $x_{22}^{(2)}=0$ | $d_{22}^{(2)}=[-2,3,8]>0$ | $q_{22}^{(2)}=[-2,3,8]>0$ |
| :--- | :--- | :--- |
| $x_{34}^{(2)}=0$ | $d_{34}^{(2)}=[-7,2,11]>0$ | $q_{34}^{(2)}=[-1,4,9]>0$ |

The indicators value $q_{i j}$ using table (3) it is exists for the fuzzy optimal conditions
Table 4: The time minimizing Fuzzy optimal solutions

|  |  |  |  |  | Source |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(-2,0,2)$ | $(0,1,2)$ | $(-2,0,2)$ | $(-2,0,2)$ | $[0112]$ |
|  | $[\mathbf{0}, \mathbf{1}, \mathbf{2}]$ | $*[-2,1,4]$ | $*[0,0,0]$ | $*[-3,-1,2]$ |  |
|  | $(4,8,12)$ | $(4,7,10)$ | $(2,4,6)$ | $(1,3,5)$ | $[246]$ |
|  | $*[-2,4,10]$ | $*[-2,3,8]$ | $[-\mathbf{3}, \mathbf{1}, \mathbf{5}]$ | $[\mathbf{1}, \mathbf{3}, 5]$ |  |
|  | $(2,4,6)$ | $(4,6,8)$ | $(4,6,8)$ | $(4,7,10)$ | $[468]$ |
|  | $[-\mathbf{1 , 2 , 5}]$ | $[-\mathbf{2} \mathbf{2} \mathbf{6}]$ | $[-\mathbf{4}, \mathbf{2}, \mathbf{6}]$ | $*[-\mathbf{1 , 4 , 9}]$ |  |
| Demand | $[135]$ | $[024]$ | $[135]$ | $[135]$ |  |

The minimum of total transportation time $\mathrm{T}^{*}=\mathrm{T}^{(2)}=22$ has multiple optimal solution $\mathrm{X}{ }^{\left({ }^{(2)}\right.}$ with $d_{13}^{(2)}=0, d_{12}^{(2)}<0$. Keeping minimum of $\mathrm{T}(\mathrm{x})$ may it's possible to fuzzy optimize some another criteria using the step (5 to 7 ) regarding changing the paths with their entering and leaving variable
$x_{13}^{(2)}=0, \mathrm{~L}_{13}^{(2)}=\{(1,3)(1,1)(3,1)(2,3)\}$
$x_{13}^{(3)}>0, \mathrm{x}_{13}^{(3)}=\min \{(0,1,2)(-4,2,6)\}=(0,1,2)=x_{11}^{(2)}$
Table 5: The time minimizing fuzzy optimal solution stage 3 is: $\left(X^{(3)}\right)$

|  |  |  |  |  | Source |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(-2,0,2)$ | $(0,1,2)$ | $\begin{gathered} (-2,0,2) \\ {[0,1,2]} \end{gathered}$ | $(-2,0,2)$ | [ 0 lll 12$]$ |
|  | ( $4,8,12)$ | $(4,7,10)$ | $\begin{gathered} (2,4,6) \\ {[-3,1,5]} \\ \hline \end{gathered}$ | $\begin{gathered} (1,3,5) \\ {[1,3,5]} \\ \hline \end{gathered}$ | [246] |
|  | $\begin{gathered} (2,4,6) \\ {[-1,3,7]} \\ \hline \end{gathered}$ | $\begin{aligned} & (4,6,8) \\ & {\left[-22^{6}\right]} \\ & \hline \end{aligned}$ | $\begin{aligned} & (4,6,8) \\ & {[-6,1,6]} \end{aligned}$ | $(4,7,10)$ | [4 6 8] |
| Demand | [135] | [024] | $\left[\begin{array}{ll}135\end{array}\right]$ | [135] |  |

$F^{(3)}=[-39,43,197]=67=61, T^{(3)}=22-1=21, \mathrm{t}^{(3)}=[4,6,8], \mathrm{Q}^{(3)}=[-2,2,6]$
On changing the path by using the step ( 5 to 7 )
$x_{14}^{(2)}=0, \mathrm{~L}_{14}^{(2)}=\{(1,4)(1,1)(3,1)(3,3)(2,3)(2,4)(1,4)\}$
$x_{14}^{(3)}>0, \mathrm{x}_{14}^{(3)}=\min \{(0,1,2)(-4,2,6)\}=(0,1,2)$
Table 6: The time minimizing fuzzy optimal solution stage 4 is: ( $X^{(4)}$ )

|  |  |  |  |  | Source |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(-2,0,2)$ | $(0,1,2)$ | $(-2,0,2)$ | $\begin{gathered} (-2,0,2) \\ {[0,1,2]} \\ \hline \end{gathered}$ | [ $\left.\begin{array}{llll}0 & 1 & 2\end{array}\right]$ |
|  | $(4,8,12)$ | $(4,7,10)$ | $\begin{gathered} (2,4,6) \\ {[-3,2,7]} \\ \hline \end{gathered}$ | $\begin{gathered} (1,3,5) \\ {[-1,2,5]} \\ \hline \end{gathered}$ | [2 4 6] |
|  | $\begin{gathered} (2,4,6) \\ {[-1,3,7]} \\ \hline \end{gathered}$ | $\begin{aligned} & (4,6,8) \\ & {[-226]} \\ & \hline \end{aligned}$ | $\begin{gathered} (4,6,8) \\ {[-6,1,6]} \\ \hline \end{gathered}$ | $(4,7,10)$ | [4 68 ] |
| Demand | [13 5] | [024] | [13 5] | [135] |  |

$\mathrm{F}^{*}=[-41,44,209]=70.6, \mathrm{~T}^{(4)}=21+0=21$,
$\mathrm{t}^{(4)}=[4,6,8], \mathrm{Q}^{(4)}=[-6,1,6]$
Since, the optimal condition exists, the solution $\left(X^{(4)}\right)$ keep $t^{(4)}=t^{(3)}=(4,6,8)$
So, $\mathrm{X}^{(3)}, \mathrm{X}^{(4)}$ is better than $\mathrm{X}^{(2)}$, but $\mathrm{X}^{(4)}$ is not better than $\mathrm{X}^{(3)}$,
The time minimizing fuzzy optimal solution obtained above in stages is summarized in the following table:

| Time <br> Minimization on <br> fuzzy criteria | $\mathbf{X}^{(\mathbf{1})}$ | $\mathbf{X}^{(\mathbf{2})}$ | $\mathbf{X}^{(\mathbf{3})}$ | $\mathbf{X}^{(4)}$ |
| :--- | :--- | :--- | :--- | :--- |
| $\mathbf{F}_{\mathbf{m i n}}(\mathbf{x})$ | 55.3 | 76.6 | 67 | 70.6 |
| $\mathbf{T}(\mathbf{x})$ | 23 | 22 | 21 | 21 |
| $\mathbf{t}(\mathbf{x})$ | $[012]$ | $[4,6,8]$ | $[4,6,8]$ | $[4,6,8]$ |
| $\mathbf{Q}(\mathbf{x})$ | $[-214]$ | $[-2,2,6]$ | $[-2,2,6]$ | $[-6,1,6]$ |

## 6 Conclusion

In this paper, we proposed a new algorithm to find the optimal solution in time minimizing fuzzy transportation problem with an illustrated with a numerical example. This would be a new attempt in solving the transportation problem in fuzzy environment. The authors value great comments and suggestion of referees.

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