# Demand estimation for a short life cycle and novel product by using modified Markov based algorithm 

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#### Abstract

This paper deals with demand prediction by combining basic exponential smoothening with Markov based algorithm. This demand prediction is suitable for novel and minimum shelf life products and is suited for small scale retailers and manufacturers, since for them acquisition of costly software tools is out of their financial reach. This paper attempts to arrive at predicting a fixed value of demand which will enhance the profit. The Exponential smoothing process involves comparing the latest observation with the previous weighted average and making a proportional adjustment, governed by the coefficient $\alpha$, known as the smoothing constant. By convention we constrain the coefficient to the range, so that only a part of the difference between the old mean and the new observation is used in the updating, $0<\alpha<1$. Initially demand is predicted by using basic exponential smoothing for two successive months and error of demand for each day is estimated. Markov based algorithm is then applied for these errors and the steady state probability is then determined for each state. The demand corresponding to the state with maximum probability is taken as the optimum one and the corresponding profit is estimated. This concept is then implemented for a baked product and the annual savings for a particular product is then established.


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## Introduction

In Short life cycle Product the demand forecast error of supplier or manufacture is generally between $40-100 \%$ [2]. Actually, with the quick development of science and technology and with continuous rise of peoples demand more and more products will have the characteristic of short life cycle. This phenomenon will play more in the commodity market of fierce competition. The main reasons are 1. Speed of technology refreshing is more and more 2 . The consumption is more and more of short life characteristic [3]. Many researchers in the field of logistics analyzed the returning goods problem of short life cycle product. Most investigated impact of supply chain management on logistical performance indicators in food supply chains especially in the case of baked products[4]. Lee found that product that has not been sold at the end of the season may be either returned to the manufacturer or processed at the discount shop [5].The above problems as stated in the literature gives as an opportunity to analyze the demand estimation of short life cycle product supply chain.

Agrawal and Smith used negative binomial distribution (NBD) for the demand model and suggested that NBD model provides a better fit than the normal or Poisson distributed data [6]. Cachon used the negative binomial distribution model to analyze the demand of the fashion goods where it is assumed that the demand process follows the Poisson distribution and demand rate varies according to a gamma distributed model [7]. Hammond studied the Quick Response policy with ski apparel (ski suits, ski pants, parkas, etc), and showed that forecast accuracy can be substantially improved by adopting QR policy [11].A Markov chain model is a stochastic process, with discrete states and continuous time in which modeling is done on observable parameters. This model can be utilized to evaluate the probability of different states with respect to time. The earlier states are irrelevant for predicting the following states, since the current state is known [19]. In 2001, Zhang and He have developed a Grey-Markov forecasting model for forecasting the total power requirement of agricultural machinery in Shangxi Province [21]. In 2007, Akay and Atak have formulated a Grey prediction model with rolling mechanism for electricity demand forecasting of Turkey [22]. A Grey-Markov forecasting model has been developed by Huang, He and Cen in 2007. The Markov-chain forecasting model is applicable to problems with random variation, which could improve the GM forecasting model [23]. Bijesh and Jayadas formulated an algorithm for short life cycle supply chain based on Markov model [24]. The above algorithm gives us a useful and financially feasible technique to determine the demand forecast whenever the demand data given is randomly distributed

## Basic Exponential Smoothing (BES)

Nice properties of a weighted moving average would be one where the weights not only decrease as older and older data are used, but one where the differences between the weights are "smooth". Obviously the desire would be for the

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weight on the most recent data to be the largest. The weights should then get progressively smaller the more periods one considers into the past. The exponentially decreasing weights of the basic exponential smoothing forecast fit this bill nicely. The forecast equation is given by:
$\mathrm{F}_{\mathrm{t}+1}=\alpha \mathrm{D}_{\mathrm{t}}+(1-\alpha) \mathrm{F}_{\mathrm{t}}$, Where $\alpha$ is a smoothing parameter between 0 and 1 . Here we assume $\alpha=.2$.
Also Forecasting Error $=E_{t}=\left(F_{t}-D_{t}\right)$

## Methodology

1. Observed demand data for a short life cycle product is collected for any two successive months.
2. Apply B.E.S to the collected data and estimate the errors in forecasting for all the days of two successive months
3. Implement the generalized algorithm for the errors of the forecasted model
4. Deduce the initial probability matrix and the Transition probability matrix for the different states of errors of demand.
5. By utilizing the above two matrices the probability of different states of demand for any future period can be determined. The evolution of the system is determined by multiplying the transition matrix by the previous state vector (probability matrix), which is a stochastic vector representing the probabilities of the system being in any one of the given states
6. Choose the state with maximum probability from the obtained current probability vector
7. Determine the annual savings by adopting the demand of the state with maximum probability.

