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Static Analysis of Simply supported plate

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ABSTRACT

This research work deals with analysis of simply supported rectangular plate under two different types of loading viz. centrally applied point load and uniformly distributed load (UDL). Four different sizes of plates were considered depending on the shape of material that allowed in the experiments. These plates were assumed to be constructed by isotropic material and subjected to point loading & UDL. The plates were supported under standard simply supported conditions at the two ends (left & right end views from the operator) so that the moment singularity is assumed at the tip of internal line support. Plate deflections were measured for all the plates under different loading conditions. These values were then validated using the ANSYS software. The results then compared and error was found out. The physical quantities of the plates and the extent of contact related to the level of loading in the case of free contact are provided in the present work.

Introduction

A flat plate is generally considered to be a thin flat component that is subjected to load conditions that cause deflections transverse of the plate. Therefore, the loads are transverse pressures, transverse forces and moment vectors lying in the plane. Those loads are resisted mainly by bending. It is assumed that in plane membrane stresses are not present and that the transverse displacements are “small”. Generally, “small” is taken to mean a deflection that is less than half the thickness of the plate.

Focusing on the problems of flat plate with single boundary condition, there are several experimental and numerical methods used to analyze the problem. The numerical methods are generally found to be unsatisfactory (Leissa *et al.*, 1969), especially at the transition point of discontinuous boundary due to the problem singularity (Williams, 1952), and then, the use of integral transform (Sneddon, 1972) is one of the appropriately analytical methods to solve the problem which leads to determine the solution of an integral equation.

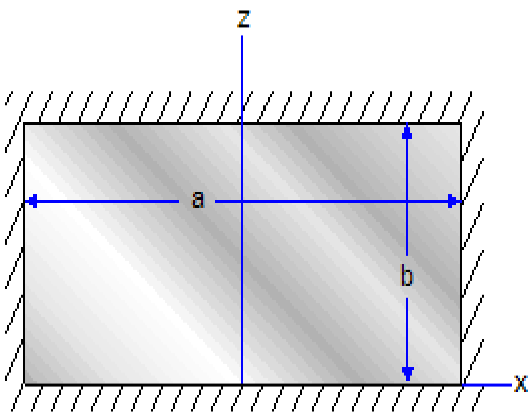
Much attention has been paid by many researchers to investigate the static bending problems (Yang, 1968; Keer and Sve, 1970; Kiattikomol *et al.*, 1974; Kiattikomol and Porn-anupapkul,1985; Kiattikomol and Sriswasdi, 1988), vibration characteristics, and stability and buckling behaviours (Leissa, 1969; Keer and Stahl, 1972; Stahl and Keer, 1972). However, the mentioned works are the problems of plate where the supports have the same level. Dundurs *et al.* (1974) first investigated the contact between the plates and the sagged supports in which the sagged supports are located at the plate edges. For the cases of sagged support placed in domain of the plate, recently, Sompornjaroensuk and Kiattikomol (2006) treated the two problems of rectangular plate simply supported on the two opposite edges and either free or clamped on the two other edges with an internal line sagged support.

However during the development stage of customized mechanical structures it becomes necessary to observe the deflections and verify the load carrying capacity of that structure. In that case application of strain gauge, Data acquisition system, sensors etc. are being used now days. But these techniques are quite expensive and creates extra burden for a small manufacturer. But at the same time it is essential to do some kind of load carrying analysis through experiment before fabricate the entire structure. This paper aims to find out the errors encountered during simple experiments after application of load for some selected simply supported plates.

Therefore, this research considers the problem of simply supported rectangular plate with two end supports. The solution technique is applied here is the experimental and results validation with ANSYS 5.4 to find out the error associated with the experimental technique.

Rectangular plate

Figure 1 shows some of the typical boundary conditions that can be applied to the edges of a plate. A segment of a plate can be fixed or enacted (left), simply supported (center), or mixed supported (right), or have a free edge. A simply supported condition usually means that the transverse displacement is zero on that segment but the rotation tangent to the segment is unknown. A fixed supported condition usually means that the rotation vector tangent to the segment is also zero. A free edge is stress free. That is, it has no moment or transverse shear resultants acting along its length.



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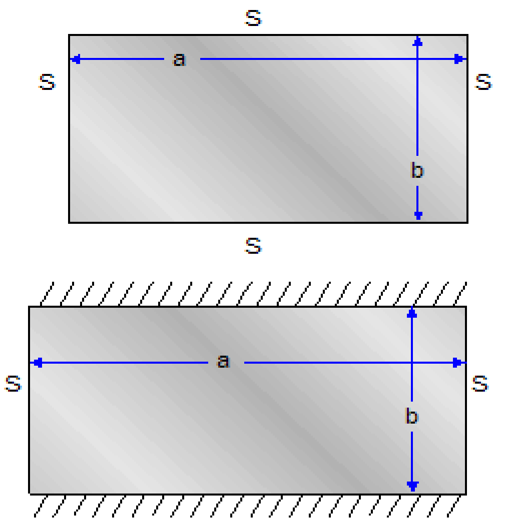

Fig.1: Some typical boundary condition options on rectangular plates

Deflection of Beams

The deformation of a beam is usually expressed in terms of its deflection from its original unloaded position. The deflection is measured from the original neutral surface of the beam to the neutral surface of the deformed beam. The configuration assumed by the deformed neutral surface is known as the elastic curve of the beam. In the present research work deformation has been measured for different plates as described afterward considering the plate equivalent to beam.

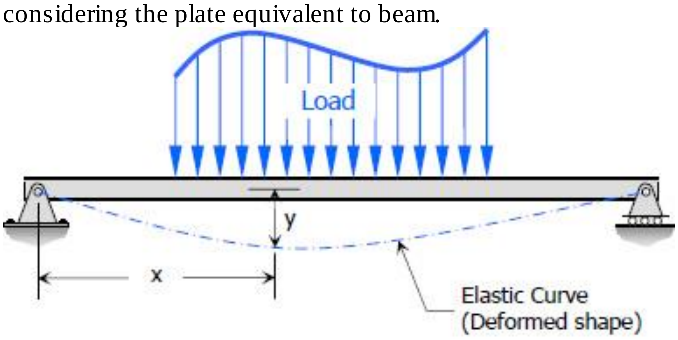

Fig.2: Typical deflection of a simply supported beam

Materials & methods

Four different sizes of plates are considered depending on the shape of material that allowed in the experiments. The plate is assumed to be constructed by isotropic material and subjected to point loading. The sizes are 550x600x4mm, 550x600x6mm, 550x600x8mm & 550x600x10mm. All are isotropic, structural steel.

Table1: Description of various sizes plates used in the experiments

Plate description	Size (a x b x t) mm	Material (isotropic)
Plate-I	550 x 600 x 4	Structural steel
Plate-II	550 x 600 x 6	Structural steel
Plate-III	550 x 600 x 8	Structural steel
Plate-V	550 x 600 x 10	Structural steel

Fig.3 shows the Plate-I with simply supports. Ten node points on the top surface of the plate were selected to find out the deformation. Similar work was done for the other plates as required.

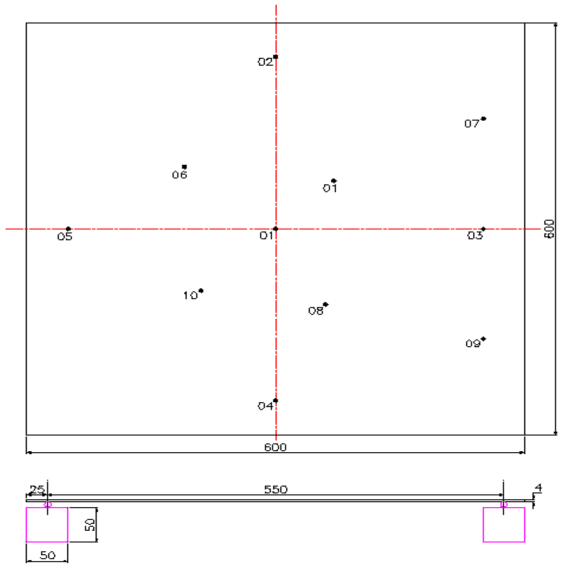

Fig.3: Rectangular Plate-I with simply supports

The experiments were conducted in the institute laboratory. All the plates were chosen as per the specified sizes of structural steel. The pates were designated as Plate-I, Plate-II, Plate-III and Plate-IV as discussed in the Table1.