## Algoritm for modification of demand forecsting of basic exponential smoothing method by using markov based algorithm

1) Collect the observed data for sales of a particular product with minimum shelf life for any two consecutive or successive months, say $t$ and $t+1$
2) Apply B.E.S to the collected data and estimate the errors in forecasting for all the days of two successive months
3).Determine the upper limit and lower limit of the errors in forecasting by B.E.S for the $t^{\text {th }}$ month. Determine the range or band width of the error as the difference between upper limit and lower limit for the $\mathrm{t}^{\text {th }}$ month
3) Discretize the obtained range into states or class intervals with minimum possible no of sample size. Let us denote these states as $\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{3} \ldots \ldots \ldots \ldots \mathrm{X}_{\mathrm{n}}$.
4) Determine the initial probability vector $P^{0}$ for the month $t$. This matrix gives the initial probability of all states say $X_{1}, X_{2}, X_{3} \ldots \ldots \ldots . X_{n}$ in month $t$.
a) List out all the days ( m ) in a month in the month t as the first column, in the ascending order of the table
b) In second column enter the state of the observed error for all the days of $\mathrm{t}^{\text {th }}$ month listed in the first column
c) Count the no of occurrence of each state in $t^{\text {th }}$ month. (For eg say state $X_{i}$ is occurring $j$ times in the month $t$ of $m$ days, then initial probability of $X_{i}=j / m$ )
d) Determine the initial probability of all states by using the formulae $\mathrm{X}_{\mathrm{i}}=\mathrm{J} / \mathrm{M}$ where J is the occurrence of $\mathrm{i}^{\text {it }}$ state in $\mathrm{t}^{\text {th }}$ month of M days and $\mathrm{i}=1,2,3 \ldots \ldots . \mathrm{n}$.
e) Represent the initial probabilities obtained from step 8 as a row vector ( $1 * \mathrm{n}$ ) with n no of entries and is called as initial probability vector denoted by $\mathrm{P}^{0}$.
5) Construct state occurrence table for $t^{\text {th }}$ month and $t+1^{\text {th }}$ month.
a) List out all the days of $\mathrm{t}^{\text {th }}$ and $\mathrm{t}+1^{\text {th }}$ month in the ascending order as the first column of the table. Assume the number of working days in both months as same
b) In the second column of the table enter the state corresponding to errors in forecasting for all the days listed in $\mathrm{t}^{\text {th }}$ month.
c) In column three enter the state corresponding to errors of forecasting for all days listed in the $t+1^{\text {th }}$ month.
6) Deduce transition probability matrix from the event occurrence table.
a ) Any current state $\mathrm{X}_{\mathrm{i}}$ in a particular day of $t^{\text {th }}$ month can transform into states $\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{3} \ldots \ldots \ldots . . \mathrm{X}_{\mathrm{n}}$ during the same day of $t+1^{\text {th }}$ month. Hence there exits $n$ probabilities which results from the probable transformation of current state $X_{i}$ to other possible states $X_{1,2} X_{2}, X_{3} \ldots \ldots \ldots . X_{n}$. Represent these probabilities as $P_{11}, P_{12} \ldots \ldots . . . P_{1 n}$
b) Form the Transition probability matrix by representing all the current states as rows and next states as columns. Now enter the probabilities as $\mathrm{P}_{11}, \mathrm{P}_{12} \ldots \ldots . . . \mathrm{P}_{1 \mathrm{n}}$ in 1st row and repeat the same procedure for other rows. Any entry say $\mathrm{P}_{\mathrm{ij}}=$ No of transformations of current state $i$ of $t^{\text {th }}$ month in a particular day to next state $j$ of $t+1$ th month in the same day/ Total no of occurrence of current state $i$ in the $t^{\text {th }}$ month
7) Deduce the current probability vector for the succeeding months $t+2, t+3$ as
$\mathrm{P}^{1}=\mathrm{P}^{0 *}$ TPM
$\mathrm{P}^{2}=\mathrm{P}^{1 *} \mathrm{TPM}$
$\mathrm{P}^{\mathrm{m}}=\mathrm{P}^{\mathrm{m}-1} * \mathrm{TPM}$
8) Choose the state with maximum probability from the obtained current probability vector for say the $\mathrm{m}^{\text {th }}$ month which is a row matrix with probability of each state during say $\mathrm{m}^{\text {th }}$ month.
9) Determine the possible profit to firm by the adoption of this state of production as indicated by the step 8

## Implementation of the Proposed Algorithm For A Baked Product

## Step-1

The data collected from a reputed baking firm is furnished below. The firm has been producing 1300 items per day and selling this item @ Rs 12. Any leftover item is sold at a rate of Rs 3 and there by occurring a possible loss of profit of Rs 9 per product for left over item. Cost of each item is Rs7. This data is manipulated by using naïve forecasting and the forecast as well as error for all the days of April and May is estimated.

Table-1

| Demand D | Mean $\mathrm{X}_{\mathrm{m}}$ | $\begin{aligned} & \mathrm{F}_{\mathrm{t}+1}= \\ & .2 \mathrm{D}_{\mathrm{t}}+.8 \mathrm{~F}_{\mathrm{t}} \end{aligned}$ | $\begin{aligned} & \hline \text { Error }= \\ & \mathrm{E}_{\mathrm{t}}= \\ & \left(\mathrm{F}_{\mathrm{t}}-\mathrm{D}_{\mathrm{t}}\right) \\ & \text { April } \\ & \hline \end{aligned}$ | Demand X | Mean $\mathrm{X}_{\mathrm{m}}$ | $\begin{aligned} & \mathrm{F}_{\mathrm{t}_{1} 1}= \\ & .2 \mathrm{D}_{\mathrm{t}}+.8 \mathrm{~F}_{\mathrm{t}} \end{aligned}$ | $\begin{aligned} & \hline \text { Error }= \\ & \left(\mathrm{F}_{\mathrm{t}}-\mathrm{D}_{\mathrm{t}}\right) \\ & \text { May } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1260 | 1263.74 | 1263.74 | 3.74 | 1260 | 1263.74 | 1265.396 | 5.396264 |
| 1267 | 1263.74 | 1262.99 | -4.008 | 1260 | 1263.74 | 1264.317 | 4.317011 |
| 1260 | 1263.74 | 1263.79 | 3.7936 | 1268 | 1263.74 | 1263.454 | -4.54639 |
| 1252 | 1263.74 | 1263.035 | 11.03488 | 1255 | 1263.74 | 1264.363 | 9.362887 |
| 1248 | 1263.74 | 1260.828 | 12.8279 | 1249 | 1263.74 | 1262.49 | 13.49031 |
| 1266 | 1263.74 | 1258.262 | -7.73768 | 1267 | 1263.74 | 1259.792 | -7.20775 |
| 1271 | 1263.74 | 1259.81 | -11.1901 | 1267 | 1263.74 | 1261.234 | -5.7662 |
| 1260 | 1263.74 | 1262.048 | 2.047887 | 1265 | 1263.74 | 1262.387 | -2.61296 |
| 1259 | 1263.74 | 1261.638 | 2.638309 | 1265 | 1263.74 | 1262.91 | -2.09037 |
| 1264 | 1263.74 | 1261.111 | -2.88935 | 1268 | 1263.74 | 1263.328 | -4.6723 |
| 1256 | 1263.74 | 1261.689 | 5.688518 | 1259 | 1263.74 | 1264.262 | 5.262164 |
| 1265 | 1263.74 | 1260.551 | -4.44919 | 1267 | 1263.74 | 1263.21 | -3.79027 |
| 1260 | 1263.74 | 1261.441 | 1.440652 | 1264 | 1263.74 | 1263.968 | -0.03222 |
| 1271 | 1263.74 | 1261.153 | -9.84748 | 1269 | 1263.74 | 1263.974 | -5.02577 |
| 1265 | 1263.74 | 1263.122 | -1.87798 | 1271 | 1263.74 | 1264.979 | -6.02062 |
| 1271 | 1263.74 | 1263.498 | -7.50239 | 1265 | 1263.74 | 1266.184 | 1.183506 |
| 1259 | 1263.74 | 1264.998 | 5.998091 | 1262 | 1263.74 | 1265.947 | 3.946805 |
| 1266 | 1263.74 | 1263.798 | -2.20153 | 1268 | 1263.74 | 1265.157 | -2.84256 |
| 1270 | 1263.74 | 1264.239 | -5.76122 | 1269 | 1263.74 | 1265.726 | -3.27405 |
| 1260 | 1263.74 | 1265.391 | 5.391023 | 1263 | 1263.74 | 1266.381 | 3.380764 |
| 1262 | 1263.74 | 1264.313 | 2.312818 | 1268 | 1263.74 | 1265.705 | -2.29539 |
| 1268 | 1263.74 | 1263.85 | -4.14975 | 1265 | 1263.74 | 1266.164 | 1.163689 |
| 1265 | 1263.74 | 1264.68 | -0.3198 | 1260 | 1263.74 | 1265.931 | 5.930951 |
| 1271 | 1263.74 | 1264.744 | -6.25584 | 1269 | 1263.74 | 1264.745 | -4.25524 |
| 1263 | 1263.74 | 1265.995 | 2.99533 | 1265 | 1263.74 | 1265.596 | 0.595809 |