Experiments with point loading

Initially Plate-I was kept on a fixture specially designed for the experiment above a surface table. Ten numbers of node points were selected randomly above the Plate-I. Ten numbers dial gauges were fixed under the plate at these nodes. All the gauges were calibrated before the experiments and set to “0” (zero) reading. Point load is applied at the centre of the plate with ranges from 4 - 20 kg. Deformations were noted after the application of load. Fig.4 shows the experimental set up for Plate-I. Experiments were carried out for Plate-I. Similar experiments repeated for the Plate-II, Plate-III and Plate-IV successively and the corresponding data were observed.

Fig.4: Experimental setup with point loading

Experiments with point UDL

In a way similar to point loading, the experiments were carried out with UDL. Here Plate-I was loaded ad shown in the fig.5. Loading was done through placement of sand bag each weigh 360 gm uniformly over the plate. Loads were increased by adding extra layers of sandbag over the first one.

Deformations were noted accordingly. Similar experiments repeated for the Plate-II, Plate-III and Plate-IV successively and the results were noted.

Fig.5: Experimental setup with UDL

Analysis using ANSYS

Analysis was done using ANSYS 5.4 for all the plates to find the deflection of a simply supported plate with a Central Point Load and UDL. The deflection was found after dividing the model of the plate in 24 elements with four node element shell63 for the model and applied appropriate boundary conditions. The deformations were taken at ten selected nodes which also exactly same nodes points taken during the experiment. The results were presented in the Tables 2-5 respectively. The error also calculated with both the results and these are given in the same table as stated above. Figs.6-9 depicts the comparison of results obtain during experiment and ANSYS. The graphical deformations achieved in ANSYS were is shown in Figs. 10-13.

Table 2: Deformations of the Plate-I under point loading & UDL

		Deformation, mm											
Coordinates		at 4 kg			at 5 kg			at 9 kg			at 10 kg		
X	Y	ANSYS	Exp.	Error%	ANSYS	Exp.	Error%	ANSYS	Exp.	Error%	ANSYS	Exp.	Error%
45.83	300	4.93E-02	0.05	1.4	6.17E-02	0.07	11.8571429	0.111	0.1	7.5	0.1233	0.1	11.928571
137.5	100	0.1	0.11	9.09091	0.1508	0.16	5.75	0.2715	0.3	9.5	0.3017	0.3	2.6774194
137.5	450	0.1259	0.13	3.15385	0.1576	0.17	7.29411765	0.2836	0.3	5.466667	0.3152	0.4	9.9428571
229.16	150	0.1748	0.2	12.6	0.2187	0.24	8.875	0.3937	0.4	1.575	0.4375	0.5	12.5
275	50	0.1664	0.17	2.11765	0.2082	0.21	0.85714286	0.3748	0.4	6.3	0.4164	0.5	7.4666667
275	300	0.2109	0.22	4.13636	0.2639	0.28	5.75	0.4751	0.5	4.98	0.5279	0.6	12.016667
275	550	0.1664	0.19	12.4211	0.2082	0.21	0.85714286	0.3748	0.4	6.3	0.4164	0.5	16.72
412.5	500	0.1205	0.13	7.30769	0.1508	0.16	5.75	0.2715	0.3	9.5	0.3017	0.3	2.6774194
504.16	100	4.38E-02	0.05	12.4	5.48E-02	0.06	8.66666667	9.86E-02	0.1	1.4	0.1095	0.1	8.75
504.16	300	4.93E-02	0.05	1.4	6.17E-02	0.07	11.8571429	0.111	0.1	7.5	0.1233	0.1	11.928571

x,y (mm)	Deformation (mm) at 35 pkts @360			Deformation (mm) at 70 pkts @360 gm		
	ANSYS	Exp.	Error%	ANSYS	Exp.	Error%
45.83,300	0.07	0.08	11.33	0.13	0.14	5.00
137.5,100	0.42	0.43	3.03	0.83	0.87	3.82
137.5,450	0.41	0.41	0.54	0.82	0.85	4.05
229.16,150	0.69	0.71	3.51	1.37	1.40	2.12
275,50	0.76	0.78	2.01	1.53	1.52	-0.58
275.0,300	0.71	0.71	-0.30	1.42	1.50	5.07
275.0,550	0.76	0.79	3.25	1.53	1.60	4.50
412.5,500	0.42	0.41	-1.46	0.83	0.90	7.56
504.16,100	0.07	0.07	0.91	0.14	0.15	7.53
504.16,300	0.07	0.07	-2.31	0.13	0.15	11.33

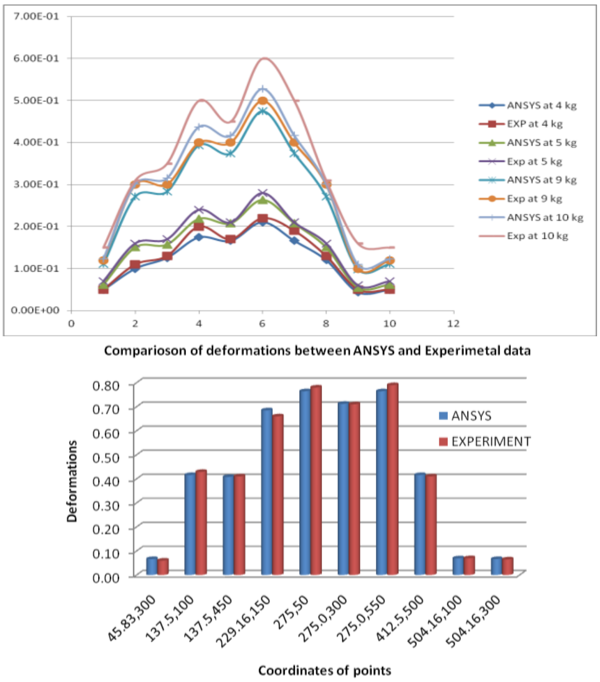

Fig.6: Comparison of results between ANSYS and Experimental for Plate-I

Table 3: Deformations of the Plate-II under point loading & UDL

		Deformation (mm)																	
Coordinates		at 4 kg			at 5 kg			at 9 kg			at 10 kg			at 14 kg			at 16 kg		
X	Y	ANSYS	Exp.	Error%	ANSYS	Exp.	Error%	ANSYS	Exp.	Error%	ANSYS	Exp.	Error%	ANSYS	Exp.	Error%	ANSYS	Exp.	Error%
45.83	300	1.46E-02	0.017	14.12	1.83E-02	0.02	8.65	3.29E-02	0.04	17.75	3.65E-02	0.04	8.75	5.11E-02	0.06	14.83	8.47E-02	0.1	15.30
137.5	100	3.57E-02	0.04	10.75	4.47E-02	0.05	10.62	8.04E-02	0.09	10.67	8.93E-02	0.1	10.70	1.25E-02	0.014	10.71	0.143	0.16	10.63
137.5	450	3.73E-02	0.04	6.75	4.67E-02	0.05	6.62	8.41E-02	0.09	6.56	9.33E-02	0.11	15.18	1.30E-02	0.015	13.33	0.1494	0.17	12.12
229.16	150	5.18E-02	0.052	0.38	6.48E-02	0.07	7.41	0.1166	0.13	10.31	0.1296	0.14	7.43	0.181	0.2	9.50	0.2074	0.24	13.58
275	50	4.93E-02	0.05	1.40	6.17E-02	0.07	11.87	0.111	0.12	7.50	0.1233	0.125	1.36	0.1727	0.19	9.11	0.1974	0.22	10.27
275	300	6.25E-02	0.07	10.71	7.82E-02	0.09	13.09	0.14	0.15	6.67	0.1564	0.18	13.11	0.219	0.25	12.40	0.2503	0.28	10.61
275	550	4.93E-02	0.05	1.40	6.17E-02	0.07	11.87	0.11	0.125	12.00	0.1233	0.14	11.93	0.1727	0.2	13.65	0.1974	0.23	14.17
412.5	500	3.57E-02	0.042	15.00	4.47E-02	0.05	10.62	8.04E-02	0.09	10.67	8.93E-02	0.1	10.70	0.1251	0.14	10.64	0.143	0.16	10.63
504.16	100	1.30E-02	0.015	13.33	1.62E-02	0.018	9.89	2.92E-02	0.031	5.81	3.24E-02	0.035	7.43	4.54E-02	0.051	10.98	5.19E-02	0.06	13.50
504.16	300	1.46E-02	0.016	8.75	1.83E-02	0.02	8.65	3.29E-02	0.035	6.00	3.65E-02	0.04	8.75	5.11E-02	0.055	7.09	5.84E-02	0.065	10.15