Error in demand data modified to nearest whole no. Table-2

| Date | Error of Demand during each day of April 2012 | Error of Demand during each day of May 2012 |
| :--- | :--- | :--- |
| $01-04-2012$ | 4 | 5 |
| $02-04-2012$ | -4 | 4 |
| $03-04-2012$ | 4 | -5 |
| $04-04-2012$ | 11 | 9 |
| $05-04-2012$ | 13 | 13 |
| $06-04-2012$ | -8 | -7 |
| $07-04-2012$ | -11 | -6 |
| $08-04-2012$ | 2 | -3 |
| $09-04-2012$ | 3 | -2 |
| $10-04-2012$ | -3 | -5 |
| $11-04-2012$ | 6 | 5 |
| $12-04-2012$ | -4 | -4 |


| $13-04-2012$ | 1 | 0 |
| :--- | :--- | :--- |
| $14-04-2012$ | -10 | -5 |
| $15-04-2012$ | -2 | -6 |
| $16-04-2012$ | -8 | 1 |
| $17-04-2012$ | 6 | 4 |
| $18-04-2012$ | -2 | -3 |
| $19-04-2012$ | -6 | -3 |
| $20-04-2012$ | 5 | 3 |
| $21-04-2012$ | 2 | -2 |
| $22-04-2012$ | -4 | 1 |
| $23-04-2012$ | 0 | 6 |
| $24-04-2012$ | -6 | -4 |
| $25-04-2012$ |  | 1 |

## Step-2

The Range $=$ Highest error in demand - Lowest error in Demand $=13-(-11)=24$.
Step 3
Discretize the range into class intervals or states with a minimum sample size of 3. I.e. Say 8 states denoted by $X_{1,} X_{2}$, $\mathrm{X}_{3} \ldots \ldots \ldots . . \mathrm{X}_{8}$.

Step 4
Determine the initial probability vector $\mathrm{P}^{0}$ for the month April. This matrix gives the initial probability of all states say $\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{3} \ldots \ldots \ldots \ldots \mathrm{X}_{8}$ in the month of April.

Table-3

| Class interval of error of demand | State | No of occurrence | Probability |
| :--- | :--- | :--- | :--- |
| $-11,-10,-9$ | $\mathrm{X}_{1}$ | 2 | .08 |
| $-8,-7,-6$ | $\mathrm{X}_{2}$ | 4 | .16 |
| $-5,-4,-3$ | $\mathrm{X}_{3}$ | 4 | .16 |
| $-2,-1,0$ | $\mathrm{X}_{4}$ | 3 | .12 |
| $1,2,3$ | $\mathrm{X}_{5}$ | 5 | .2 |
| $4,5,6$ | $\mathrm{X}_{6}$ | 5 | .2 |
| $7,8,9$ | $\mathrm{X}_{7}$ | 0 | 0 |
| $10,11,12,13$ | $\mathrm{X}_{8}$ | 2 | .08 |