X,Y	at 35 pkts @360 gm			at 70 pkts @360 gm		
	ANSYS	Exp.	Error%	ANSYS	Exp.	Error%
45.83,300	0.020	0.022	10.455	0.039	0.040	1.500
137.5,100	0.123	0.125	1.600	0.246	0.250	1.600
137.5,450	0.120	0.130	7.692	0.241	0.250	3.600
229.16,150	0.203	0.205	0.976	0.406	0.450	9.778
275,50	0.226	0.230	1.739	0.453	0.500	9.400
275.0,300	0.211	0.240	12.083	0.422	0.480	12.083
275.0,550	0.226	0.250	9.600	0.453	0.480	5.625
412.5,500	0.123	0.140	12.143	0.247	0.250	1.200
504.16,100	0.021	0.023	8.696	0.041	0.046	10.870
504.16,300	0.019	0.020	5.000	0.039	0.040	1.500

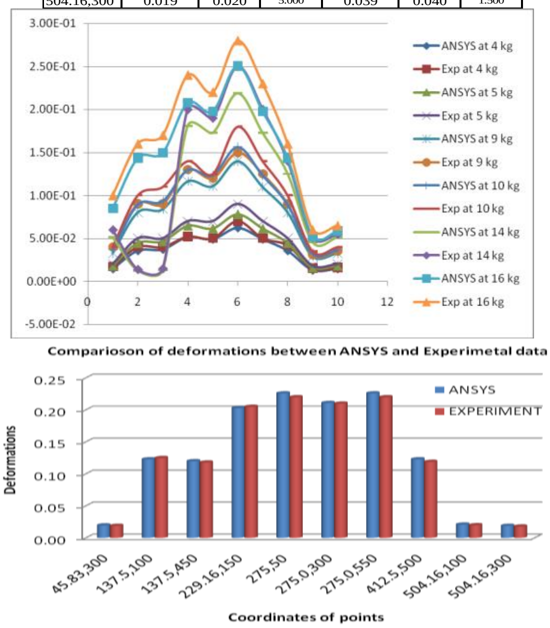

Fig.7: Comparison of results between ANSYS and Experimental for Plate-II

Table 4: Deformations of the Plate-III under point loading & UDL

		Deformation (mm)																				
Coordinates		at 4 kg			at 6 kg			at 10 kg			at 12 kg			at 15 kg			at 18 kg			at 20 kg		
X	Y	ANSYS	Exp.	Error%	ANSYS	Exp.	Error%	ANSYS	Exp.	Error%	ANSYS	Exp.	Error%	ANSYS	Exp.	Error%	ANSYS	Exp.	Error%	ANSYS	Exp.	Error%
45.83	300	6.02E-03	0.007	14.00	9.25E-03	0.01	7.50	1.54E-02	0.017	9.41	1.85E-02	0.02	15.91	2.31E-02	0.027	14.44	2.78E-02	0.03	7.33	3.08E-02	0.04	12.00
137.5	100	1.51E-02	0.017	11.18	2.26E-02	0.025	9.60	3.77E-02	0.04	5.75	4.53E-02	0.05	9.40	5.66E-02	0.06	5.67	6.79E-02	0.08	15.13	7.54E-02	0.09	11.29
137.5	450	1.57E-02	0.018	12.78	2.36E-02	0.026	9.23	3.94E-02	0.04	1.50	4.73E-02	0.06	14.00	5.91E-02	0.065	9.08	7.09E-02	0.08	5.47	7.88E-02	0.09	12.44
229.2	150	2.19E-02	0.024	8.75	3.28E-02	0.035	6.29	5.47E-02	0.06	8.83	6.56E-02	0.07	6.29	8.20E-02	0.09	8.89	9.84E-02	0.11	10.55	0.10938	0.12	8.85
275	50	2.08E-02	0.022	5.45	3.17E-02	0.033	3.94	5.21E-02	0.06	13.17	6.25E-02	0.07	10.71	7.81E-02	0.09	13.22	9.37E-02	0.1	6.30	0.10411	0.11	5.35
275	300	2.64E-02	0.03	12.00	3.96E-02	0.045	12.00	6.60E-02	0.075	12.00	7.92E-02	0.09	12.00	9.90E-02	0.11	10.00	0.1188	0.13	8.62	0.132	0.14	5.71
275	550	2.08E-02	0.022	5.45	3.12E-02	0.035	10.86	5.21E-02	0.06	13.17	6.25E-02	0.07	10.71	7.81E-02	0.09	13.22	9.37E-02	0.1	6.30	0.10411	0.12	13.24
412.5	500	1.51E-02	0.017	11.18	2.26E-02	0.025	9.60	3.77E-02	0.041	8.05	4.53E-02	0.05	9.40	5.66E-02	0.06	5.67	6.79E-02	0.08	15.13	7.54E-02	0.08	5.75
504.2	100	5.47E-03	0.006	8.83	8.22E-03	0.009	8.67	1.37E-02	0.016	14.38	1.64E-02	0.02	3.53	2.05E-02	0.022	6.82	2.46E-02	0.03	12.14	2.74E-02	0.03	8.67
504.2	300	6.16E-03	0.007	12.00	9.25E-03	0.01	7.50	1.54E-02	0.016	3.75	1.85E-02	0.02	7.50	2.31E-02	0.025	7.60	2.78E-02	0.03	7.33	3.08E-02	0.04	12.00

	at 35 pkts @360 gm			at 70 pkts @360 gm		
X,Y	ANSYS	Exp.	Error%	ANSYS	Exp.	Error%
45.83,300	0.008	0.009	7.778	0.017	0.018	7.222
137.5,100	0.052	0.055	5.455	0.104	0.120	13.333
137.5,450	0.051	0.055	7.273	0.102	0.110	7.273
229.16,150	0.086	0.090	4.444	0.171	0.180	5.000
275,50	0.096	0.100	4.000	0.191	0.210	9.048
275.0,300	0.089	0.100	11.000	0.178	0.190	6.316
275.0,550	0.096	0.100	4.000	0.191	0.210	9.048
412.5,500	0.052	0.060	13.333	0.104	0.120	13.333
504.16,100	0.087	0.090	3.333	0.173	0.180	3.889
504.16,300	0.008	0.009	7.778	0.016	0.019	15.789

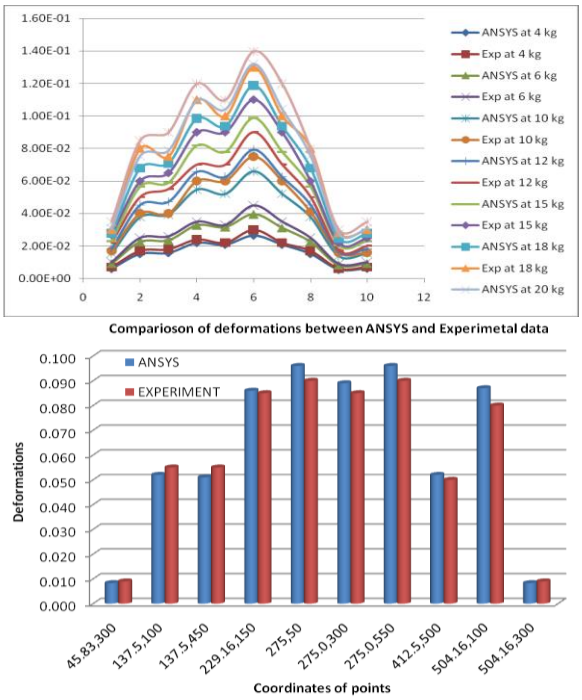

Fig.8: Comparison of results between ANSYS and Experimental for Plate-III

Table 5: Deformations of the Plate-IV under point loading & UDL

		Deformation (mm)								
Coordinates		at 10 kg			at 15 kg			at 20 kg		
X	Y	ANSYS	Exp.	Error %	ANSYS	Exp.	Error %	ANSYS	Exp.	Error %
45.83	300	7.89E-02	0.09	12.33	1.18E-02	0.013	9.23	1.57E-02	0.017	7.65
137.5	100	1.93E-02	0.022	12.27	2.89E-02	0.03	3.67	3.86E-02	0.04	3.50
137.5	450	9.33E-02	0.1	6.70	3.02E-02	0.033	8.48	4.03E-02	0.045	10.44
229.16	150	2.01E-02	0.021	4.29	4.20E-02	0.045	6.67	5.60E-02	0.06	6.67
275	50	2.80E-02	0.03	6.67	3.99E-02	0.043	7.21	5.33E-02	0.06	11.17
275	300	2.66E-02	0.03	11.33	5.06E-02	0.06	15.67	6.75E-02	0.08	15.63
275	550	3.37E-02	0.035	3.71	3.99E-02	0.045	11.33	5.33E-02	0.06	11.17
412.5	500	1.93E-02	0.021	8.10	2.89E-02	0.03	3.67	3.86E-02	0.04	3.50
504.16	100	7.01E-03	0.008	12.38	1.05E-02	0.012	12.50	1.40E-02	0.015	6.67
504.16	300	7.89E-03	0.009	12.33	1.18E-02	0.013	9.23	1.58E-02	0.018	12.22