The Fourth row of the above table gives initial probability vector $\mathrm{P}^{0}$ for the month April. This matrix gives the initial probability of all states say $\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{3} \ldots \ldots \ldots \ldots \mathrm{X}_{8}$ in the month of April.
$\mathrm{P}^{0}=[.08, .16,16, .12, .2,2,0,0,08]$
Step 5 - State transition table
Table-4

| Day | Current state (April) | Subsequent state (May) |
| :--- | :--- | :--- |
| 1 | 6 | 6 |
| 2 | 4 | 6 |
| 3 | 6 | 3 |
| 4 | 8 | 7 |
| 5 | 8 | 8 |
| 6 | 2 | 2 |
| 7 | 1 | 2 |
| 8 | 5 | 3 |
| 9 | 5 | 4 |
| 10 | 4 | 6 |
| 11 | 6 | 3 |
| 12 | 3 | 4 |
| 13 | 5 | 3 |
| 14 | 1 | 2 |
| 15 | 4 | 6 |
| 16 | 2 |  |
| 17 | 6 |  |


| 18 | 4 | 3 |
| :--- | :--- | :--- |
| 19 | 2 | 3 |
| 20 | 6 | 5 |
| 21 | 5 | 4 |
| 22 | 3 | 5 |
| 23 | 4 | 6 |
| 24 | 2 | 3 |
| 25 | 5 | 5 |

Step 6
Deduction of Transition Probability Matrix (T P M)
. Any current state $X_{i}$ in a particular day of $t^{\text {th }}$ month can transform into states $X_{1}, X_{2}, X_{3} \ldots \ldots \ldots . X_{n}$ during the same day of $t+1^{\text {th }}$ month. Hence there exits $n$ probabilities which results from the probable transformation of current state $X_{i}$ to other possible states $X_{1}, X_{2}, X_{3}$ $\qquad$ .$X_{n}$ Represent these probabilities as $P_{11}, P_{12 \ldots \ldots . . . . . . .} P_{1 n}$

- Form the Transition probability matrix by representing all the current states as rows and next states as columns. Now enter the probabilities as $\mathrm{P}_{11}, \mathrm{P}_{12 \ldots \ldots \ldots . .} \mathrm{P}_{1 \mathrm{n}}$ in 1st row and repeat the same procedure for other rows.
- Any entry say $P_{i j}=$ No of transformations of current state $i$ of $t$ th month in a particular day to next state $j$ of $t+1^{\text {th }}$ month in the same day/ Total no of occurrence of current state $i$ in the $t^{\text {th }}$ month
- Transition Probability Matrix,

Table-5

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| 1 | 0 | .5 | .5 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | .25 | .5 | 0 | .25 | 0 | 0 | 0 |
| 3 | 0 | 0 | .5 | 0 | .5 | 0 | 0 | 0 |
| 4 | 0 | .2 | .2 | .2 | 0 | .4 | 0 | 0 |
| 5 | 0 | 0 | .2 | .6 | .2 | 0 | 0 | 0 |
| 6 | 0 | 0 | .2 | 0 | .2 | .6 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | .5 | .5 |

Step 7
Deduce the current probability vector for the succeeding month's $t+2, t+3 \ldots . t+18$ as