	at 35 pkts @360 gm			at 70 pkts @360 gm		
X,Y	ANSYS	Exp.	Error%	ANSYS	Exp.	Error%
45.83,300	0.004	0.005	14.000	0.009	0.010	14.000
137.5,100	0.027	0.030	11.333	0.053	0.060	11.167
137.5,450	0.026	0.030	13.333	0.052	0.060	13.000
229.16,150	0.044	0.045	2.222	0.088	0.090	2.222
275,50	0.049	0.050	2.000	0.098	0.100	2.000
275.0,300	0.046	0.050	8.000	0.091	0.100	9.000
275.0,550	0.049	0.050	2.000	0.098	0.100	2.000
412.5,500	0.026	0.027	3.704	0.053	0.054	1.296
504.16,100	0.044	0.046	4.348	0.089	0.092	3.261
504.16,300	0.043	0.050	14.000	0.085	0.100	15.000

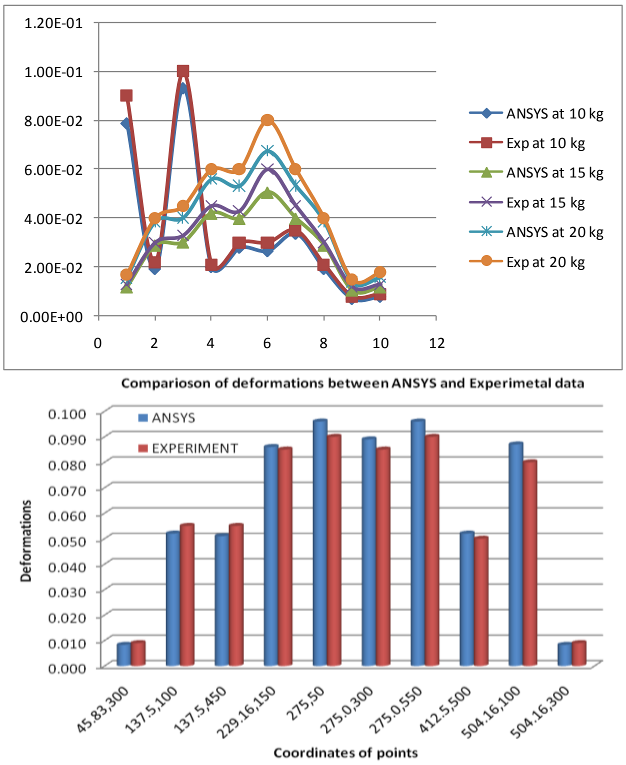

Fig.9: Comparison of results between ANSYS and Experimental for Plate-IV

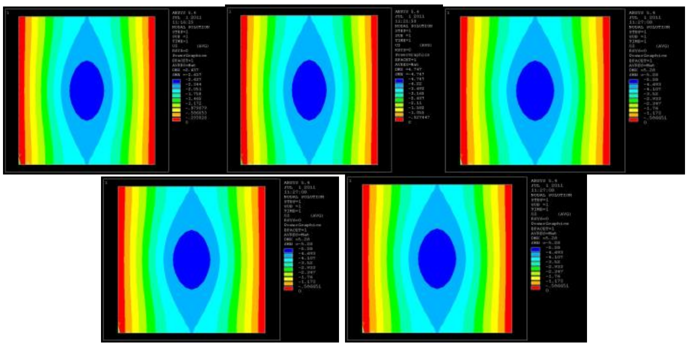

Fig.10: Deformation of Plate-I at different loads

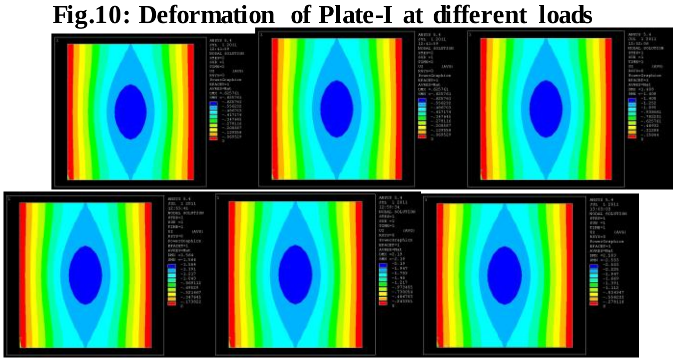

Fig.11: Deformation of Plate-II at different loads

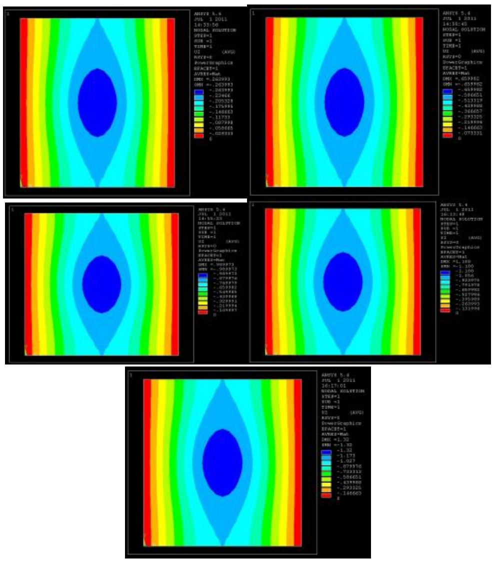

Fig.12: Deformation of Plate-III at different loads

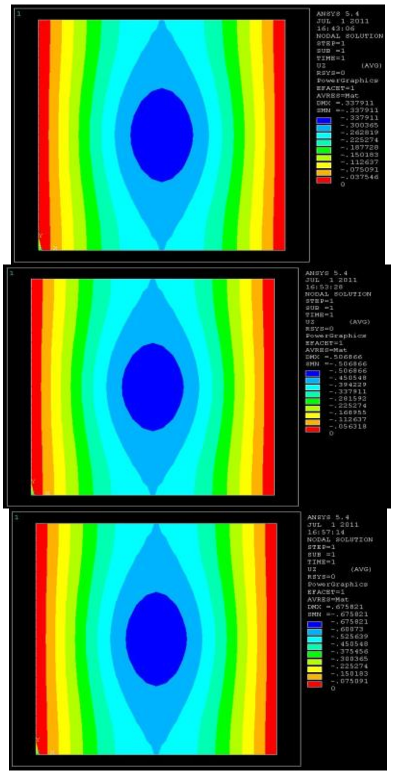

Fig.13: Deformation of Plate-IV at different loads

Results and Discussion

From the above results it has been observed that deformations patterns were quite similar for experiment and ANSYS. But experimental values are little bit higher. The error ranges minimum 0.2% to maximum 16%. Though most error is within 10% except very few values gives more error where as normal range for this type of experiment is within 10% error. This is quite acceptable as dial gauges reading may differ in some cases i.e. why few readings not within the limit. However further fine tuning of experimental setup may give better results. During the experiment with central point loading, the maximum deformation was found out as 0.6 mm in case of Plate-I with 10 kg load and minimum as 0.006 mm in case of Plate-IV with 4 kg load. In same loading condition, analysis in ANSYS results maximum deformation as 0.5279 of Plate-I with 4 kg load and minimum as 0.0130 mm for Plate-II with 4 kg load.

While doing experiment with UDL, maximum deformation was observed in Plate-I as 1.6 mm with 70 packet sand bag (each bag weighing 360 gm) and minimum as 0.005 mm in case of Plate-IV with 35 packet sand bag. In same loading condition, analysis in ANSYS gives maximum deformation as 1.53 mm in Plate-I with 70 packet sand bag and minimum as 0.004 mm for Plate-IV with 35 packet sand bags. In all cases the Young's Modulus was taken 2.1×10^4 N/mm^2 and the Poisson's ratio taken as 0.3.

Conclusions

The experimental investigations presented in this work are concerned with the bending problem of plates having different thickness and under simply supported at two opposite ends. The deflections were measured at selected nodes randomly. The results obtained using ANSYS and the experimental results were compared and the error is presented graphically. The deflection and stress resultants of the plates are determined using ANSYS and presented graphically. From the obtained results, it found that the magnitude of deflection for all the cases of plates is varied with application of load.

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