| $\mathrm{P}^{1}=\mathrm{P}^{0}$ * | TPM= [0 | 0.1040 | 0.3040 | 0.1440 | 0.2000 | 0.1680 | 0.0400 | 0.0400 ] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}^{2}=[0$ | 0.0548 | 0.3064 | 0.1488 | 0.2516 | 0.1584 | 0.0200 | 0.0200 ] |  |
| $\mathrm{P}^{3}=[0$ | 0.0435 | 0.2924 | 0.1807 | 0.2489 | 0.1546 | 0.0100 | 0.0100 ] |  |
| $\mathrm{P}^{4}=[0$ | 0.0470 | 0.2847 | 0.1855 | 0.2377 | 0.1650 | 0.0050 | $0.0050]$ |  |
| $\mathrm{P}^{5}=[0$ | 0.0488 | 0.2835 | 0.1797 | 0.2347 | 0.1732 | 0.0025 | 0.0025 ] |  |
| $\mathrm{P}^{6}=[0$ | 0.0482 | 0.2837 | 0.1768 | 0.2356 | 0.1758 | 0.0013 | 0.0013 |  |
| $\mathrm{P}^{7}=[0$ | 0.0474 | 0.2836 | 0.1767 | 0.2362 | 0.1762 | 0.0006 | 0.0006 ] |  |
| $\mathrm{P}^{8}=[0$ | 0.0472 | 0.2833 | 0.1770 | 0.2361 | 0.1764 | 0.0003 | 0.0003 ] |  |
| $\mathrm{P}^{9}=[0$ | 0.0472 | 0.2831 | 0.1771 | 0.2359 | 0.1766 | 0.0002 | 0.0002 ] |  |
| $\mathrm{P}^{10}=[0$ | 0.0472 | 0.2831 | 0.1770 | 0.2359 | 0.1768 | 0.0001 | 0.0001 |  |
| $\mathrm{P}^{11}=[0$ | 0.0472 | 0.2831 | 0.1769 | 0.2359 | 0.1769 | 0.0000 | $0.0000]$ |  |
| $\mathrm{P}^{12}=[0$ | 0.0472 | 0.2831 | 0.1769 | 0.2359 | 0.1769 | 0.0000 | $0.0000]$ |  |
| $\mathrm{P}^{13}=[0$ | 0.0472 | 0.2831 | 0.1769 | 0.2359 | 0.1769 | 0.0000 | 0.0000] |  |
| $\mathrm{P}^{14}=[0$ | 0.0472 | 0.2831 | 0.1769 | 0.2359 | 0.1769 | 0.0000 | 0.0000] |  |
| $\mathrm{P}^{15}=[0$ | 0.0472 | 0.2831 | 0.1769 | 0.2359 | 0.1769 | 0.0000 | $0.0000]$ |  |
| $\mathrm{P}^{16}=[0$ | 0.0472 | 0.2831 | 0.1769 | 0.2359 | 0.1769 | 0.0000 | $0.0000]$ |  |
| $\mathrm{P}^{17}=[0$ | 0.0472 | 0.2831 | 0.1769 | 0.2359 | 0.1769 | 0.0000 | 0.0000] |  |
| $\mathrm{P}^{18}=[0$ | 0.0472 | 0.2831 | 0.1769 | 0.2359 | 0.1769 | 0.0000 | $0.0000]$ |  |
| $\mathrm{P}^{19}=[0$ | 0.0472 | 0.2831 | 0.1769 | 0.2359 | 0.1769 | 0.0000 | $0.0000]$ |  |
| $\mathrm{P}^{20}=[0$ | 0.0472 | 0.2831 | 0.1769 | 0.2359 | 0.1769 | 0.0000 | 0.0000] |  |

Step 8
The maximum Probability is for state 3 and as time passes we can see that probability of demand for state 1 is approaching steady state probability. The Probability V/s Time curve for state one is shown below

Figure-2


Step 9
The maximum probability is for state 3 with errors $-5,-4$, and -3 . The corresponding demands are $1267,1264,1265$ and1268. To determine the single point estimate of demand, the annual profit by adopting each of the above figures of demand are determined. The demand with maximum profit is selected as the optimum forecast. For this purpose we are assuming an overstocking cost of (Rs 7-3) =Rs 4 per item and under stocking cost of Rs 5 per piece. The firm was producing 1300 items per day.

Case-1
Assuming predicted demand as 1267 items per day.

| Actual Demand for April. | Predicted demand based on algorithm for April | Error for demand in April | $\begin{array}{ll} \hline \text { Overstocking } & \text { cost } \\ \text { (Rs) } \end{array}$ | Under stocking cost (Rs) |
| :---: | :---: | :---: | :---: | :---: |
| 1260 | 1267 | -7 | 28 |  |
| 1267 | 1267 | 0 |  |  |
| 1260 | 1267 | -7 | 28 |  |
| 1252 | 1267 | -15 | 60 |  |
| 1248 | 1267 | -19 | 72 |  |
| 1266 | 1267 | -1 | 4 |  |
| 1271 | 1267 | 4 |  | 20 |
| 1260 | 1267 | -7 | 28 |  |
| 1259 | 1267 | -8 | 32 |  |
| 1264 | 1267 | -3 | 12 |  |
| 1256 | 1267 | -11 | 44 |  |
| 1265 | 1267 | -2 | 8 |  |
| 1260 | 1267 | -7 | 28 |  |
| 1271 | 1267 | 4 |  | 20 |
| 1265 | 1267 | -2 | 8 |  |
| 1271 | 1267 | 4 |  | 20 |
| 1259 | 1267 | -8 | 32 |  |
| 1266 | 1267 | -1 | 4 |  |
| 1270 | 1267 | 3 |  | 15 |
| 1260 | 1267 | -7 | 28 |  |
| 1262 | 1267 | -5 | 20 |  |
| 1268 | 1267 | 1 |  | 5 |
| 1265 | 1267 | -2 | 8 |  |
| 1271 | 1267 | 4 |  | 20 |
| 1263 | 1267 | -4 | 16 |  |
|  | 1267 |  | TOTAL-460 | TOTAL-100 |

Annual Savings by adopting a demand of 1267 items $=\{(1300-1267)(7-3) 24-100-460\} 12=$ Rs 32880

Case-2
Assuming predicted demand as 1264 items per day.

| Actual Demand for April. | Predicted demand based on algorithm for April | Error for demand in April | Overstocking cost (Rs) | Under stocking cost (Rs) |
| :---: | :---: | :---: | :---: | :---: |
| 1260 | 1264 | -4 | 16 |  |
| 1267 | 1264 | 3 |  | 15 |
| 1260 | 1264 | -4 | 16 |  |
| 1252 | 1264 | -12 | 48 |  |
| 1248 | 1264 | -16 | 64 |  |
| 1266 | 1264 | 2 |  | 10 |
| 1271 | 1264 | 7 |  | 35 |
| 1260 | 1264 | -4 | 16 |  |
| 1259 | 1264 | -5 | 20 |  |
| 1264 | 1264 | 0 |  |  |
| 1256 | 1264 | -8 | 32 |  |
| 1265 | 1264 | 1 |  | 5 |
| 1260 | 1264 | -4 | 16 |  |
| 1271 | 1264 | 7 |  | 35 |
| 1265 | 1264 | 1 |  | 5 |
| 1271 | 1264 | 7 |  | 35 |
| 1259 | 1264 | -5 | 20 |  |
| 1266 | 1264 | 2 |  | 10 |
| 1270 | 1264 | 6 |  | 30 |
| 1260 | 1264 | -4 | 16 |  |
| 1262 | 1264 | -2 | 8 |  |
| 1268 | 1264 | 4 |  | 20 |
| 1265 | 1264 | 1 |  | 5 |
| 1271 | 1264 | 7 |  | 35 |
| 1263 | 1264 | -1 | 4 |  |
|  |  |  | TOTAL-276 | TOTAL-240 |

Annual Savings by adopting a demand of 1264 items $=\{(1300-1264)(7-3) 24-276-240\} 12=$ Rs 35280 Case-3
Assuming predicted demand as 1265 items per day.

| Actual Demand for April. | Predicted demand based on algorithm for April | Error for demand in April | $\begin{aligned} & \text { Overstocking cost } \\ & \text { (Rs) } \end{aligned}$ | Under stocking cost (Rs) |
| :---: | :---: | :---: | :---: | :---: |
| 1260 | 1265 | -5 | 20 |  |
| 1267 | 1265 | 2 |  | 10 |
| 1260 | 1265 | -5 | 20 |  |
| 1252 | 1265 | -13 | 52 |  |
| 1248 | 1265 | -17 | 68 |  |
| 1266 | 1265 | 1 |  | 5 |
| 1271 | 1265 | 6 |  | 30 |
| 1260 | 1265 | -5 | 20 |  |
| 1259 | 1265 | -6 | 24 |  |
| 1264 | 1265 | -1 | 4 |  |
| 1256 | 1265 | -9 | 36 |  |
| 1265 | 1265 | 0 |  |  |
| 1260 | 1265 | -5 | 20 |  |
| 1271 | 1265 | 6 |  | 30 |
| 1265 | 1265 | 0 |  |  |
| 1271 | 1265 | 6 |  | 30 |
| 1259 | 1265 | -6 | 24 |  |
| 1266 | 1265 | 1 |  | 5 |
| 1270 | 1265 | 5 |  | 25 |
| 1260 | 1265 | -5 | 20 |  |
| 1262 | 1265 | -3 | 12 |  |
| 1268 | 1265 | 3 |  | 15 |
| 1265 | 1265 | 0 |  |  |


| 1271 | 1265 | 6 |  | 30 |
| :--- | :--- | :--- | :--- | :--- |
| 1263 | 1265 | -2 | 8 |  |
|  |  |  | TOTAL-328 | Total-180 |

Annual Savings by adopting a demand of 1264 items $=\{(1300-1265)(7-3) 24-328-180\} 12=$ Rs 34224
Case-4
Assuming predicted demand as 1268 items per day.

| Actual Demand for April. | Predicted demand based on algorithm for April | Error for demand in April | Overstocking cost (Rs) | Under stocking cost (Rs) |
| :---: | :---: | :---: | :---: | :---: |
| 1260 | 1268 | -8 | 32 |  |
| 1267 | 1268 | -1 | 4 |  |
| 1260 | 1268 | -8 | 32 |  |
| 1252 | 1268 | -16 | 64 |  |
| 1248 | 1268 | -20 | 80 |  |
| 1266 | 1268 | -2 | 8 |  |
| 1271 | 1268 | 3 |  | 15 |
| 1260 | 1268 | -8 | 32 |  |
| 1259 | 1268 | -9 | 36 |  |
| 1264 | 1268 | -4 | 16 |  |
| 1256 | 1268 | -12 | 48 |  |
| 1265 | 1268 | -3 | 12 |  |
| 1260 | 1268 | -8 | 32 |  |
| 1271 | 1268 | 3 |  | 15 |
| 1265 | 1268 | -3 | 12 |  |
| 1271 | 1268 | 3 |  | 15 |
| 1259 | 1268 | -9 | 36 |  |
| 1266 | 1268 | -2 | 8 |  |
| 1270 | 1268 | 2 |  | 10 |
| 1260 | 1268 | -8 | 32 |  |
| 1262 | 1268 | -6 | 24 |  |
| 1268 | 1268 | 0 |  |  |
| 1265 | 1268 | -3 | 12 |  |
| 1271 | 1268 | 3 |  | 15 |
| 1263 | 1268 | -5 | 20 |  |
|  |  |  | TOTAL-540 | TOTAL-70 |

Annual Savings by adopting a demand of 1264 items $=\{(1300-1268)(7-3) 24-540-70\} 12=$ Rs 31080.

## Result

The maximum probability is for state 3 with errors $-5,-4$, and- 3 . The corresponding demands are $1267,1264,1265$ and1268.To determine the single point estimate of demand, the annual profit by adopting each of the above figures of demand are determined. The demand with maximum profit is selected as the optimum forecast. For this purpose we are assuming an overstocking cost of (Rs 7-3) =Rs 4 per item and under stocking cost of Rs 5 per piece. The firm was producing 1300 items per day. The annual savings for each product is determined for each demand stated above by using the formulae $\left\{\left(P-D_{f}\right)(E-S) * 25-\sum\right.$ Under stocking costs- $\sum$ Overstocking costs $\} * 12$, where $\mathrm{P}=$ Production rate per day, $\mathrm{D}_{\mathrm{f}}$ is the predicted value of demand by using algorithm, E is the cost of making unit quantity, S is the price of the product after discount to be sold after 8 pm . The demand with maximum savings is chosen as the optimal one.

Annual Savings by adopting a demand of 1267 items $=\{(1300-1267)(7-3) 24-100-460\} 12=$ Rs 32880
Annual Savings by adopting a demand of 1264 items $=\{(1300-1264)(7-3) 24-276-240\} 12=$ Rs 35280
Annual Savings by adopting a demand of 1265 items $=\{(1300-1265)(7-3) 24-328-180\} 12=$ Rs 34224
Annual Savings by adopting a demand of 1268 items $=\{(1300-1268)(7-3) 24-540-70\} 12=$ Rs 31080.
The optimal predicted demand with maximum savings is 1264 items.

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